CREATIVE MATH. & INF.
15 (2006), 137 - 155

From non-Euclidean geometries to Picasso, Stravinski, Ionesco... Otherwise: About the route to maximum freedom of creative thinking in the European sciences and arts of 19^{th} and 20^{th} century

DAN I. PAPUC

ABSTRACT. In this article some special aspects of the European creative thinking in sciences and arts are discussed. In the XIX^{th} and XX^{th} centuries the European creative thinking in sciences and arts attained a very high level of freedom, creating the first non-Euclidean geometry, the relativity theory, the nonconformist arts. These results of European culture became of utmost importance in the progress of human civilization.

Contents

Received: 15.10.2006. In revised form: 18.10.2006.
2000 *Mathematics Subject Classification.* 00A30, 01A55, 01A60.
Key words and phrases. *Creative thinking; thinking pattern; the freedom of creative thinking in mathematics, sciences and arts; academic arts; nonconformist arts.*

ACKNOWLEDGES: The author thanks to Professor Vasile Berinde for his proposals to translate into English and to publish this essay and, essentially, for the help to realize these two proposals.

The author expresses his gratitude to Mrs. Ada Kranjec who translated into English the first form of this essay. Without this help, the essay would never appear in English.

Also the author express his gratitude to Mrs. Emilia Petrisor and to Mrs. Adara Blaga for the help to improve the language of this changed form of essay.

Introduction.

The history of mathematics, as a part of the history of human civilization, [3], reveals the existence of a general phenomenon hidden in numerous patterns. This phenomenon deserves to be known and understood. What exactly is it? *Beginning with the 3rd decade of the 19th century, and continuing with the first decades of the 20th century, the creative thinking in European culture* (more exact, in *sciences* and *arts*) *comes up to access its maximum freedom, i.e. the greatest freedom that this thinking can possess in the many specific instances in which it works*. The result was remarkable. For European sciences, it was an amazing progress that extended to the entire humankind. For European arts, it entailed the existence of everlasting masterpieces, like those done by Picasso, Brâncuşi, Stravinski, Rimbaud, Joyce, Ionesco, Le Corbusier and many others. This essay is *an attempt* to decode this general phenomenon and understand some of its specific instances.

The route followed by the European creative thinking in sciences and arts towards its maximum freedom, in the 19 th century and the first half of 20th century will be illustrated by means of three particular cases:

– *Mathematics*. It is the oldest modern science (2300 years old!), the science of everlasting truth. It is also, historically speaking, *the first field in European culture in which the creative thinking has used the maximum freedom in its creative processes.* The result was *the creation* of the first Non-Euclidean geometry.

– *Sciences*. It will be explained what means *maximum freedom for the scientific creative thinking*. The phenomenon will be considered and exemplified in Physics. The maximum freedom, used by physical creative thinking will be emphasized in the achievement of the Theory of Relativity.

– *Arts*.In the first half of 20th century, the artistic creative thinking claims for *a freedom without limits*. It will be explained what this means. More illustrative considerations will concern all the Arts of the 19th and of 20th century.

* * *

In this attempt, a special terminology is necessary to be introduced. The meaning of some concepts, belonging to this terminology, are specified here.

The unit-pair Human – Cosmos. The Cosmos is the Physical world, i.e. the set of all things that are perceived by the five human senses or by their technical extensions, a set endowed with an ordering relation. The lack of order leads to Chaos. The Human is a double entity: a physical one and thinking and emotional one. In the pair Human – Cosmos, the Human is a thinking and emotional being. The Human, a physical entity, belongs to the Cosmos.

From the Greek antiquity until now, the unit-pair Human – Cosmos is the basic subject of all activities of creative thinking. These activities lead to the appearance and development of human culture (philosophy, sciences and arts), the essential part of the human civilization. More precisely: The Cosmos, as topics of research of creative thinking, generated the sciences. The Arts are the result of attempts made by the creative thinking to understand and express in works of art *the emotion* caused by the thoughts, the emotions and the historical life (material, social and cultural) of the Human being or by some aspects of the Cosmos.

The creative thinking is the solitary mental mechanism (it is the talent!) used by the Humankind in order *to know* and *to understand* the unit-pair Human – Cosmos. The results of action of *the creative thinking* (of the talent) are *the creations*. *A creation* is confirmed by its *originality* and by its *value*. The value is determined by the contribution at the knowledge of the unit-pair Human – Cosmos.

