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DEFINITION OF AN INSTRUMENTAL SURFACE, GENERATING GEARS OF A PARALLEL, CIRCULAR AND CYCLOID LINE OF THE TOOTH

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The progress in synthesis and manufacture of gear trains was possible due to the creation and application of the gearing theory. In the mathematics literature this theory is considered of a particular practical importance and some of the greatest mathematicians and mechanics like Olivie, Hoffman, Oiler, Willis and others take pride in its development.

The created theory and geometry of involute gearing, the introduced method of enveloped curves for the description of conjugated teeth surfaces, the use of matrix methods for coordinates transition, methods of description of curves of enveloped curves, etc. are considered „peaks" in this theory.

The Russian scientist F.L. Litvin generalizes all results and builds a nice theory of gearing [1] applicable for the research of all two- and three-dimensional gear mechanisms. He uses the so called „cinematic" method, according to which the teeth surfaces are coming as a result of the relative movement of one simpler surface (called instrumental) and the blank. The widest application of cylindrical and cone gears are those of a parallel, circular and cycloid line of the teeth. They can be cut with coping or by a combined continuous rotary motion of the cutter and the blank, using the method of interrupted or non-interrupted division.

In this paper will be reviewed the method of a combined continuous rotary motion of the cutter and the blank, with which the cutter is a head (type "Gleason","Oerlicon" or another), containing few groups of knives (each group must have one knife for cutting of the concave part and one for the protruding part of the teeth). The instrumental (cutting) surface is formed as a result of the knives cutting edges in the head, performing different movements with each method of cutting.

The goal is common parameters and a mechano-mathematical model to be involved, summerizing all possible movements of the instruments in cutting cylindrical and cone gears, excluding gears of straight teeth. In the process of recording the instrumental surface through an exclusion of some movements

$S_q(x_qO_qy_qz_q)$ - a coordinate system attached to the generating comb where the plane $y_qO_qz_q$ coincides with the plane of symmetry of the machined gear;

u - a parameter of the cutting edge p;

θ - rotation angle of the cutting head around the axis O_nz_n in the process of machining;

m - gear module;

z_w – number of groups of knives on the cutting head.

The angle between the tangent line in the middle of the cutting edge of the knife and the axis of rotation of the cutting head is α and the plane of this angle is tangent to a cylinder of $r_0=\frac{m.z_w}{2}$. If R_n is the nominal radius, then $\delta=\arcsin(r_0/R_n)$.

Let the equations of the generating straight edge in the system S_p are of the form [1]:

$$\vec{r}_0=x_p.\vec{i}+y_p.\vec{j}+z_p.\vec{k}\ , \quad (1)$$

where

$x_p=$ u.sinα

$y_p=0$

$z_p=$ u.cosα

are coordinates of a random point M on the forming line and $\vec{i}$, $\vec{j}$, $\vec{k}$ - single vectors of axes x_p, z_p, y_p.

Besides the rotary movement the cutting head does also a parallel movement along the unfolded plane of the primary cylinder of the gear and let it is S. The coordinates of the point M in S_q will be defined with the matrix equation:

$$r_q=M_{qp}.r_p \quad (2)$$

where M_{qp} is a matrix of a fourth row, defining the transition of the coordinate system S_p into S_q and r_q and r_p – matrixes – columns, formed from the coordinate of the point M, defined accordingly in the coordinate systems S_q and S_p.

After some transformations from (2) we get:

$$\begin{cases} x_q = u.\sin\alpha.\cos(\theta-\delta)+R_n.\cos\theta \\ y_q = u.\sin\alpha.\sin(\theta-\delta)+R_n.\sin\theta - S \\ z_q = u.\cos\alpha \end{cases} \quad (3)$$

Except as a straight line the cutting edge can be described as a non-specified curve k in Fig. 2:

$$\begin{cases} x_p = x_p(L) \\ y_p = y_p(L) \\ z_p = z_p(L)\ . \end{cases} \quad (4)$$

and parameters, the different special cases (for the different methods of cutting) to be got and to specify if there are some other instrumental surfaces.

