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An extension of some results on the degree of progress to goal in self-organization process

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ABSTRACT. In this paper, we establish some results for the degree of progress to goal at any stage during a self-organization process by considering several self-organizing systems with their corresponding distance functions, $g_k(t),\ k = 1, 2, ..., s$. We employ some properties of curve as well as the convex combination of the distance functions to determine the degree of progress to goal at any stage of the resulting self-organization process. The probability of reaching the goal at any stage of the resulting self-organization process is considered as its degree or level of progress to goal during this process.

The results obtained are in agreement with the axiomatic properties of probability.

1. INTRODUCTION

Adeagbo-Sheikh [1] in his model for self-organizing systems employed the concepts of a *distance function*, $g(t)$, and that of a *controlled-disturbance function*, $h(g(t))$,(where t is the time variable) in explaining the views of some notable thinkers as Ashby [2] and Beer [3].

In Olatinwo [8], the degree of progress to goal at any stage during self-organization process was considered. The transition probabilities at various time intervals were evaluated and then subsequently interpreted as the degrees of progress to goal at such time intervals.

Given a set of self-organizing systems with each system self-organizing to a distinct desired state of affairs, is it possible for all these systems to interact and become a system that is self-organizing to a particularly desired state of affairs? This question is answered in the affirmative in the next section.

In this paper, we generalize the results of Olatinwo [8] by considering a convex combination of the various distance functions characterizing the various self-organizing systems. Our results are established by using elementary concepts of the probability and the curve theory as well as the idea of convex functions (see [5] for detail). It is found that the results obtained are in agreement with the axiomatic properties of probability.

The study becomes pertinent for its possible applications in diverse areas, especially in learning, adaptive control and pattern recognition systems. Literature abounds with the theories of learning and invariably use statistical techniques. See Fu and Mendel [6].

However, we shall require the following Lemmas in the sequel.

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Lemma 1. *Let $\{g_k(t)\}_{k=1}^{s}$ be a set of the distance functions for s different self-organization processes. Then, $\sum_{k=1}^{s}\lambda_k g_k(t)$ is a distance function for the resultant self-organization process, where $\lambda_1,\ \lambda_2,\ \ldots,\lambda_s \in [0,1]$ and $\sum_{k=1}^{s}\lambda_k = 1$.*

Proof. Let

$$u(t) = \sum_{k=1}^{s}\lambda_k g_k(t) = \lambda_1 g_1(t) + \lambda_2 g_2(t) + \ \ldots\ + \lambda_s g_s(t). \tag{1}$$

We now show that $u(t)$ is a distance function by showing that it satisfies all the properties of a distance function stated in Olatinwo [8].

We show that $u(t) > 0,\ t_0 \le t < t_f < \infty$, where t_f is the final time for the completion of the self-organization process:
Since each $g_k(t),\ k = 1, 2, \ldots, s$, is a distance function, then each
$g_k(t) > 0,\ t_0 \le t < t_f < \infty$, and so each $\lambda_k g_k(t) > 0,\ k = 1, 2, \ldots, s$, noting that each $\lambda_k > 0$. Hence, $u(t) > 0$.

We now show that $u'(t) < 0,\ t_0 \le t < t_f < \infty$.
Differentiating $u(t)$ in (1) with respect to t yields

$$u'(t) = \lambda_1 g_1'(t) + \lambda_2 g_2'(t) + \ldots + \lambda_s g_s'(t) = \sum_{k=1}^{s}\lambda_k g_k'(t). \tag{2}$$