There are two fundamental forms of *creative thinking: scientific creative thinking* (it used fundamentally the information; the Sciences are created in this way) and *artistic creative thinking* (the Arts are created to understand the human emotionality).

There exist many special types of *creative thinking*: philosopher's creative thinking, mathematician's creative thinking, physicist's creative thinking, poet's creative thinking, painter's creative thinking, architect's creative thinking and so on.

The thinking pattern (Denkstructur)is a syntagm introduced by Werner Heisenberg in an interesting essay from [2]: *"Transformations of thinking patterns towards the progress of science (Änderungen der Denkstructur in Fortschritt der Wissenschaft)"*. Werner Heisenberg does not explain what he means by *a thinking pattern*, allowing its significance to be understood from the context. In our opinion, a thinking pattern is the *"apparatus"* used by the *creative thinking* (by the talent) in the realization of the great creations in a well-determined domain of the sciences or the arts. *A thinking pattern*, associated to a well-determined domain of sciences or arts, is materialized by *a language* with a special semantics, a special *symbolism*, a set of special and effective *tools*, *methods* of use these tools, *"the familiarity"* with a significant set of results from the considered domain, *the integration* of the new results in the precedent set etc.

Thinking patterns exist in two hypostasis: in the case of sciences, the thinking patterns make possible the activity of scientists in their domain of scientific activity. In the case of arts, the thinking patterns assure the activity of artists in their domain of artistic activity. In the case of arts, the thinking pattern determines *the style* and conversely, *the style* determines the thinking pattern. A scientist or an artist uses, in its activity, a precise *thinking pattern*.

An important aspect pointed out by Werner Heisenberg is the following: *Any transformation* (i.e. any change) in a thinking pattern, which ensures the knowledge of a phenomenon or the creation of a work of art, is *a real revolution* in the knowledge of the phenomenon or in the style of work of art. These revolutions determine usually the progress in sciences and a plus of originality and of value in arts.

The freedom of creative thinking is the freedom to eliminate certain "restrictions" in the creative processes, to enforce new creative ideas, to make certain changes that imply profoundly the used thinking patterns, much more, it may choose a new thinking pattern, all these in order to obtain better results in creations. It is also the freedom to disregard traditional paths and canons in the creative processes, as well as the accomplishments of creations implied by these paths and canons. The problem whether this freedom in creation is favorable or not is "solved" in two manners: *for the scientific thinking* by means of the "true" or "false" test, or through the confrontation with the physical world. *For the artistic thinking*, up to a certain degree, with the help of a specialized critique, but especially with the aid of the cruel test of survival over time.

1. THE ACCESS TO MAXIMUM FREEDOM ACHIEVED BY MATHEMATICAL CREATIVE THINKING

1.1. **The maximum freedom of mathematical creative thinking.** The science called Mathematics is the "favorite daughter" of the Greek Philosophy from the Classic (Helen) period (the 6^{th}- 4^{th} century BC). The Greek philosophers from this period, studying the Cosmos, established two important results: *the principle of causality* (the relationship cause-effect) and they stated (a hard to justify then) assertion: *the Cosmos is generated by the interaction of a finite number of fundamental generators* (assertion brilliantly confirmed by the modern Physics). By means of these two results, the Greek philosophers of Classical period created the oldest modern science, *the Mathematics*. The principle of causality leaded the Greek philosophers to the achievement of the concept of *proof* (*the demonstration*). The idea, that the Cosmos is generated by the interaction of a finite number of fundamental generators, obviously inspired Euclid in the axiomatization of the geometry of his time. Then, a first *mathematical structure* had been stated, which is called today the Euclidean space.

An arbitrary mathematical structure is determined by a defining system, i.e. by a finite system of fundamental (sometime called primary) notions and relations, which fulfill a non-contradictory finite system of hypotheses (sometime called axioms).

Studying a mathematical structure determined by a given defining system, by means of only two logical tools, *the definition* and *the proof*, there were elaborated axiomatized mathematical theories associated to this structure.

Beginning with the third decade of the 19th century, the mathematical structures gradually became the principal element in the architectural construction of the Mathematical Universe.

The maximum freedom of the creative thinking in Mathematics is the liberty to determine arbitrary mathematical structures and to define new mathematical entities in these structures, observing only the mathematical rules that have already existed in Mathematics.

