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ASUPRA UNOR INTEGRALE DIN GAZETA MATEMATICĂ

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In cele ce urmeaza ne propunem sa calculam integralele definite din functii construite cu ajutorul altor functii care sunt solutiile unor ecuatii functionale sau din functii care satisfac anumite conditii .

Propozitia 1 . Daca $a, b \in \mathbf{R}$, $a < b$ si $f : [a, b] \to \mathbf{R}$ este o functie continua (integrabila) , atunci :

$$\int_a^b f(x)\mathrm{d}x = \int_a^b f(a+b-x)\mathrm{d}x \tag{1}$$

Demonstratie . Facem schimbarea de variabila $t = u(x) = a + b - x$ cu $u'(x) = -1$, $u(a) = b$, $u(b) = a$ si atunci :

$$\int_a^b f(x)\mathrm{d}x = \int_b^a -f(a+b-x)\mathrm{d}x = \int_a^b f(a+b-x)\mathrm{d}x \tag{2}$$

Propozitia 2 . Daca $a, b \in \mathbf{R}$, $a < b$ iar $f : [a, b] \to \mathbf{R}$, $g : [a, b] \to \mathbf{R}_+^*$ au proprietatile $f(a+b-x) = f(x)$, $g(a+b-x) \cdot g(x) = 1$, $\forall x \in [a, b]$, atunci oricare ar fi functia continua $h : \mathbf{R} \to \mathbf{R}$ are loc egalitatea :

$$2 \cdot \int_a^b \frac{(h \circ f)(x)}{1 + g(x)}\mathrm{d}x = \int_a^b (h \circ f)(x)\mathrm{d}x \tag{3}$$

Demonstratie . Este evident ca in conditiile date prin enunt , functia $\dfrac{h \circ f}{1+g} : [a,b] \to \mathbf{R}$ este integrabila si conform propozitiei 1 rezulta

$$I = \int_a^b \frac{(h \circ f)(x)}{1+g(x)}\mathrm{d}x = \int_a^b \frac{(h \circ f)(a+b-x)}{1+g(a+b-x)}\mathrm{d}x =$$

$$= \int_a^b \frac{h(f(a+b-x))}{1+\dfrac{1}{g(x)}}\mathrm{d}x = \int_a^b \frac{(h \circ f)(x)) \cdot g(x)}{1+g(x)}\mathrm{d}x .$$

Prin urmare ,

$$2I = \int_a^b \frac{(h \circ f)(x)}{1+g(x)}\mathrm{d}x + \int_a^b \frac{(h \circ f)(x) \cdot g(x)}{1+g(x)}\mathrm{d}x = \int_a^b (h \circ f)(x)\mathrm{d}x .$$

Propozitia 3 . Daca $a, b \in \mathbf{R}$, $a < b$, $f : \mathbf{R} \to \mathbf{R}$, $g : \mathbf{R} \to \mathbf{R}_+^*$ sunt functii continue astfel incât

$$f(a+b-x) = -f(x), \quad g(a+b-x) \cdot g(x) = 1 , \quad \forall x \in \mathbf{R}$$

atunci

$$2 \cdot \int_a^b f(x) \cdot \ln^2(1+g(x))\mathrm{d}x = 2 \cdot \int_a^b f(x) \cdot \ln(1+g(x)) \cdot \ln g(x)\mathrm{d}x -$$

$$- \int_a^b f(x) \cdot \ln^2 g(x)\mathrm{d}x \qquad (4).$$

Demonstratie . Avem

$$I = \int_a^b f(x) \cdot \ln^2(1+g(x))\mathrm{d}x = \int_a^b f(a+b-x) \cdot \ln^2(1+g(a+b-x))\mathrm{d}x =$$

$$= -\int_a^b f(x) \cdot \ln^2\left(1+\frac{1}{g(x)}\right)\mathrm{d}x = -\int_a^b f(x)(\ln(1+g(x)) - \ln g(x))^2\mathrm{d}x =$$

$$= -\int_a^b f(x) \cdot (\ln^2(1+g(x)) - 2\ln(1+g(x)) \cdot \ln g(x) + \ln^2 g(x))\mathrm{d}x =$$

$$= -I + 2 \cdot \int_a^b f(x) \cdot \ln(1+g(x)) \cdot \ln g(x)\mathrm{d}x - \int_a^b f(x) \cdot \ln^2 g(x)\mathrm{d}x$$

de unde deducem enuntul .