Since each $g_k(t)$, is a distance function, we have $g_k'(t) < 0$. Again, since each $\lambda_k > 0$, we have each $\lambda_k g_k'(t) < 0$. It follows from (2) that $u'(t) < 0$.
Using (1), we obtain
$u(t_f) = \lambda_1 g_1(t_f) + \lambda_2 g_2(t_f) + \ldots + \lambda_s g_s(t_f) = 0,\ t_0 < t_f < \infty$, since for the distance functions $g_k(t)$, we have $g_k(t_f) = 0,\ k = 1, 2, \ldots, s$.
Finally, we have using (2) and triangle inequality that

$$|u'(t)| = |\sum_{k=1}^{s}\lambda_k g_k'(t)| \le \sum_{k=1}^{s}\lambda_k |g_k'(t)| < \infty,$$

since $\lambda_k > 0,\ |g_k'(t)| < \infty,\ k = 1, 2, \ldots, s,\ t_0 < t < t_f < \infty$.
Therefore, $u(t)$ is a distance function. This completes the proof of the Lemma.

Lemma 2. *Let $\delta(x)$ be continuous on $[a, b] \subset \mathbb{R}$. Then, $\int_a^x ||\delta(u)||du$ is the length of a certain curve from a to x.*

Proof. Since $\delta(x)$ is continuous on $[a, b]$, there exists a differentiable function $\rho(x)$ on (a, b) such that

$$\rho(x) = \int_a^x \delta(u)du,\ x \in [a, b]. \tag{3}$$

Applying the Fundamental Theorem of Integral Calculus in equation (3) yields

$$\rho'(x) = \frac{d}{dx}\int_a^x \delta(u)du = \delta(x),$$

from which we obtain

$$||\delta(x)|| = ||\rho'(x)||. \tag{4}$$

Integrating both sides of eqn(4) yields

$$\int_a^x ||\delta(u)||du = \int_a^x ||\rho'(u)||du.$$

Since $\int_a^x ||\rho'(u)||du$ is the length of the curve $\rho(x)$ from a to x, then $\int_a^x ||\delta(u)||du$ is indeed the length of a certain curve from a to x.
This completes the proof of the Lemma.

Remark 1. The proof of this Lemma is also contained in Olatinwo [9].

2. Main Results

We recall that the distance function, $g(t)$, according to Adeagbo-Sheikh [1], is the distance from the goal at any time satisfying the following properties:

$$(i)\ g(t) > 0,\ t_0 \leq t < t_f < \infty,\ (ii)\ g'(t) < 0,\ t_0 < t < t_f < \infty,$$
$$(iii)\ g(t_f) = 0,\ t_0 < t_f < \infty,\ (iv)\ |g'(t)| < \infty,\ t_0 < t < t_f < \infty,$$

where t_f is the final time.

Without loss of generality, our self-organizing systems are considered to be in the sense of Ramon-Margalef (see Beer [3]). The property(ii) of $g(t)$ shows that it is (strictly)monotone decreasing. The system begins to self-organize towards some desired state of affairs at time t_0 and the self-organization process reaches completion at time t_f (i.e. $g(t_f) = 0$), see property (iii) and Olatinwo[8] for detail.

Recall that the length $l(t)$ of a curve $f(t)$ (see Bruce and Giblin [4] as well as Olatinwo [8, 9]) is given by

$$l(t) = \int_{t_0}^t ||f'(u)||du. \tag{5}$$

We see easily that $l(t_0) = 0$ and $l(t) > 0$, for $t > t_0$.
We assume that the curve $f(t)$ is regular.

Definition 1. Let X_k be the event that a self-organizing system attains a stage P_k at time t_k during self-organization process. Then, the probability of this event is given by

$$Prob\,\{X_k\} = \frac{l(t_k)}{l(t_n)},\ k = 0, 1, 2, ..., n\ . \tag{6}$$

This definition is also contained in Olatinwo [8, 9].

