CREATIVE MATH.
12 (2003), 117 - 119

Preliminaries for the teaching of non-Euclidean geometries

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Abstract. According to contemporary principles of methodology, there are good reasons for teaching the three geometries - namely Euclidean, spherical and hyperbolic - in parallel, despite the belief of some teachers that it makes little sense in contemporary conditions. I have been looking for historical data that prove that this parallelism is natural, and precedents can be found not only in ancient Greek history, but also in the Hungarian history of science.

The acknowledgment of Euclidean geometry as the only true geometry was only a short deviation in the history of European science. Following the romantic ideas of the 19^{th} century, we are inclined to believe that the Greeks claimed that the angles of the triangle sum up to 180 degree. That is a false belief. Euclid and the other great Greek mathematicians claimed rather that if we suppose that the axioms are true, then different statements follow. For example, we can say that if we postulate that the axiom of parallelism holds, then it follows that the angles of the triangle sum to 180 degrees.

According to the studies of Imre Tóth, we can suppose - and this assumption is based upon more than mere analogies - that the acceptance of the Fifth Postulate was a positive choice. We know that even Archimedes consciously decided in favor of the Archimedean axiom, although we have unquestionable evidence that his computation of the area of the section of the parabola is based upon infinitesimals. The acceptance of the existence of infinitesimals means the negation of the Archimedean axiom. Thus, Archimedes used two different, mutually incompatible, mathematical architectures, indicating a clear difference between them.

Imre Tóth has used philology as a tool to show that there exist data in Greek geometry which refer to several types of geometry: those that we call Euclidean, and also the hyperbolic type.

The records about hyperbolic geometry were found only in the 20^{th} century. On the other hand, knowledge of the spherical geometry has existed for millennia, not as an alternative geometry, but rather as the science describing the geographical environment on a cosmic scale. The Greeks were aware of several data indicating the Earth's spherical shape, but had no clear

Received: 21.08.2003; In revised form: 30.12.2003
Key words and phrases. *Non-Euclidean geometries, Bolyai, teaching.*

evidence to prove that fact. Eratosthenes, who had calculated the Earth's radius based upon a clever, simple and easily reproducible measurement, was convinced that the Earth is spherical, but this view was not generally accepted. It could be said that the Greeks did not know that the Earth is spherical, but they did know that, if it were so, they could determine the value of the radius.

Knowledge of the Earth's spherical shape has been preserved, in the most picturesque way, in the map constructed in the 3^d century by Ptolemy, which plots the Mediterranean while showing the Earth's Grid.

A tract summarizing spherical trigonometry was also written, so spherical geometry was developed in the West-European culture long ago.

In the beginning, an odd kind of duality existed.

Archaeological and ethnographical records show that, in the early stage of cultures, people thought of the Earth as a flat disc. Their views referring to the universe can be symbolized by the tree of life. Let us recall our own personal memories. In childhood, we all thought of the world as a flat thing, and when first we learned that the Earth was spherical, we knew that it was impossible: people living on the opposite side would have fallen off.

This duality - the result of personal perception on one hand, and knowledge coming from antiquity on the other hand - was characteristic of the European, and consequently of the Hungarian, way of thinking in the Middle Ages.

There are also records of views accepting the spherical nature of the Earth from very early times. In about 1000CE, St. Stephen founded schools where scholars were educated for the Church. The curriculum included *computus* - that is, the method calculating the date of Easter - as an important discipline. The Council of Nicea determined a method for calculating the date of Easter based upon planetary motion. Hungary's first, 15^{th} century, map and its further revisions preserving the Earth Grid of longitude and latitude were made using Ptolemy's map.

The calculations of the *computus* could be made mechanically.

It is likely that the meaning of the signs at the map's margin was not clear either to the constructors or to the users.

King Matthew had obtained a globe from Regiomontanus. (May be only few people understood the meaning of this object.)

This is clear evidence that there were signs in Hungary also, roughly contemporary with the efforts of Columbus, that some people were aware of the Earth's sphericity.

Interestingly enough, this knowledge spread almost without attracting any attention, and certainly did not cause as many problems as did the dilemma of geo- and heliocentricity.

Spherical geometry has continued to be developed in Hungary since that time. Maps have been plotted, based no longer upon Ptolemy but rather upon measurements. István Hatvani was the first to carry out measurements on latitudes, which then became standard in the practice of cartography.

The collection of problems of high school mathematics published by Beke and Reif at the end of the 19^{th} century also contained problems on astronomy and geography.

In the 20^{th} century, Lóránd Eötvös made measurements with his torsional pendulum, to determine the exact geoidal shape of the Earth.

János Bolyai created a new world from the void.

Riemann was the first who studied the consequences of the fact that several geometries coexist - although he did not mention his sources. It was Baltzar who first cited Bolyai's name. His theory started to be taught in Graz, in the 1870-71 academic year.

Gauss - not by chance - was afraid to let mankind learn about the hyperbolic geometry. The coexistence of different geometries led to the revision of the axiomatic structure of mathematics. The results of Gödel and Church seemed to shatter the basic principles of mathematics, but by now, the problems have been solved within the framework of mathematics. The representatives of other sciences were mostly at a loss if they ever faced this question, as Sokal's joke proves. It would be valuable to study comprehensively the similarities and differences between contemporary mathematics and the post-modern sciences. One way to do this would be to follow the historical approach, and to learn the real nature of axiomatic development through studying geometry.

This teaching and learning process might even start at the level of general education. For this, not only the scientific basis but also child-friendly teaching tools are at hand.

Acknowledgment. Many thanks to Tony Mann for his help.

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