Assuming that in the common case the formative surface is created by a rotation of the cutting knives (whose cutting edges are straight or of an unspecified curve line) around the axis of the head and a translating movement parallel to the axis of the blank (if it is a cylindrical one).

For defining generalized equations of the generating surface we shall introduce the following coordinate systems, symbols and parameters (Fig. 1)

$S_p(x_p O_p y_p z_p)$ - a coordinate system attached to the cutting edge with a center O_p in the middle of the straight edge p;

$S_n(x_n O_n y_n z_n)$- a coordinate system attached to the cutting head with a center on the axis of rotation of the instrument and coordinate planes coinciding with those in S_p ;

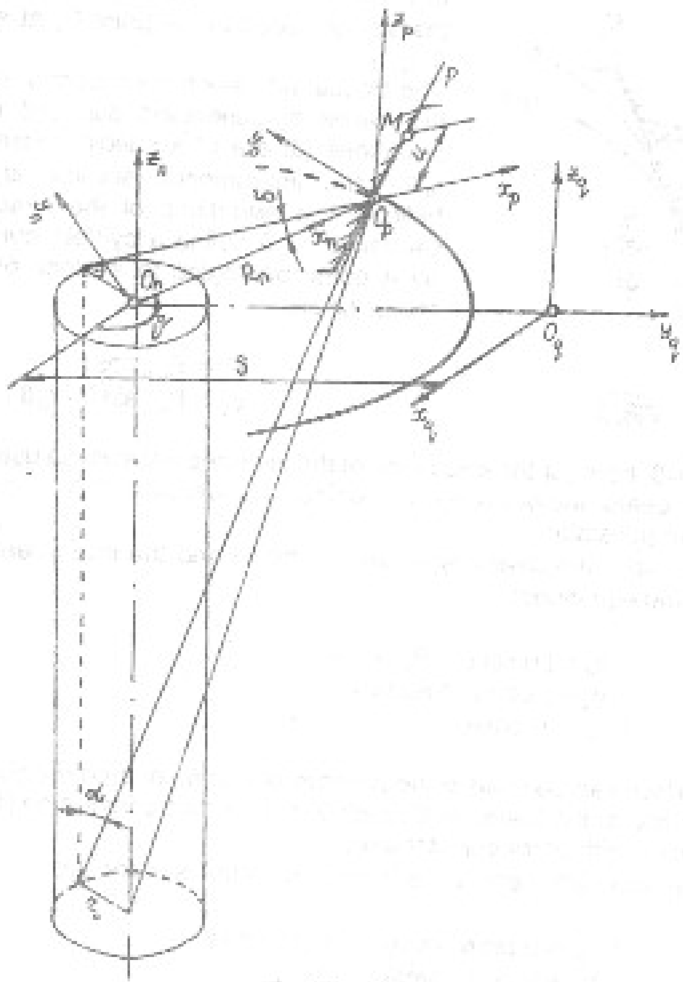

Fig. 1

where L_i ($i = 1,2$) are parameters of the curve.

The generating (instrumental) surface is derived in the same way and it is suitable for creating of an appropriate method for modification in cutting of the small gear out of the semi-rolling gear transmission [2]. By optimization of the function of the gear mechanism position, parameters L_i of the specified curve (4) are studied. This curve must fulfill the following requirements:

- to cross the point O_p (Fig. 2) of the straight cutting edge **k'**;
- **k'** to be tangent to the curve **k** in point O_p;
- in the point O_p to be fulfilled the condition for a conjugation according to the equation of Oiler-Savary [1], giving the relation between the radiuses of curves of the profiles and sentries.

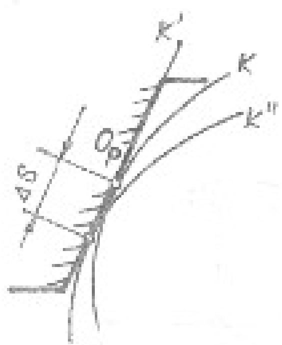

Fig. 2

The most appropriate solution seems to be an arc of a circle **k"** (with a specified radius) tangent to the straight edge in the point O_p at a distance ΔS (Fig.2).