Our main results are the following:

Theorem 1. *Suppose that* $[t_0, t_k]$ *and* $[t_0, t_n]$ *are two given time intervals such that* $[t_0, t_k] \subseteq [t_0, t_n]$. *Let* $\sum_{i=1}^s \lambda_i g_i(t)$, *with* $\sum_{i=1}^s \lambda_i = 1,\ \lambda_i \in [0, 1]$,
$i = 1, 2, \ldots, s$, *be as in Lemma 1, where* $g_1(t), g_2(t), \ldots, g_s(t)$ *are the individual distance functions for* s *different self-organization processes. Let* X_k *be the event that the*

self-organizing system having the distance function $\sum_{i=1}^{s}\lambda_i g_i(t)$ *attains a stage* P_k *at time* t_k *during self-organization. Then,*

$$\mathrm{Prob}\left\{X_k\right\}=\frac{\sum_{j=1}^{k}\int_{t_{j-1}}^{t_j}[\sum_{i=1}^{s}\lambda_i||g_i'(u)||-||\delta(u)||]du}{\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}[\sum_{i=1}^{s}\lambda_i||g_i'(u)||-||\delta(u)||]du}, \tag{7}$$

where $\delta(t)$ *is some continuous function on* $[t_0,t_n]$ *such that*

$$0\leq\delta(t)\leq||\delta(t)||<\sum_{i=1}^{s}\lambda_i||g_i'(t)||,\ k\leq n,\ k,n\in\{1,2,\ldots\}.$$

Proof. Let $f(t)=\sum_{i=1}^{s}\lambda_i g_i(t)$. By Lemma 1, we have that $f(t)$ is a distance function. We obtain by (5) and (6) that

$$\mathrm{Prob}\left\{X_k\right\}=\frac{\int_{t_0}^{t_k}||f'(u)||du}{\int_{t_0}^{t_n}||f'(u)||du}=\frac{\int_{t_0}^{t_k}||\sum_{i=1}^{s}\lambda_i g_i'(u)||du}{\int_{t_0}^{t_n}||\sum_{i=1}^{s}\lambda_i g_i'(u)||du}, \tag{8}$$

where

$$\frac{df}{dt}=f'(t)=\sum_{i=1}^{s}\lambda_i g_i'(t).$$

We obtain by the triangle inequality that

$$||\sum_{i=1}^{s}\lambda_i g_i'(t)||\leq\sum_{i=1}^{s}\lambda_i||g_i'(t)||, since \lambda_i\in[0,1],\ i=1,2,\ldots,s. \tag{9}$$

Addition of $||\delta(t)||$ to the left-hand side of (9) yields

$$||\sum_{i=1}^{s}\lambda_i g_i'(t)||=\sum_{i=1}^{s}\lambda_i||g_i'(t)||-||\delta(t)||. \tag{10}$$

Integrating both sides of (10) from t_0 to t_k yields

$$\int_{t_0}^{t_k}||\sum_{i=1}^{s}\lambda_i g_i'(u)||du\int_{t_0}^{t_k}(\sum_{i=1}^{s}\lambda_i||g_i'(u)||-||\delta(u)||)du \tag{11a}$$

$$=\int_{t_0}^{t_k}\sum_{i=1}^{s}\lambda_i||g_i'(u)||du-\int_{t_0}^{t_k}||\delta(u)||du.$$

Similarly, we obtain from (10) that

$$\int_{t_0}^{t_n}||\sum_{i=1}^{s}\lambda_i g_i'(u)||du=\int_{t_0}^{t_n}\sum_{i=1}^{s}\lambda_i||g_i'(u)||du-\int_{t_0}^{t_n}||\delta(u)||du. \tag{11b}$$

By Lemma 2, $\int_{t_0}^{t_k}||\delta(u)||du$ and $\int_{t_0}^{t_n}||\delta(u)||du$ are lengths of a certain curve $\delta(t)$ from t_0 to t_k and from t_0 to t_n respectively, so that both $\int_{t_0}^{t_k}||\sum_{i=1}^{s}\lambda_i g_i'(u)||du$ and $\int_{t_0}^{t_n}||\sum_{i=1}^{s}\lambda_i g_i'(u)||du$ in (11a) and (11b) are well-defined as the lengths of curve $f(t)$ from t_0 to t_k and from t_0 to t_n respectively. Substituting both (11a) and (11b) in (8) yields