Until the 3^{rd} decade of 19^{th} century, the creative thinking in Mathematics was constrained to observe, in the problem of development of Mathematical Universe (i.e. in the problem to determine new mathematical structures and to define new mathematical entities) a principle outside to Mathematics. It represents the point of view of Aristotle (384-322 BC) about the Mathematics, and here it is called *the Aristotelian principle*.

1.2. **Mathematical creative thinking under the Aristotelian principle.** At the beginning of the 19^{th} century, Mathematics comprised by an impressive system of mathematical theories. Among these theories, only two had been axiomatized. The first was *the Euclidean space* axiomatized in the work *Elements* (*Stoiheia*) by the

Greek Euclid from Alexandria, at the end of the 4^{th} century and the beginning of the 3^{rd} century BC. The second is *the Newtonian Dynamics*, axiomatized in 1686 in the work *Philosophiae Naturalis Principia Mathematica* (*Mathematical Principles of Philosophy of Nature*) written by the English mathematician Isaac Newton (1642-1727). The rest of the mathematical sciences from the beginning of the 19^{th} century was composed by studies of well-determined problems in already existent mathematical theories, or, overwhelmingly, of the studies of "mathematical models" of phenomena from the real world (the most studied within Physics or Technique).

At the beginning of the 19^{th} century, the majority of mathematical theories were in the late category. This is obvious for those two axiomatized mathematical theories, the *Geometry of Euclidean Space* and the *Newtonian Dynamics theory*. The situation was the same for all mathematical theories developed by mathematicians in the 18^{th} century and the first two decades of the 19^{th} century (Euler, Lagrange, Laplace, Monge, Fourier and others). All these mathematical theories had been created strictly observing an old principle. It represented the point of view of Aristotle concerning Mathematics. It connected the Mathematics to the Physical world like an umbilical cord and it has become an unbreakable and untouchable dogma. This fundamental principle, named below "Aristotelian principle", stated that:

Mathematics emerged and developed in a continuous process of modelling of physical phenomena in mathematical patterns, and studying these mathematical patterns, Mathematics serves to understand these phenomena.

The creative mathematical thinking was "under the heel of the Aristotelian principle" i.e. all progresses in Mathematics must be done by solving problems from Mathematics or modelling physical phenomena in mathematical patterns. The Aristotelian principle was the most used, without facing any objections. Perhaps this principle perhaps was brought into discussion until the first two decades of the 19^{th} century only to underline its great value (and this great value exists!) in mathematicians' creativity. Here it is what Jean Joseph Fourier (1768-1830) said about Mathematics in his speech during the welcoming ceremony into the French Academy:

"There is no other language more universal and simple, with less mistakes and confusions, i.e. more worthy of expressing invariable links between natural realities. From this point of view, this science (N/A Mathematics) is as encompassing as nature itself. It defines all sensitive relations, measures time, spaces, forces and temperatures. This difficult science was developed in time, but keeps all its principles once it has acquired them. It grows and consolidates without a break, in the midst of so many errors done by the human spirit."

From antiquity until the beginning of 19^{th} century, was considered that defying this principle is an "unpardonable" error. Indeed, within these two axiomatized mathematical theories (Euclid's Space and Newtonian Dynamics), the Aristotelian principle led to the creative thinking. *All primary concepts and hypotheses about these concepts are obviously exact copies of elements from the Physical world*. When, after presenting the law of universal gravitation in his work *Philosophiae Naturalis Principia Mathematica* from 1686, Newton was accused of "making up" this hypothesis, in the second edition of his work, in 1713, he strongly objected: "Hypothesis non fingo!" (I do not make up hypotheses!). He was right; this law could be proved experimentally within the physical world. (It all started with the multitude of astronomic observations done by Tycho Brahé (1546-1601), which led to the formulation of laws by Johannes Kepler (1571-1630), then Newton determined and proved with their help the law of universal gravitation).