The traditional gear transmissions are rolling and the cases of generating surfaces described by equations (3) are of a special interest.When $S = r_0.\theta$ the instrumental surface is a rotational helicoid.The equations of the directing curve in section **z** $= 0$ define a cycloid curve (described by a point, belonging to a plane of a circle of a radius r_0) are

$$\begin{cases} x_q = R_n \,.\, \cos\theta \\ y_q = R_n \,.\, \sin\theta - r_0.\theta \,. \end{cases} \qquad (5)$$

When S=0, from (3) the equations of the instrumental surface used for cutting of arcoidal gears without tangent feeding, are derived. With this condition two cases are possible:

a) with $\delta = 0°$ - the directory curve is a circle and the instrumental surface is a cone of the equations:

$$\begin{cases} x_q = (u.\sin\alpha + R_n)\cos\theta \\ y_q = (u.\sin\alpha + R_n)\sin\theta \\ z_q = u.\cos\alpha \end{cases} \qquad (6)$$

This surface can be used in the process of cutting of arcoidal gears of a parallel circular line of the teeth, participating in a worm transmission [3] (for a motion transferring with perpendicular axes);

b) with $\delta \neq 0°$ - equations of the generating surface are:

$$\begin{cases} x_q = u.\sin\alpha \,.\cos(\theta - \delta) + R_n.\cos\theta \\ y_q = u.\sin\alpha \,.\sin(\theta - \delta) + R_n.\sin\theta \\ z_q = u.\cos\alpha \end{cases} \qquad (7)$$

For finding of the canonic type of this surface are necessary some transitions and the result is:

$$\frac{x_q^2}{a^2}+\frac{y_q^2}{a^2}-\left(\frac{z_q+2}{b}\right)^2=1, \qquad (8)$$

where

$a = R_n\sqrt{1-\cos^2\delta}$

$b = R_n.\mathrm{cotg}\,\alpha.\sqrt{1-\cos^2\delta}$

$c = R_n.\mathrm{cotg}\,\alpha.\cos\delta$

The equation (8) defines a single strip rotational hyperboloid (Fig. 3), the generating surface is hyperbolic, respectively. Besides, with cutting of cylindrical gears the generating surface of this type (3) can be used with forming profiles of the teeth of cone gears.

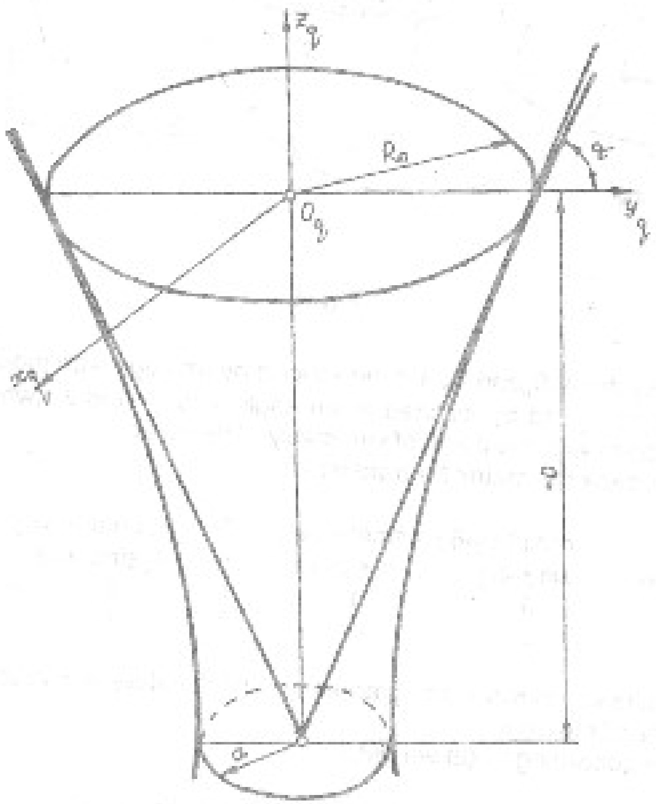

Fig. 3

On Fig. 4 is shown the relative position of the instrumental surface according to the blank. The axis of the cutting head is inclined at γ equal to the angle of the dividing cone of the cutted gear.