Consecinta. Daca $g(x) = \mathbf{e}^{f(x)}$ atunci avem :

$$2\cdot\int_a^b f(x)\cdot\ln^2(1+\mathbf{e}^{f(x)})\mathrm{d}x = 2\cdot\int_a^b f^2(x)\cdot\ln(1+\mathbf{e}^{f(x)})\mathrm{d}x - \int_a^b f^3(x)\mathrm{d}x \quad (5)$$

Propozitia 4 . Daca $a, b \in \mathbf{R}$, $a < b$ si $f, g : [a, b] \to \mathbf{R}$ sunt functii continue cu proprietatea ca exista $c \in \mathbf{R}$, $d \in \mathbf{R}\setminus\{1\}$ astfel incât $f(a+b-x) = = d\cdot f(x)$, $g(a+b-x)+g(x) = c, \forall x \in [a, b]$, atunci

$$\int_a^b f(x)\cdot g(x)\mathrm{d}x = \frac{c\cdot d}{1+d}\cdot\int_a^b f(x)\mathrm{d}x \quad (6)$$

Demonstratie . Este evident ca

$$I = \int_a^b f(x)\cdot g(x)\mathrm{d}x = \int_a^b f(a+b-x)\cdot g(a+b-x)\mathrm{d}x =$$
$$= \int_a^b d\cdot f(x)\cdot(c-g(x))\mathrm{d}x = c\cdot d\cdot\int_a^b f(x)\mathrm{d}x - d\cdot\int_a^b f(x)\cdot g(x)\mathrm{d}x =$$
$$= c\cdot d\cdot\int_a^b f(x)\cdot\mathrm{d}x - d\cdot I$$

de unde deducem ca

$$I = \frac{c\cdot d}{1+d}\cdot\int_a^b f(x)\cdot g(x)\cdot\mathrm{d}x\ .$$

Propozitia 5 . Daca $a, b \in \mathbf{R}$, $a < b$, $s = a+b$ si $f : \mathbf{R} \to \mathbf{R}$, $g, h : \mathbf{R} \to \mathbf{R}_+^*$ sunt functii continue astfel incât $f(s-x) = f(x)$, $\forall x \in [a, b]$, , atunci

$$2\cdot\int_a^b \frac{f(x)\cdot g(x)\cdot h(s-x)}{g(x)\cdot h(s-x)+g(s-x)\cdot h(x)}\cdot\mathrm{d}x = \int_a^b f(x)\cdot\mathrm{d}x \quad (7)$$

Demonstratie . Fie

$$\begin{aligned} I &= \int_a^b \frac{f(x)\cdot g(x)\cdot h(s-x)}{g(x)\cdot h(s-x)+g(s-x)\cdot h(x)}\cdot\mathrm{d}x = \\ &= \int_a^b \frac{f(s-x)\cdot g(s-x)\cdot h(x)}{g(s-x)\cdot h(x)+g(x)\cdot h(s-x)}\cdot\mathrm{d}x = \\ &= \int_a^b \frac{f(x)\cdot g(s-x)\cdot h(x)}{g(x)\cdot h(s-x)+g(s-x)\cdot h(x)}\cdot\mathrm{d}x \end{aligned}$$

Prin urmare ,

$$\begin{aligned}2I &= \int_a^b \frac{f(x)\cdot g(x)\cdot h(s-x)}{g(x)\cdot h(s-x)+g(s-x)\cdot h(x)}\cdot \mathrm{d}x+\\ &+ \int_a^b \frac{f(x)\cdot g(s-x)\cdot h(x)}{g(x)\cdot h(s-x)+g(s-x)\cdot h(x)}\cdot \mathrm{d}x = \int_a^b f(x)\cdot \mathrm{d}x\end{aligned}$$

ceea ce demonstreaza enuntul .