$$\mathrm{Prob}\left\{X_k\right\} = \frac{\int_{t_0}^{t_k} \sum_{i=1}^{s} \lambda_i ||g_i'(u)||du - \int_{t_0}^{t_k} ||\delta(u)||du}{\int_{t_0}^{t_n} \sum_{i=1}^{s} \lambda_i ||g_i'(u)||du - \int_{t_0}^{t_n} ||\delta(u)||du}$$
$$= \frac{\int_{t_0}^{t_k} (\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| \ - \ ||\delta(u)||)du}{\int_{t_0}^{t_n} (\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| \ - \ ||\delta(u)||)du}. \tag{12}$$

Application of the fact that finite union of intervals can be split up into disjoint ones (see Kai Lai [7] and Olatinwo [8, 9]) yields from (12)

$$\mathrm{Prob}\left\{X_k\right\} =$$
$$= \frac{\int_{t_0}^{t_1} [\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| - ||\delta(u)||]du + ... + \int_{t_{k-1}}^{t_k} [\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| - ||\delta(u)||]du}{\int_{t_0}^{t_1} [\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| - ||\delta(u)||]du + ... + \int_{t_{n-1}}^{t_n} [\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| - ||\delta(u)||]du}$$
$$= \frac{\sum_{j=1}^{k} \int_{t_{j-1}}^{t_j} (\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| \ - \ ||\delta(u)|| \ )du}{\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} (\sum_{i=1}^{s} \lambda_i ||g_i'(u)|| \ - \ ||\delta(u)|| \ )du}$$

This completes the proof of the Theorem.

Remark 2. Theorem 1 is a generalization of Theorem 2A in Olatinwo [8], since it reduces to Theorem 2A when $s = 1$ and $||\delta(t)|| = 0$.
Suppose that we have s self-organizing systems such that there are m subsystems in each self-organizing system. Suppose that

$$\left\{A_{11}, A_{12}, ..., A_{1m}\right\}, \left\{A_{21}, A_{22}, ..., A_{2m}\right\}, . . ., \left\{A_{s1}, A_{s2}, ..., A_{sm}\right\}$$

are the corresponding sets of activities for the m subsystems in each of the self-organizing systems and

$$(y_{11}(t), y_{12}(t), ..., y_{1m}(t)), \ (y_{21}(t), y_{22}(t), ..., y_{2m}(t)), ..., (y_{s1}(t), y_{s2}(t), ..., y_{sm}(t))$$

are the vectors whose components measure the level or aggregate effects of respective activities from time $t_0 \geq 0$ to time t.

In this paper, we shall assume a convex combination of the corresponding components of the vectors. Thus, if $\{B_1, B_2, ..., B_m\}$ is the corresponding overall set of activities for the resulting self-organizing system, then we have

$$(\alpha_1 y_{11}(t) + \alpha_2 y_{21}(t) + ... + \alpha_s y_{s1}(t), \ \alpha_1 y_{12}(t) + \alpha_2 y_{22}(t) + ...+$$
$$+ ... + \alpha_s y_{s2}(t), ..., \alpha_1 y_{1m}(t) + \alpha_2 y_{2m}(t) + ... + \alpha_s y_{sm}(t)),$$

with $\sum_{j=1}^{s} \alpha_j = 1$, as the corresponding vector whose components measure the level or aggregate effects of respective activities from time $t_0 \geq 0$ to time t in the resulting self-organizing system. We are interested in finding the level of contribution or efficiency of each subsystem from time t_0 to time t_n during self-organization process. We then employ it to find the probability for the overall level of the self-organization process. This idea is summarized in the next two results.