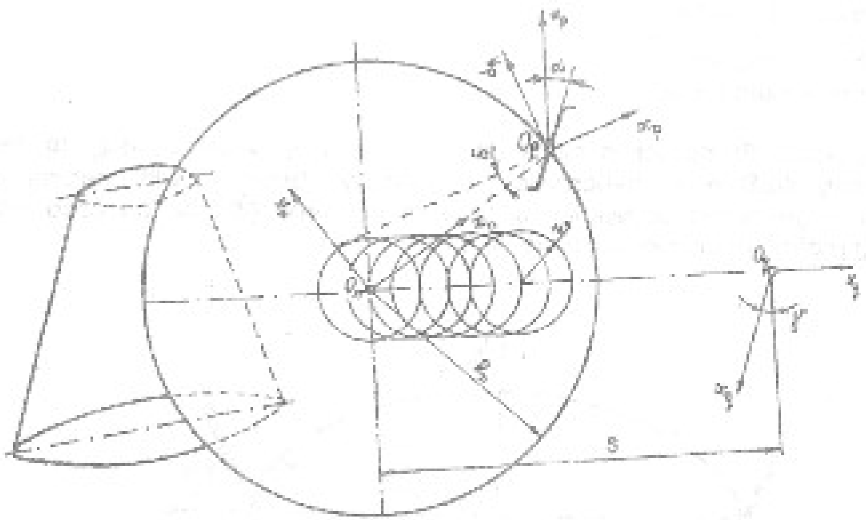

Fig. 4

Coordinate systems S_p and S_n are introduced by analogy with those (Fig. 1) and S_q is with axes x_q and z_q, inclined at an angle γ to x_n and z_n, where the plane $x_qO_qz_q$ coincides with the plane of symmetry of the gear.

In this case the matrix for transfer is:

$$M_{qp} = \begin{Vmatrix} \cos(\theta+\gamma-\delta) & \sin(\theta+\gamma-\delta) & 0 & R_n.\cos(\theta+\gamma) \\ \sin(\theta-\delta) & \cos(\theta-\delta) & 0 & R_n.\sin\theta - r_o.\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix} \qquad (9)$$

where the already introduced symbols and parameters are valid, and m is the face module of the gear.

Then according to (2) we get:

$$\left| \begin{aligned} x_q &= u.\sin\alpha.\cos(\theta+\gamma-\delta)+R_n.\cos(\theta+\gamma) \\ y_q &= u.\sin\alpha.\sin(\theta-\delta)+R_n.\sin\theta - r_o.\theta \\ z_q &= u.\cos\alpha \end{aligned} \right. \qquad (10)$$

Obviously with $\gamma = 0^{\circ}$ the equations (10) will assume the form of (3) and it can be considered that equations (10) are a summarized record of the observed instrumental surfaces.

Besides, there are other surfaces of variable nominal radiuses of the instrument. That case is for example the instrumental surface of the face spiral-disc cutter [4] but it is out of interest in this paper.

The instrumental surfaces reviewed so far will form the concave part of the arcoidal teeth. To form the convex part in equations (3), (6), (7) and (10) we must exchange the angle α with $(180+\alpha)$.

With the summerized record of their instrumental surfaces the problems of synthesis and analysis of arcoidal gear transmissions will be solved.

Obviously with annulment of some parameters can be got as all known till now instrumental surfaces as well as some new ones (for instance, a hyperboloid surface).

There is a possibility for generating another interesting surfaces (using different types of curved cutting edges of the instrument) which will be an object of another future research.

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DEFINITION OF AN INSTRUMENTAL SURFACE. GENERATING GEARS OF A PARALLEL, CIRCULAR AND CYCLOID LINE OF THE TOOTH

Abstract.

The geometry of different types of instrumental surfaces, used in forming the profiles of the teeth of arcoidal gear wheels, is known.This geometry depends mostly on the peculiarities in the kinematics of cutting for getting a specified longitudinal tooth line.

In the paper are summerized all main parameters, introduced by the study of the various methods of forming of the teeth profiles and the producing surface (called a summerized one) is presented in a general type.From the got equations, trought annulment of some parameters are derived the instrumental surfaces knowntill now and some new ones.

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