Propozitia 6. Daca $a, b \in \mathbf{R}$, $a < b$, $a+b = \frac{\pi}{2}$ si $f : [a,b] \to \mathbf{R}$ este o functie integrabila astfel incât $f(x) + f\left(\frac{\pi}{2} - x\right) = c \in \mathbf{R}_+$, $\forall x \in [a,b]$; atunci

$$2\cdot \int_a^b \frac{f(x)+\cos x+\sin^2 x}{1+c+\sin x+\cos x}\cdot \mathrm{d}x = b-a \qquad (8)$$

Demonstratie . Este evident ca

$$I = \int_a^b \frac{f(x)+\cos x+\sin^2 x}{1+c+\sin x+\cos x}\cdot \mathrm{d}x = \int_a^b \frac{f\left(\frac{\pi}{2}-x\right)+\cos x+\cos^2 x}{1+c+\cos x+\sin x}\cdot \mathrm{d}x.$$

Prin urmare ,

$$\begin{aligned}2I &= \int_a^b \left(\frac{f(x)+\cos x+\sin^2 x}{1+c+\sin x+\cos x}+\frac{f\left(\frac{\pi}{2}-x\right)+\sin x+\cos^2 x}{1+c+\sin x+\cos x}\right)\cdot \mathrm{d}x =\\ &= \int_a^b \frac{1+\sin x+\cos x+f(x)+f\left(\frac{\pi}{2}-x\right)}{1+c+\sin x+\cos x}\cdot \mathrm{d}x = \int_a^b \mathrm{d}x = b-a.\end{aligned}$$

Aplicatia 1 . Daca $c \in (0,1)\cup(1,\infty)$ si $h : \mathbf{R} \to \mathbf{R}$ este o functie continua, atunci

$$2\cdot \int_{-1}^1 \frac{h(x^2)}{1+c^x}\cdot \mathrm{d}x = \int_{-1}^1 h(x^2)\cdot \mathrm{d}x \qquad (9)$$

Ovidiu Gabriel Dinu, (23320), G.M. 7/1995, pag. 333

Solutie . In propozitia 2 consideram $a=-1$, $b=1$, $f(x)=x^2$, $g(x)=c^x$ si obtinem rezultatul dorit .

Aplicatia 2 . Fie $f : \mathbf{R} \to \mathbf{R}$ o functie continua pe $\mathbf{R}$ cu proprietatea ca $f(x) + f(-x) = k \in \mathbf{R}$, $\forall x \in \mathbf{R}$. Sa se calculeze :

$$\int_{-\pi/4}^{\pi/4} \frac{f(x)}{\cos^2 x} \cdot \mathrm{d}x \qquad (10)$$

V. Gorgota , (24132), G.M. 5-6/1999, pag. 252.

Solutie . Avem

$$I = \int_{-\pi/4}^{\pi/4} \frac{f(x)}{\cos^2 x} \cdot \mathrm{d}x = \int_{-\pi/4}^{\pi/4} \frac{f(-x)}{\cos^2 x} \cdot \mathrm{d}x$$

de unde deducem ca

$$2I = \int_{-\pi/4}^{\pi/4} \frac{f(x) + f(-x)}{\cos^2 x} \cdot \mathrm{d}x = k \cdot \int_{-\pi/4}^{\pi/4} \frac{1}{\cos^2 x} \cdot \mathrm{d}x = k \cdot \mathrm{tg}\ \ x\Big|_{\pi/4}^{\pi/4} = 2k$$

si prin urmare $I = k$.

Aplicatia 3 . Fie $a, b \in \mathbf{R}_+$, $a < b$, $a + b = s \in \mathbf{R}_+^*$ si $f : \mathbf{R} \to \mathbf{R}$ o functie integrabila astfel incât $f(s - x) = f(x)$, $\forall x \in [a, b]$. Daca $c \in \mathbf{R}_+^*$, si $2cs < \pi$, atunci

$$2 \cdot \int_a^b f(x) \cdot \ln(1 + \mathrm{tg} : cs \cdot \mathrm{tg} : cx) \cdot \mathrm{d}x = \ln(1 + \mathrm{tg}^2 cs) \cdot \int_a^b f(x) \cdot \mathrm{d}x \ , \ (11).$$

