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About the study of Romanian Black Sea Shore Marine Currents

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Abstract. The paper refers to the fundamental system of motion equations for the theoretical study of marine water circulation. There are presented some assumptions for the case of Romanian shore marine currents. Some results regarding Ekman currents on the Romanian Black Sea shore finish the work.

1. Introduction

The Black Sea is a nearly-enclosed marginal sea. The Strait of Bosphorus links it to the Marmara Sea and the Strait of Kerch links it to the Azov Sea. The river runoff, dominated by Danube, Dnepr and Dnestr dilutes the surface water in the northwestern Black Sea. The exchange with the Marmara Sea is small and the ratio between the Black Sea and the amount of the water exchanged with the other seas show that the Black Sea is one of the most isolated ocean areas.

Black Sea possesses many processes and features of major ocean basins. But there are some notable features of this marine basin. Black Sea has not an open boundary in contrast with the other ocean basins. The combination of wind stress and buoyancy forcing, the lateral exchange of mass through the Bosphorus drive very complex flows.

The exchange between other seas and the river runoff does not determinate a real conservation of mass and salt for Black Sea. We can observe an increase of water volume, which gives an increase of Black Sea level with 1,8-2,0 mm/year and a negative budget of salt, which drives to a dilution of Black Sea waters. But the very small differences allow use the equations for conservation of mass and salt for the Black Sea basin.

The Black Sea is located in the northern part of the subtropical latitudes, its northern end is in the temperate zone. The Azore and Siberia centers of high pressure affect the atmospheric fields over the Black Sea. The winter storms of Mediterranean origin determinate main tracks which pass in the southern part of the sea. The depressions of Atlantic origin propagate to East and Southeast, crossing Romania and Bulgaria. Generally, the field of wind is cyclonic.

The annual mean atmospheric temperature over the Black Sea is about 14^0, but there are some differences between the variations of temperature in summer or in winter. The horizontal temperature gradients are about twice less in summer than in winter. The precipitation regime is very different in the western and northwestern

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part and in the eastern part (the range of precipitation is 300-500 mm/year for the first and 1800-2500 mm/year for the second).

Half of the river runoff due to the Danube and the river discharge is one of the most important components of the Black Sea fresh water balance.

A very important element of water circulation is the wind. Winds over the Black Sea are, in 80% of the cases, below 5 m s^{-1}, only in about 20 to 40 days each year wind speed exceeds 15 m s^{-1}. In the northern and in the western part of the Black Sea dominant are the winds from NE, N and NW. There is, in the warm part of the year, an important influence of the Azore anticyclone.

2. THE EQUATIONS OF MOTION

The equations for conservation of mass and salt for Black Sea waters are:

$$\begin{array}{l} Q_B^S + Q_B^C + Q_R + P + E = 0 \\ Q_B^S S_B + Q_B^C S_M = 0 \end{array} \tag{1}$$

where:

Q_B^S −volume of the water flowing in Bosphorus Strait from the Black Sea,
Q_B^C −volume of the water flowing in the Black Sea basin from Marmara Sea,
Q_R −volume of river runoff,
P – precipitation, E – evaporation, S_M – Marmara Sea salinity, S_B – Black Sea salinity.

The continuity equation:

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

has a simple form for the marine water circulation, because the density is nearly constant in the ocean. Boussinesque assumed that the density is constant except when it is multiplied by g in calculation of pressure in the ocean. This assumption, named Boussinesque approximation, greatly simplifies the equations of motion and allow to use the continuity equation for the incompressible fluid:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

Four forces are important for the marine water circulation: pressure gradients, Coriolis force, gravity and friction. The Navier-Stokes equations, in Cartesian coordinates, are:

$$\begin{array}{l} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + 2\,\omega\,v\,\sin\varphi + F_x \\ \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial w}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} - 2\,\omega\,u\sin\varphi + F_y \\ \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} + 2\,\omega\,u\cos\varphi - g + F_z \end{array} \tag{3}$$

where u, v, w are the components of velocity, p is the pressure, ω is the rotation rate of Earth, g– the acceleration of gravity, φ - the latitude and F_x, F_y, F_z the components of frictional force. Generally, it assume that $w \ll u, v$ and $2\,\omega\,u\cos\varphi$ has been dropped from the last equation because it is small compared with g.

Diffusion equations for heat and salt are:

$$\begin{aligned} -\mu_T\,\Delta T+\gamma_T\,w &= \frac{\partial}{\partial z}\nu_T\frac{\partial T}{\partial z}\\ -\mu_S\,\Delta S+\gamma_S\,w &= \frac{\partial}{\partial z}\nu_S\frac{\partial S}{\partial z}\end{aligned} \tag{4}$$

where μ_T, γ_T, ν_T, μ_S, γ_S, ν_S are coefficients for temperature and salinity.

An important equation for the study of marine water circulation is the equation of state. For the sea water, this equation has the form:

$$\rho=\alpha_T T+\alpha_S S \tag{5}$$

All these equations form a system which the unknowns are u, v, w, p, ρ, S and T. This system needs boundary and initial conditions. The general form of those conditions is:

$$\begin{aligned}
&\frac{\partial u}{\partial z}=\frac{-\tau_x}{v\rho_0},\quad \frac{\partial v}{\partial z}=\frac{-\tau_y}{v\rho_0},\quad \frac{\partial T}{\partial z}=f_1,\\
&\frac{\partial S}{\partial z}=f_2,\ w=0\quad \text{for}\quad z=0,\\
&\frac{\partial u}{\partial z}=0,\quad \frac{\partial v}{\partial z}=0,\quad T=f_3,\quad S=f_4,\\
&w=0\quad \text{for}\quad z=-H\\
&u=0,\quad v=0,\quad \frac{\partial T}{\partial n}=0,\\
&\frac{\partial S}{\partial n}=0\quad \Gamma\quad \text{frontier},\\
&u=u^0,\quad v=v^0,\quad T=T^0,\quad S=S^0\quad \text{at}\quad t=0.
\end{aligned} \tag{6}$$

Here τ_x, τ_y are the two components of wind stress, f_i, $i=\overline{1,4}$ are functions, H is the depth of the basin.