Theorem 2. *Let X_q be the event that the subsystems have aggregate effects*

$$\sum_{j=1}^{s} \alpha_j y_{jq}(t),\ q = 1, 2, \ldots, m,\ \sum_{j=1}^{s} \alpha_j = 1,$$

in the time interval $[t_0, t_n]$ during self-organization process. If B_q, $q = 1, 2, \ldots, m$ are the corresponding activities over the same time interval, then,

$$\mathrm{Prob}\,\{\mathrm{X_q}\} = \frac{\sum_{\mathrm{j=1}}^{\mathrm{n}} \int_{\mathrm{t_{j-1}}}^{\mathrm{t_j}} (\ \sum_{\mathrm{i=1}}^{\mathrm{s}} \alpha_{\mathrm{j}} ||\mathrm{y}'_{\mathrm{iq}}(\mathrm{u})|| - ||\varphi(\mathrm{u})||\ )\ \mathrm{du}}{\sum_{\mathrm{j=1}}^{\mathrm{n}} \int_{\mathrm{t_{j-1}}}^{\mathrm{t_j}} (\ \sum_{\mathrm{i=1}}^{\mathrm{s}} \lambda_{\mathrm{i}} ||\mathrm{g}'_{\mathrm{i}}(\mathrm{u})|| - ||\delta(\mathrm{u})||\ )\ \mathrm{du}}, \tag{13}$$

where $\varphi(t) \geq 0$ and $\delta(t) \geq 0$ are continuous functions on $[t_0, t_n]$ such that

$$0 \leq \delta(t) \leq ||\delta(t)|| < \sum_{i=1}^{s} \lambda_i ||g_i'(t)||$$

and

$$0 \leq \varphi(t) \leq ||\varphi(t)|| \leq \sum_{i=1}^{s} \alpha_i ||y'_{iq}(t)||,\ q = 1, 2, ..., m.$$

Proof. Let $h_q(t) = \sum_{i=1}^{s} \alpha_i y_{iq}(t)$. Then,

$$||h_q'(t)|| = ||\sum_{i=1}^{s} \alpha_i y'_{iq}(t)|| \leq \sum_{i=1}^{s} ||\alpha_i y'_{iq}(t)|| = \sum_{i=1}^{s} |\alpha_i|\ ||y'_{iq}(t)|| = \sum_{i=1}^{s} \alpha_i ||y'_{iq}(t)||, \tag{14}$$

since $\alpha_i \in [0, 1],\ i = 1, 2, ..., s$.
Addition of $||\varphi(t)||$ to the left-hand side of the inequality (14) yields

$$||h_q'(t)|| = \sum_{i=1}^{s} \alpha_i ||y'_{iq}(t)|| - ||\varphi(t)|| \tag{15}$$

Integrating both sides of (15) from t_0 to t_n yields

$$\int_{t_0}^{t_n} ||h_q'(u)||du = \int_{t_0}^{t_n} \sum_{i=1}^{s} \alpha_i ||y'_{iq}(u)||du - \int_{t_0}^{t_n} ||\varphi(u)||du. \tag{16}$$

By applying Lemma 2, we have that $\int_{t_0}^{t_n} ||\varphi(u)||du$ is the length of a certain curve $\varphi(t)$ from t_0 to t_n so that $\int_{t_0}^{t_n} ||h_q'(u)||du$ in (16) is well-defined as the length of curve $h_q(t)$ from t_0 to t_n. Hence, substituting both (11b) and (16) in (6), we have

$$\begin{aligned} \mathrm{Prob}\,\{\mathrm{X_q}\} &= \frac{\int_{\mathrm{t_0}}^{\mathrm{t_n}} ||\mathrm{h}'_{\mathrm{q}}(\mathrm{u})||\mathrm{du}}{\int_{\mathrm{t_0}}^{\mathrm{t_n}} ||\sum_{\mathrm{i=1}}^{\mathrm{s}} \lambda_{\mathrm{i}} \mathrm{g}'_{\mathrm{i}}(\mathrm{u})||\mathrm{du}} \\ &= \frac{\int_{t_0}^{t_n} \sum_{i=1}^{s} \alpha_i ||y'_{iq}(u)||du\ -\ \int_{t_0}^{t_n} ||\varphi(u)||du}{\int_{t_0}^{t_n} \sum_{i=1}^{s} \lambda_i ||g_i'(u)||du\ -\ \int_{t_0}^{t_n} ||\delta(u)||du} \\ &= \frac{\int_{t_0}^{t_n} (\sum_{i=1}^{s} \alpha_i ||y'_{iq}(u)||\ -\ ||\varphi(u)||)du}{\int_{t_0}^{t_n} (\sum_{i=1}^{s} \lambda_i ||g_i'(u)||\ -\ ||\delta(u)||)du} \end{aligned} \tag{17}$$