Solutie . Avem

$$\begin{aligned}
I &= \int_a^b f(x) \cdot \ln(1 + \mathrm{tg}\ \ s \cdot \mathrm{tg}\ \ cx) \cdot \mathrm{d}x = \\
&= \int_a^b f(s - x) \cdot \ln(1 + \mathrm{tg}\ \ cs \cdot \mathrm{tg}\ \ c(s - x)) \cdot \mathrm{d}x = \\
&= \int_a^b f(x) \cdot \ln\left(1 + \mathrm{tg}\ \ cs \cdot \frac{\mathrm{tg}\ \ cs - \mathrm{tg}\ \ cx}{1 + \mathrm{tg}\ \ cs \cdot \mathrm{tg}\ \ cx}\right) \cdot \mathrm{d}x = \\
&= \int_a^b f(x) \cdot \ln \frac{1 + \mathrm{tg}\ \ cs \cdot \mathrm{tg}\ \ cx + \mathrm{tg}^2 cs - \mathrm{tg}\ \ cs \cdot \mathrm{tg}\ \ cx}{1 + \mathrm{tg}\ \ cs \cdot \mathrm{tg}\ \ cx} \cdot \mathrm{d}x = \\
&= \ln(1 + \mathrm{tg}^2 cs) \cdot \int_a^b f(x) \cdot \mathrm{d}x - \int_a^b f(x) \cdot \ln(1 + \mathrm{tg}\ \ cs \cdot \mathrm{tg}\ \ cx) \cdot \mathrm{d}x
\end{aligned}$$

de unde rezulta enuntul .

Aplicatia 4 . Daca $t \in \left(0, \frac{\pi}{4}\right]$, sa se calculeze

$$\int_0^t (x^2 - tx + t^2) \cdot \ln(1 + \mathrm{tg}\ \ t \cdot \mathrm{tg}\ \ x) \cdot \mathrm{d}x \qquad (12)$$

V.Gorgota, (C: 2040), G.M. 4/1998, pag. 186.

Solutie . In aplicatia 3 consideram $a = 0$, $b = t$, $c = 1$, $f(x) = x^2 - tx + t^2$ si astfel obtinem

$$\int_0^t (x^2 - tx + t^2) \cdot \ln(1 + \mathrm{tg}\ \ t \cdot \mathrm{tg}\ \ x) \cdot \mathrm{d}x = \frac{5t^3}{12} \cdot \ln(1 + \mathrm{tg}^2 t).$$

Aplicatia 5 . Se considera functia continua $f : \mathbf{R} \to \mathbf{R}$ astfel incqt $\alpha \cdot f(x) + \beta \cdot f(-x) = \gamma$, $\forall x \in \mathbf{R}$ unde $\alpha, \beta, \gamma \in \mathbf{R}$, $\alpha + \beta \neq 0$. Sa se calculeze

$$\int_{-a}^{a} \cdot f(x) \cdot \mathrm{d}x\ , \quad a \in \mathbf{R}_+^* \ .$$

Anton Riederer , (16383), G.M. 1/1987, pag. 23

Solutie . Este evident ca

$$I = \int_{-a}^{a} f(-x) \cdot \mathrm{d}x = -\int_{a}^{-a} f(x)\mathrm{d}x = \int_{-a}^{a} f(x) \cdot \mathrm{d}x\ .$$

Integrând egalitatea din enunt deducem $\alpha \cdot I + \beta \cdot I = \gamma \cdot \int_{-a}^{a} \cdot \mathrm{d}x = = 2a\gamma$ si atunci $I = \dfrac{2a\gamma}{\alpha + \beta}$.

Aplicatia 6 . Sa se calculeze

$$\int_0^{\pi/2} \frac{\sin^{1998} x + \cos^2 x}{1 + \sin^{1998} x + \cos^{1998} x} \cdot \mathrm{d}x \tag{13}$$

Florin Nicolaescu , (23852), G.M. 1/1998, pag. 37

Solutie. Notând $f(x) = \dfrac{\sin^{1998} x + \cos^2 x}{1 + \sin^{1998} x + \cos^{1998} x}$ avem de calculat

$$I = \int_0^{\pi/2} f(x)\mathrm{d}x = \int_0^{\pi/2} f\left(\frac{\pi}{2} - x\right) \cdot \mathrm{d}x = \int_0^{\pi/2} \frac{\cos^{1998} x + \sin^2 x}{1 + \sin^{1998} x + \cos^{1998} x} \cdot \mathrm{d}x$$

si deci $2I = \int_0^{\pi/2} \mathrm{d}x$. Prin urmare $I = \dfrac{\pi}{4}$.