It obtains a system which contains seven differential equations (2-5), with the conditions (6). It is impossible to obtain analytical solutions for the general case. Great difficulties appear due to the non-linear terms, friction, real shapes of basin and coastlines. Hence, the only possibility for study the marine waters circulation is numerical model. The practical models must be simpler than the real ocean, the initial conditions are not well known, that why numerical models drive to many problems. The models can never give a real and complete description of marine waters circulation in a basin. The use of numerical models
necessitate many assumptions.

Some of those assumptions are:
- vertical velocity is small, $w \ll u, v$;
- the flow is steady, homogeneous, horizontal, with friction on a rotating Earth;
- the vertical eddy viscosity is constant;
- the first derivatives are assumed to be null:

$$\frac{\partial}{\partial\, t}\;=\;\frac{\partial}{\partial\, x}\;=\frac{\partial}{\partial\, y}\;=\;0$$

bulk formula for the wind stress, that is:

$$\tau \;=\; \rho_{air}\; C_D\; U_{10}^2 \tag{7}$$

where ρ_{air} is the density of air, C_D is the drag coefficient and U_{10} the wind speed at 10 m above the sea.

3. The Black Sea waters circulation

We use the Ekman theory for the study of water circulation in the Black Sea basin. It is assumed that the sea surface is plane, the viscosity coefficient is constant and the wind is blowing above the sea with a constant stress and a constant direction given by α_1 angle. Because the vertical component of velocity is null, we use an horizontal coordinate system with the standard convention in geophysical fluid mechanics: Ox to the East, Oy to the North, Oz up. We assume that the mean flow is steady and that the two horizontal components u, v varies only in the z direction. The momentum equations have the simple form:

$$\begin{aligned} \alpha\mu\frac{d^2u}{dz^2} + 2\omega\, v\, \sin\varphi = 0 \\ \alpha\mu\frac{d^2v}{dz^2} - 2\omega\, u\, \sin\varphi = 0 \end{aligned} \tag{8}$$

The boundary conditions are:

$$\mu\left(\frac{du}{dz}\right)_{z=0} = \tau\,\sin\alpha_1 \tag{9a}$$

$$\mu\left(\frac{dv}{dz}\right)_{z=0} = \tau\,\cos\alpha_1 \tag{9b}$$

where τ is the wind stress. At $z=-H$, for the basin bottom the velocity is nul:

$$u = v = 0 \quad \text{at} \quad z = -H.$$

With some assumptions [3], we obtained the two components of velocity given by the formulas:

$$\begin{aligned} u = {} & \frac{\tau}{\sqrt{2}a\mu(ch2aH + \cos 2aH)}\left[\left(\frac{\pi}{4} - \alpha_1\right)(\cos\alpha(\zeta + H)sha(\zeta - H) + \right. \\ & + \cos a(\zeta - H)sha(\zeta + H) - \sin\left(\frac{\pi}{4} - \alpha_1\right)\times \\ & \left.\times\left(\sin a(\zeta + H)cha(\zeta - H) + \sin a(\zeta - H)cha(\zeta + h)\right)\right] \end{aligned} \tag{10}$$

$$\begin{aligned} v = {} & \frac{\tau}{\sqrt{2}a\mu(ch2aH + \cos 2aH)}\left[\cos\left(\frac{\pi}{4} - \alpha_1\right)\big(\sin a(\zeta - H)cha(\zeta + H) + \right. \\ & + \sin a(\zeta + H)cha(\zeta - H)\big) - \sin\left(\frac{\pi}{4} - \alpha_1\right)\times \\ & \left.\times\big(\cos a(\zeta - H)sha(\zeta + H) + \cos a(\zeta + H)sha(\zeta - H)\big)\right] \end{aligned}$$

where $\zeta = +z$.
The magnitude of velocity is given by:

$$V = \frac{\tau}{\sqrt{2}a\mu}\sqrt{\frac{ch2a(z+H) - \cos 2a(z+H)}{ch2aH + \cos 2aH}} \tag{11}$$

We made a network for the Romanian Black Sea shore and we used the formulas above to calculate the currents driven only by wind, gravity and Coriolis force. On used real different depths for each rectangular of the network. We calculated, with (10), the two components of velocity in a program using C++, for the wind velocity between 7 and 10 m s $^{-1}$. We obtained some maps presented in the figure below.

4. Conclusions

It is obvious that the maps represent only a component of marine water circulation, because there are too many simplifications in the system of equations of motion. The currents are the result of wind blowing above sea surface. The real currents are more complicated and a very important element which is not taken in consideration is the boundary condition in the sea shore region. The influence of Coriolis force can be observed by the differences between the velocities directions. To obtain better results we must construct a Romanian Black Sea shore fine network. It is also necessary to use continuity conditions passing from a rectangle to other. We consider that the work is a beginning for the analytical study of the water circulation on the Romanian Black Sea shore.

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