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Models of management of capital

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Abstract. The goal of this contribution is to explain modeling of particular investment projects of a company at varying conditions. The conditions could change net present value of the project. Various changing conditions can result in optimalisation of investing activity of the company

1. Model of Management of Capital with Possibility of Grant a Credits

Linear model for capital budgeting is based on the standard linear programming model. The problem is to find the right portfolio capital projects that result in maximization of the objective function - total net present value of the money was invested in projects, subject to the constraints. The constraints in the capital budgeting process can include limited supplies of capital and other resources (personnel, raw material, materials, etc.) in individual years. In this model we assume that we have possibility to borrow money for investment projects where was not used in one year, can be carried over to future years. In case when the firm has no invest money in determinate year, than we can this money carry the other economic subjects and in next year we have new capital budget for invest projects where is heighten about the interest from original capital. In other word firm cannot of grant a credit and transform unused budget to future years.

We assume that interest rate is zero. In this case firm cannot have benefit from interest of grant a credits.

Let's define this model as follows:

$$z = \sum_{i=1}^{n} P_i x_i \to \max$$

subject to

$$\begin{aligned}
&-\sum_{i=1}^{n} f_{i1} x_i + S_1 = b_1 \\
&-\sum_{i=1}^{n} f_{i1} x_i - S_{t-1} + S_t = b_1 \text{ for } t = 2, 3, ..., m, \\
&0 \leqslant x_i \leqslant 1 \; for i = 1, 2, ..., n, \\
&S_t \geqslant 0 \; for t = 1, 2, ..., m.
\end{aligned}$$

where

$n-$ number of invest projects,

P_i-present value of individual projects for i = *1,2,...,n,*

$b_t-$the available money of firm on the invest projects in year t,

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f_{it}−the cash flow in year t resulting from approval project i,

S_t−not used money in year t.

Condition $S_t \geqslant 0$ ensure that money from the present year t cannot be used in the past year $t-1$.

Now let's create mathematical model for real data but we will not refer the name of firm. We contemplate about the firm, where want invest own money to five investment projects. In the table 1. below we have cash flow during the last five years and we have 6% discount rate:

Project	Cash	flow	in	mil.	Sk
	1.year	2.year	3.year	4.year	5.year
A	-10	-4	6	1	13
B	-12	-8	3	12	19
C	-6	-6	4	11	16
D	-14	-5	5	13	22
E	0	-7	2	6	11
Butget constraint	20	17	15	10	5

We can define mathematical model as follow:

$$10,25980419x_A + 8,24803051x_B + 13,80891913x_C +$$

$$+14,07411234x_D + 8,92696529x_E \to \max$$

subject to

$$10x_A + 12x_B + 6x_C + 14x_D + s_1 = 20$$

$$4x_A + 8x_B + 6x_C + 5x_D + 7x_E + s_2 - s_1 = 17$$

$$-6x_A - 3x_B - 4x_C - 5x_D - 2x_E + s_3 - s_2 = 15$$

$$-10x_A - 12x_B - 11x_C - 13x_D - 6x_E + s_4 - s_3 = 10$$

$$-13x_A - 19x_B - 16x_C - 22x_D - 11x_E + s_5 - s_4 = 5$$

$$3x_A + 2x_B + x_D + x_E \leqslant 5$$

$$2x_A + 2x_B + 3x_C + 2x_D + x_E \leqslant 5$$

$$x_C \leqslant x_B$$

$$0 \leqslant x_i \leqslant 1$$

$$s_i \geqslant 0$$

The solution by means of the Simplex method is shown in the table 2.:

Project	Solution (%)
A	0
B	37,73
C	37,73
D	94,34
E	100

The solution for money where can be carry are shown in the table 3.:

Year	Solution(%)
1	0
2	0
3	24,358
4	61,302
5	111,264

The value of objective function is 30 526 569,97 Sk. It means, that firm will fully invest to the project E, does not take part in the project A, in the project D will accept with 94,34% and in the projects B, C equally 37,73 %.

In the table 3. we can see, that investment firm in the first and second years do not carry any money, because in this years need money for own building and own growth. From 3rd year to 4th year is the firm the able carry 24,358 mil. Sk, from 4th to 5th year 61,302 mil. Sk and from 5th to 6th year 111,264 mil. Sk.

2. Model of Management of Capital with Possibility of Grant and Receive Credits

We can create model, where firm has approach to the free capital market. It means, that firm can of grant and receives noninvest funds and transform there to future years. Because we want operate in the capital market it is needed to set out interest rate for grant and receive a credits.

For simplification of model, we assume, that interest rate are equal and in course of decision time are constant. Now we will define mathematical model, where we have define in the preceding example, when we assume, that invest projects can make positive cash flow to the year m. This cash flow we will discount to the year m for interest rate r.