Aplicatia 7. Daca functia integrabila $f : [-1, 1] \to \mathbf{R}$ este para, atunci

$$\int_{-1}^{1} f(x) \cdot \arccos x \cdot \mathrm{d}x = \frac{\pi}{2} \cdot \int_{-1}^{1} \cdot f(x) \cdot \mathrm{d}x \tag{14}$$

Doru Isac, (22573) , G.M. 11-12(1991), pag. 445

Solutie. Este evident ca

$$\begin{aligned} I &= \int_{-1}^{1} f(x)\cdot \arccos x \mathrm{d}x = \int_{-1}^{1} f(-x)\cdot \arccos(-x)\mathrm{d}x = \\ &= \int_{-1}^{1} \cdot(\pi - \arccos x)\mathrm{d}x = \pi\cdot\int_{-1}^{1} f(x)\mathrm{d}x - I \end{aligned}$$

de unde rezulta enuntul .

Aplicatia 8 . Fie $a, b \in \mathbf{R}$, $a < b$ si $s = a + b > 0$. Sa se calculeze .

$$\int_a^b (4x^3 - 6sx^2 + 4s^2x - s^3)\cdot \operatorname{arctg}\,(x^2 - sx + s^2)\cdot \mathrm{d}x \tag{15}$$

D.M. Batinetu-Giurgiu , (24292), G.M. 3/2000, pag. 135

Solutie . Daca notam

$$f(x) = 4x^3 - 6sx^2 + 4s^2x - s^3\ ,\ g(x) = \operatorname{arctg}\,(x^2 - sx + s^2)$$

atunci $f(s-x) = -f(x)$, $g(s-x) = g(x) \quad \forall x \in [a,b]$ si deci

$$I = \int_a^b f(x)\cdot g(x)\cdot \mathrm{d}x = \int_a^b -f(x)\cdot g(x)\cdot \mathrm{d}x = -I$$

de unde rezulta $I = 0$.

Aplicatia 9 . Daca $a \in \left(0, \frac{\pi}{2}\right]$, $b \in \left[\frac{\pi}{2}, \pi\right)$ si $a + b = \pi$. Sa se calculeze

$$\int_a^b \frac{x}{\sin x}\cdot \mathrm{d}x \tag{16}$$

Dan M. Brad-Gorun , (C: 2149), G.M. 3/1999, pag. 140

Solutie .

$$\begin{aligned} I &= \int_a^b \frac{x}{\sin x}\cdot \mathrm{d}x = \int_a^b \frac{a+b-x}{\sin(a+b-x)}\cdot \mathrm{d}x = \\ &= \int_a^b \frac{\pi - x}{\sin x}\cdot \mathrm{d}x = \pi\cdot\int_a^b \frac{1}{\sin x}\cdot \mathrm{d}x - I \end{aligned}$$

de unde rezulta

$$\begin{aligned} I &= \frac{\pi}{2}\cdot\int_a^b \frac{1}{\sin x}\cdot \mathrm{d}x = \frac{\pi}{2}\cdot \ln\left|\operatorname{tg}\frac{\pi}{2}\right|\Big|_a^b = \\ &= \frac{\pi}{2}\cdot \ln \operatorname{tg}\frac{x}{2}\Big|_a^b = \frac{\pi}{2}\cdot \ln\left(\operatorname{ctg}\frac{a}{2}\cdot \operatorname{tg}\frac{b}{2}\right). \end{aligned}$$

Aplica'tia 10 . Sa se calculeze

$$\int_{\pi/2}^{3\pi/2} \frac{x\cdot \cos x}{1 + \sin^2 x}\cdot \mathrm{d}x \tag{17}$$

Gh. Szöllösy, (17212), G.M. 5/1978, pag. 215.

Solutie . Avem

$$\begin{aligned} I &= \int_{\pi/2}^{3\pi/2} \frac{x\cdot\cos x}{1+\sin^2 x}\cdot \mathrm{d}x = \int_{\pi/2}^{3\pi/2} \frac{(2\pi - x)\cos\ (2\pi-x)}{1+\sin^2(2\pi-x)}\cdot \mathrm{d}x = \\ &= \int_{\pi/2}^{3\pi/2} \frac{(2\pi-x)\cos x}{1+\sin^2}\cdot \mathrm{d}x = 2\pi\int_{\pi/2}^{3\pi/2}\frac{\cos x}{1+\sin^2 x}\cdot \mathrm{d}x - I \end{aligned}$$

de unde rezulta

$$\begin{aligned} I &= \pi\cdot\int_{\pi/2}^{3\pi/2}\frac{\cos x}{1+\sin^2 x}\cdot \mathrm{d}x = \pi\cdot \mathrm{arctg}\ \ (\sin x)\Big|_{\pi/2}^{3\pi/2} = \\ &= \pi(\mathrm{arctg}\ \ (-1) - \mathrm{arctg}\ \ 1) = -\frac{\pi^2}{2}\,. \end{aligned}$$