We remark that at the beginning of 19^{th} century, there were mathematicians expressing their agreement for the freedom of mathematical creative thinking, i.e. for *the right to make up hypotheses and to defy, if it necessary, the Aristotelian principle*. Replying to Fourier, who thought that a paper written by the German mathematician Karl Jacob Jacobi (1804-1851) had a doubtful value because it had no practical application, Jacobi wrote:

"It is true that Mr Fourier considers the public utility and the explanation of natural phenomena as the main purpose of Mathematics; a philosopher like him should nevertheless know that **the only purpose of science is the honor of human thinking**, *and that from this point of view a problem concerning numbers is as important as a problem concerning the world system."*

At the beginning of the 19^{th} century, in Mathematics there existed and something else. By means of mathematical theories, which made up the Mathematics of that time, researches, some of which had become traditional, continued to be realized and tried to solve problems Mathematics, problems that had nothing to do with the Physical world, problems that were difficult to be solved, defying the thinking ability of mathematicians: *the three famous problems from Antiquity, the problem of parallel postulate, the issue of solving with radicals algebraic equations of degree greater than 4, the issue of defining real numbers,* etc. **But nobody tried to determine and study a mathematical structure which has no practical application**. Even the "princeps mathematicorum" of those times, the German mathematician Carl Friedrich Gauss (1777-1855), did not it.

1.3. **A first non-Euclidean geometry. The maximum liberation of mathematical creative thinking.** In solving *t*he problem of postulate of parallel lines, [in 1823 by the Hungarian Janos Bolyai (1802-1860), technical officer in the garrison of Timişoara, and in 1826 by the Russian Nikolai Lobachevski (1792-1856), professor at the University at Kazan on Volga], *for the first time a mathematical structure was determined* (i.e. a defining system of this structure was chosen) *the Aristotelian principle being disregarded, equivalently the physical reality being not taken into account*. Thus, the umbilical cord, which connected the Mathematics with the Physical world, was broken. Even more, then, *in the third decade of the* 19^{th} *century, with the solution of the problem of postulate of parallel lines the mathematical creative thinking had acquired* the maximum freedom *in its activities*. More exactly, it was established that **in the problem of determining an arbitrary mathematical structure (the choice of a defining system, i.e. a finite system of concepts, relations and non-contradictory hypotheses) the mathematicians must rigorously observe only mathematical rules**. This was an intensely used freedom that immediately led to an impressive array of significant mathematical accomplishments, proving thus, what the Jewish mathematician Georg Cantor (1845-1918), creator of the Set theory, stated around 1874-75: **"The essence of Mathematics lies within its freedom."**

Briefly, we shall explain how it happened.

In the work *"the Elements"* of Euclid, a defining system of the mathematical structure, called today Euclidean plane, was constituted by 23 "definitions", 5 postulates and 5 axioms. The 5^{th} postulate, denoted here by P_5, is the postulate of parallel line. Its enunciation (a logically equivalent form with that of Euclid!) is: *for any line and for any point which does not belong to the line, the point belongs to at the most a parallel line to the initially considered line*.

Which is the problem of the postulate of parallel lines? If we denote by A the composed proposition obtained only from the conjunction of the 5 axioms and 4 postulates, the postulate of parallel lines being excluded, then the problem of the fifth postulate states: *the logical implication* $A \Longrightarrow P_5$ *is true or not*? All attempts to give an answer to this question, made during two millennia, by mathematicians of great repute, were unsuccessful. We note: all attempts that tried to solve this problem, used only direct proof.

Both, Lobachevski and Bolyai, tried to solve the problem of fifth postulat by means of *"reductio ad absurdum"* method (which was known by Greeks from the 6^{th} century BC). They tried to prove that the logical implication [A and not P_5] $\Longrightarrow$ *it exists a proposition denoted by R such that the proposition* [R and not R] *is true*. If this logical implication is true, the proposition [R and not R] being a false proposition, then the proposition [A and notP_5] is false. However A is a true proposition. It follows that the proposition notP_5 must be false, hence the proposition P_5 is true. It follows that A implies P_5, i.e. P_5 is a theorem in the geometry of Euclidean plane. However... both mathematicians, Lobachevski and Bolyai, cannot prove that the hypotheses [A and notP_5] implies a contradiction (i.e. the existence of a contradictory proposition R) despite of all the bizarre logical consequences (of the hypothesis A and notP_5) which they established. Both, they understood that the implication [A and notP_5] $\Longrightarrow$ *it exists a proposition denoted by R such that the proposition* [R and not R] *is false*. The proposition [A and notP_5] **do not imply a contradiction**. (We note, they do not prove the last proposition. It will be proved first time in 1868, by the Italian mathematician Eugenio Beltrami, 1835-1900). **With daring, they affirmed that they determined and studied a new mathematical structure, that they 'built up" a new Geometrical plane, a non-Euclidean plane**. *A defining system of this new mathematical structure, i.e. a system of concepts and hypotheses resembled that of Euclid's plane, with one difference: the Euclidean postulate of parallelism was replaced, as a hypothesis, by its negation: "there is a line and there is a point not on this line, for which there are at least two lines which contain the given point and which are parallel lines to the given line".*