The mathematical model is follow:

$$z \sum_{i=l}^{n} P_i x_i + \alpha_m - \beta_m \to \max$$

subject to

$$\sum_{i=l}^{n} f_{i1} x_i + \alpha_1 - \beta_1 \leq b_i$$

$$\sum f_{it} x_i - (1+r)\alpha_{t-1} + \alpha_t + (1+r)\beta_{t-1} - \beta_t \leq b_i$$

$$\begin{array}{rl} & \text{for} \quad t = 2, 3, ..., m, \\ 0 \leqslant x_i \leqslant 1 & \text{for} \quad i = 1, 2, ..., n, \\ \alpha_t,\, \beta_t \geqslant 0 & \text{for} \quad t = 1, 2, ..., m, \\ \beta_t \leqslant C_t & \text{for} \quad t = 1, 2, \ldots, m, \end{array}$$

where

P_i - net present value of all capital cash flow from invest project for $i = 1, 2, \ldots, n$;
α_t - multitude of grant a credits in the years t;
β_t - multitude of receive a credits in the years t;
f_{it}- the cash flow in year t resulting from approval project i;
b_t- the available funds of firm on the invest projects in year t;
C_t - maximum value of receive a credits in the year t;
r - interest rate

If the firm exceed the limit of maximum value of receive a credits in the year t, than in case, if we borrow another credits reach to make more expensive of credit and this fact is express in the heighten interest rate r. Interest rate we can compensate average of capital cost this firm.

Now we will define mathematical model where we assume both grant a credit (α) and receive a credit (β). Interest rate for grant credit is 5 % and for receive credit 10 %. In this case in the objective function was used net present value of cash flow in our projects bring to five years period with discounting to fifth year. For cash flows after five years period was estimation cash flows during the space of five years with assume that is growing 1 %.

The mathematical model is as follow:

$$10,25980419x_A + 8,24803051x_B + 13,80891913x_C +$$

$$+14,07411234x_D + 8,92696529x_E + \alpha_5 + \beta_5 \rightarrow \max$$

subject to

$$10x_A + 12x_B + 6x_C + 14x_D + \alpha_1 - \beta_1 \leqslant 20$$

$$4x_A + 8x_B + 6x_C + 5x_D + 7x_E - (1 + 0,05)\alpha_1 + \alpha_2 + (1 + 0,1)\beta_1 - \beta_2 \leqslant 17$$

$$-6x_A - 3x_B - 4x_C - 5x_D - 2x_E - (1 + 0,05)\alpha_2 + \alpha_3 + (1 + 0,1)\beta_2 - \beta_3 \leqslant 15$$

$$-10x_A - 12x_B - 11x_C - 13x_D - 6x_E - (1 + 0,05)\alpha_3 + \alpha_4 + (1 + 0,1)\beta_3 - \beta_4 \leqslant 10$$

$$-13x_A - 19x_B - 16x_C - 22x_D - 11x_E - (1 + 0,05)\alpha_4 + \alpha_5 + (1 + 0,1)\beta_4 - \beta_5 \leqslant 5$$

$$3x_A + 2x_B + x_D + x_E \leqslant 5$$

$$2x_A + 2x_B + 3x_C + 2x_D + x_E \leqslant 5$$

$$x_C \leqslant x_B$$

$$0 \leqslant x_i \leqslant 1$$

$$\beta_i \leqslant 5$$

$\alpha_i, \beta_i \geqslant 0$ for $i = 1, 2, 3, 4, 5$.

The solution this model is shown in table 4.:

Project	Solution(%)
A	15,79
B	42,11
C	42,11
D	100
E	100

The solution for grant and receive credit is shown in the table 5.:

Year	Grant credit (mil. Sk)	Receive credit (mil. Sk)
1	0	3,158
2	0	5
3	20,395	0
4	61,678	0
5	119,551	0

The value of objective function is augment on 153 460 282,10 Sk. If investment firm want have this value objective function – maximum net present value – must fully invest to the projects D and E and in project A with 15,79 %, projects B and C with 42,11 %. In the first year firm receive credit in the value 3,158 mil. Sk, and in the second year 5 mil. Sk. In the 3rd, 4th and 5th years firm grant credit, what is logical, because in the first two years firm needs capital for own growth and in the next years has free capital for any economic subjects. When investment firm can operate in the capital markets than value of objective function is augment.

3. Conclusion

The value of objective function we have growth, because to the grant credit we are join assume about the receive credit. Input of the investment firm on the capital market gives noninvest funds and transforms these funds to the capital in next period.

References

[1] STEIGAUF, S.*Investiní matematika* , Grada Publishing v Prahe 1999. ISBN80-7169-429-0
[2] PITEL, J. at al, *Ekonomicko-matematické metódy* Vydavatestvo Príroda v Bratislave v roku 1988 EMM 064-054-88
[3] KRÁLOVÁ, L *Lineárne programovanie a investiné rozhodovanie* TU EkF, Košice, 2003 Diplomová práca
[4] PENJAK, V., *Capital and investment decision by means of financial optimization,*, Proceedings III. International Scientific Conference, Miskolc (2001), str. 351-356
[5] VRBENSKÁ, LPoužitie niektorých kvantitatívnych metód pri finannom rozhodovaní Diplomová práca TU EkF, Košice, 2001

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