Aplicatia 11 . Sa se calculeze

$$\int_0^1 (4x^3 - 6x^3 + 8x - 3)\cdot h(x^2 - x + 1)\cdot \mathrm{d}x \qquad (18)$$

unde $h : \mathbf{R}_+^* \to \mathbf{R}$ este o functie continua.

Dragu Atanasiu, (19160), G.M. 2/1982, pag. 89

Solutie . Notând, $f(x) = 4x^3 - 6x^2 + 8x - 3$ rezulta ca $f(1-x) = -f(x)$ si de asemenea $g(1-x) = g(x)$, $\forall x \in [0,1]$ unde $g(x) = x^2 - x + 1$. Prin urmare,

$$\begin{aligned} I &= \int_0^1 f(x)\cdot h(g(x))\cdot \mathrm{d}x = \int_0^1 f(1-x)\cdot h(g(1-x))\cdot \mathrm{d}x = \\ &= -\int_0^1 f(x)\cdot h(g(x))\cdot \mathrm{d}x = -I \end{aligned}$$

de unde deducem c a $I = 0$.

Aplicatia 12. Sa se calculeze

$$\int_{-1}^1 \frac{\arccos x}{1+x^2}\cdot \mathrm{d}x \qquad (19)$$

ing. Alexandru Constantinescu, (22430), G.M. 7/1991, pag. 269

Solutie.

$$\begin{aligned} I &= \int_{-1}^1 \frac{\arccos x}{1+x^2}\cdot \mathrm{d}x = \int_{-1}^1 \frac{\arccos\ \ (-x)}{1+x^2}\cdot \mathrm{d}x = \\ &= \int_{-1}^1 \frac{\pi - \arccos x}{1+x^2}\cdot \mathrm{d}x = \pi\cdot\int_{-1}^1 \frac{1}{1+x^2}\cdot \mathrm{d}x - I. \end{aligned}$$

Prin urmare

$$I = \frac{\pi}{2} \cdot \int_{-1}^{1} \frac{1}{1+x^2} \cdot \mathrm{d}x = \frac{\pi}{2} \cdot \operatorname{arctg}\ x \Big|_{-1}^{1} = \frac{\pi^2}{4}\,.$$

Aplicatia 13 . Daca $a > 0$ si $f : \mathbf{R} \to \mathbf{R}$ este continua si impara , atunci

$$\int_{-a}^{a} f(x) \cdot \ln^2(1+\mathbf{e}^x)\mathrm{d}x = \int_{-a}^{a} x \cdot f(x) \cdot \ln(1+\mathbf{e}^x)\mathrm{d}x \qquad (20)$$

D.M. Batinetu-Giurgiu , (XII. 1) ARHIMEDE-2000, nr.4/2000, pag. 20

Solutie . In propozitia 3 luam $g(x) = \mathbf{e}^x$ si obtinem

$$I = \int_{-a}^{a} f(x) \cdot \ln^2(1+\mathbf{e}^x)\mathrm{d}x = \int_{-a}^{a} x \cdot f(x) \cdot \ln(1+\mathbf{e}^x)\mathrm{d}x - \int_{-a}^{a} x^2 \cdot f(x)\mathrm{d}x\,.$$

Deoarece functia $h : \mathbf{R} \to \mathbf{R}$, $h(x) = x^2 \cdot f(x)$ este impara rezulta imediat ca $\int_{-a}^{a} h(x) \cdot \mathrm{d}x = 0$ si astfel rezulta enuntul .

Observatie . Daca $f(x) = x$ obtinem problema 24092 din Gazeta Matematica nr. 3/1999, pag. 137 , având drept autor pe *Sabin Tabârca*.

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ABOUT SOME INTEGRALS PROPOSED IN GAZETA MATEMATICA

Abstract. In this paper unitary methods concerning the approach of the calculus for some integrals of functions obtained by means of other functions which satisfy certain conditions are given .

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