Application of the fact that finite union of intervals can be split up into disjoint ones(as in Theorem 1) yields from (17) the desired result given by (13).

Remark 3. Theorem 2 is a generalization of Theorem 2B in Olatinwo [8], since this reduces to Theorem 2B when $s = 1$ and $||\varphi(t)|| = ||\delta(t)|| = 0$.

Theorem 3. *Suppose that the overall subsystems are independent with corresponding activities $B_r,\ r = 1, 2, \ldots, m$ and let $\sum_{j=1}^{s} \alpha_j y_{jr}(t)$, $r = 1, 2, \ldots, m,\ \sum_{j=1}^{s} \alpha_j = 1$, be the corresponding aggregate effects in the time interval $[t_0, t_n]$ during self-organization. Let $X_r,\ r = 1, 2, \ldots, m$ be the event that the subsystems have aggregate effects $\sum_{j=1}^{s} \alpha_j y_{jr}(t)$ over the time interval $[t_0, t_n]$ during self-organization process. Then,*

$$\mathrm{Prob}\left\{\bigcap_{r=1}^{m} X_r\right\} = \frac{\prod_{r=1}^{m}\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}\left(\sum_{i=1}^{s}\alpha_i||y_{ir}'(u)|| - ||\varphi(u)||\right)du}{\left[\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}\left(\sum_{i=1}^{s}\lambda_i||g_i'(u)|| - ||\delta(u)||\right)du\right]^m}, \tag{18}$$

where the functions $\varphi(t)$ and $\delta(t)$ are as in Theorem 2.

Proof. By Theorem 2, we have that

$$\mathrm{Prob}\{X_r\} = \frac{\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}\left(\sum_{i=1}^{s}\alpha_i||y_{ir}'(u)|| - ||\varphi(u)||\right)du}{\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}\left(\sum_{i=1}^{s}\lambda_i||g_i'(u)|| - ||\delta(u)||\right)du},\ r = 1, 2, \ldots, m. \tag{19}$$

Since the subsystems are independent, then $X_1, X_2, \ldots, X_m$ are independent events. Therefore,

$$\mathrm{Prob}\left\{\bigcap_{r=1}^{m} X_r\right\} = \mathrm{Prob}\{X_1\}\,\mathrm{Prob}\{X_2\}\ \ldots\ \mathrm{Prob}\{X_m\}. \tag{20}$$

Substituting (19) in (20) yields the desired result given by (18).

Remark 4. Theorem 3 is a generalization of Theorem 2C in Olatinwo [8], as it reduces to Theorem 2C when $s = 1$ and $||\delta(t)|| = ||\varphi(t)|| = 0$.

Remark 5. However, if the subsystems are mutually exclusive rather than being independent, with the same aggregate effects $\sum_{j=1}^{s} \alpha_j y_{jr}(t)$ over the same time interval during self-organization process, then it is obvious that

$$\mathrm{Prob}\left\{\bigcup_{r=1}^{m} X_r\right\} = \frac{\sum_{r=1}^{m}\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}\left(\sum_{i=1}^{s}\alpha_i||y_{ir}'(u)|| - ||\varphi(u)||\right)du}{\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}\left(\sum_{i=1}^{s}\lambda_i||g_i'(u)|| - ||\delta(u)||\right)du},$$

where $\bigcap_{r=1}^{m} X_r = \phi$ (i.e. empty set) and $\mathrm{Prob}\{\bigcap_{r=1}^{m} X_r\} = 0$.

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