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BOLYAI WAS BORN 200 YEARS AGO

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As everyone knows, János Bolyai was a great mathematician.

But what did he create? Why can't we meet his name more often?

Those who study mathematics are familiar the Rolle theorem, the L'Hospital rule, the Taylor formula, but they don't come into contact with concepts introduced by Bolyai during daily school practise. Several works on the history of mathematics mention his name only in small print or not at all.

The explanation for that could be found in two domains: one of them is Bolyai's personal fate, and the other is the characteristic of mathematics field examined by him.

Let's start the acquaintance with the latter problem. It is the question of the basis of geometry.

On the one hand János Bolyai generated significant contributions in many fields of mathematics, but these things became public only after his death. On the other hand, while he approached the state of the art in analysis, in number theory and in theory of complex numbers, he represented the state of the art in geometry.

The examination of the basis of geometry did not originate in the problems of practical life. Its effect was first felt in abstract theoretical mathematics and later in other sciences – for instance in astronomy – and only in the past decades did it arise philosophers' and mathematics teachers' interest. Einstein's theory of relativity is also based on non-Euclidean geometries.

There are possibilities of several geometries in our mind – the ancient maps and myths refer to this – but these ideas seem to be remote from mathematics. In mathematics only one fundamental geometry has existed for centuries, and this was the Euclidean one, which has been familiar to all of us since primary school. János Bolyai had to break through this petrified tradition. He solved the problem by building up a totally correct, faultless theory, which radically differs from the known geometry. This is strange and hardly comprehensible for our approach to traditional geometry.

The name of the new theory is Bolyai's absolute geometry. This theory contains the hyperbolic geometry as a particular case. So, the name of hyperbolic geometry – as it was established at the International Bibliography Congress of Mathematical Sciences directed by Poincaré, in Paris 1894 – is Bolyai-Lobatchevsky geometry.

Let's direct some attention to the cognition of this geometry. Without visualisation and demonstration this is a very difficult task. Earlier it was only mathematicians and university students who learnt about different sorts of non-Euclidean geometries. Imagine, how difficult it would be to study "normal", school-type or the Euclidean geometry on the Himalayas where nothing is plane, and moreover, it is hard to get hold off a sheet of paper. Nowadays the situation is different: maths teachers who are also involved in research have invented many equipment which make the demonstration simple also in case of non-Euclidean geometries. These are the drawing sphere set, models made of paper, wood or textile, which can demonstrate certain details of the theory. And a brand new device, the computational model of hyperbolic plane is also appropriate for this.

In this geometry it is not true that only one straight line can be drawn parallel to a straight line through an outer point. Neither is it true that the sum of the angles of the triangle is 180°, nor that the distance of parallel straight lines is constant. Is there a sense of this strangeness? Yes, there is, and these results are important in many areas. Also, they exerted a significant effect on inner development of mathematics.

The natural sciences revealed that it is absolutely not certain that the world is like the Euclidean geometry describes it. Our scope has been widening. We examine mathematics in other ways. Mathematicians returned to ancient Greek mathematicians' wording. Mathematics doesn't state unappealable facts, and it is also not a science of perfect theories. It is a developing science, its truths are conditional, they have a form like "if this and this condition is fulfilled, than this and this statement follow from this". Naturally, this doesn't change the fact that nowadays Pythagoras' theorem stays as true as it was 2000 years ago, whereas only the opportunities of mathematics have expanded.

The life of János Bolyai

János Bolyai was born on 15 December 1802 in Kolozsvár, Transylvania, in his grandfather's house.

He spent his childhood on the Bolyai family farm in Domáld and in Marosvásárhely.

His father, Farkas Bolyai was a considerable mathematician, who taught in the college of Marosvásárhely. He gave his son a share in a very careful upbringing. Since Farkas Bolyai had been himself an infant prodigy, he was deeply aware of the

disadvantages of this conditions, and this is why he kept his son away from formal, school education as long possible, but he directed a great attention to his physical and psychic development.

János Bolyai studied at Marosvásárhely College and as a student he also took a part in teaching. From the age of 17 he studied at the Vienna Academy of Military Engineering, with excellent results.

He started his service as a military engineer in 1823, his first stop was Temesvár, then came Arad, Lemberg and Olmütz.

As a military engineer he wrote his main piece of work, which unfortunately was lost, but his account letter of this remained. On 3 November 1823 he wrote his famous letter in which he stated with pride: "I created a new world from nothing". And this was his absolute geometry. Fortunately after the lost versions written in German, the new geometrical theory, the Science of Space has been published in Latin in 1831-32. It was an appendix of Farkas Bolyai's higher mathematics book mentioned as Tentamen:

APPENDIX,
SCIENTAM SPATII
ABSOLUTE VERAM EXIBENS;
A VERITATE AUT FALSITATE AXIOMATIS XI EUCLIDEI
(A PRIORI HAUD DECIDENDA) INDEPENDENTEM;

ADJECTA AD CASUM FALSITATIS QUADRATURA
CIRCULI GEOMETRICA

ACTORE

JOHANNE BOLYAI DE EADEM,

GEOMETRARUM IN EXERCITU CAESAREO REGIO
AUSTRIACO CASTRENSIUM CAPITANO

In 1833 he asked for retirement on account of his bad health condition and then he lived on a modest pension. First he moved to Domáld then to Marosvásárhely. He spent his life in poverty and bad health. He worked a lot, he also entered some competitions, he was planning the publication of his results, but in the end there wasn't time for it. He wrote his philosophical and mathematical works in Latin, German and Hungarian. His family life was not harmonic, so he divorced his children's mother, then he lived lonely.

He died in Marosvásárhely on 27 January 1860. According to register note: "His talent was wasted." Luckily this statement was a mistake. Not only did he create a wonderful work but this world became a new research area for mathematicians. Several books was published on Bolyai and his work, some of them in English. The aftermath of Appendix would make the subject of another compelling story.

We are left with no genuine portrait of János Bolyai. The often published Adler portrait is a mistake, but Attila Zsigmond's art reconstruction has been made.

Awards, conferences, institutes with his name, museums, libraries and archives cherish János Bolyai's memory.

Mathematics researchers still find inspiration in János Bolyai, and his life's work is making his way into classrooms more and more.

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Abstract. János Bolyai was born 200 years ago. Awards, conferences, institutes with his name, museums, libraries and archives cherish his memory.

What did he create?

In mathematics only one fundamental geometry has existed for centuries, and this was the Euclidean one, which has been familiar to all of us since primary school. János Bolyai had to break through this petrified tradition by building up a totally correct, faultless theory, which radically differs from the known geometry. The name of this new theory is Bolyai's absolute geometry. It contains the hyperbolic geometry as a particular case. The name of hyperbolic geometry – as it was decided at the International Bibliography Congress of Mathematical Sciences directed by Poincaré, in Paris 1894 – is Bolyai-Lobatchevski geometry.

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