Through the study of this new mathematical structure, called today the Lobachevski-Bolyai plane, a new axiomatized mathematical theory emerged. Lobachevski initially named it "an imaginary geometry". Bolyai gave it no name. He suggestively characterized it, in a letter (to his father) from Timişoara in 1823, as "a new world" created by him "out of nothing". Today this new theory is called *the geometry of Lobachevski-Bolyai plane*. Many of its propositions strongly contradict our physical intuition and fundamental results of Euclidean geometry. It follows that the Lobachevski-Bolyai plane disobeys the Aristotelian principle and it is not a good model for the Physical space. **By the determination of the Lobachevski-Bolyai plane, the mathematical creative thinking reached its maximal freedom,**

i.e. the liberty to choose defining systems in order to determine of mathematical structures, observing only rules from Mathematics.

Carl Friedrich Gauss (1777-1855) obtained the same solution to the problem of the postulate of parallel lines. In a letter, Gauss confessed what he did, but he did not dare to publish his result for fear of the obtuse people.

1.4. **Consequences of maximum liberation of mathematical creative thinking.** The liberation of mathematical creative thinking of the constraint to obey the Aristotelian principle provoked a significant departure of Mathematics from the Physical world and thus an essential development of abstraction in Mathematics. By this, *the importance of Aristotelian principle did not decrease at all,* **but the domain of action of the mathematical creative thinking became infinite.**

Based on this new freedom, an effort of axiomatization for all mathematical theories raised. Until the end of the 19^{th} century, a number of new mathematical structures had been determined. They are: *numeric structures* (natural numbers, integers, rational numbers, real numbers, complex numbers, quaternions), *fundamental algebraic structures* (group, ring, field and vector space), *fundamental geometrical structures* (Riemannian manifolds, Erlangen Program spaces, and others), *fundamental analysis structures* (real functions with real variables, complex functions with complex variables, the both producing a wide range of other mathematical theories). At the beginning of the 20^{th} century appear*d topological structures*, *mixed structures* (differentiable manifolds, topological algebras, algebraic topology etc) and many other types of important structures. At the end of the 20^{th} century, the number of distinct mathematical theories and issues studied by mathematicians was somewhere between 4000 and 5000 (see the classification in 2000, used in the Mathematical Reviews Journal, edited by AMS), confirming again *the infinite creativity of the free mathematical thinking*. These many abstract theories build *a complicated system* (from the point of view of its internal connections) and *autonomous* (with regard to the unit-pair Human – Cosmos) from the point of view of its extension. It can be stated that *the mathematical creative thinking,* **using its freedom**, *has become the demiurge of a new independent Universe, the Mathematical Universe* ([3], p. 5). For all these, this abstract scientific Universe continues to be a fundamental tool for a further understanding of the unit-pair Human-Cosmos.

2. The access to maximum freedom achieved by scientific creative thinking

2.1. **The maximum freedom of scientific creative thinking.** Obviously, the scientific creative thinking is a creative thinking that assures the progress of one from the sciences: *sciences of nature, technical sciences, social sciences, humanistic sciences*. What does *the maximum freedom of scientific creative thinking* mean? *When, in a phenomenon, from one of the above mentioned sciences, which is considered by scientists completely known and understood (maybe it is modelled in Mathematics!), an aspect that can not be understood come up, the scientists have the liberty to replace the initial way of understanding of that phenomenon by another one, so that all the aspects of this phenomenon to be understood.*

For a phenomenon that is considered *"completely known and understood"* there is a uniquely determined thinking pattern that served to be accomplished this result. *"Replacing the initial way of understanding by another one"* is equivalent with changing of the thinking pattern by another one which will assure *"that all aspects of this phenomenon to be understood"*.

In order to exemplify all these, we shall consider a phenomenon from Physics. It is a historical moment when the creative thinking in Physics used its *maximum freedom*. The result was *the realization of a new mathematical model for the Physical space*, a model that generated *the Special Relativity Theory*. This moment marked the access to a maximum freedom of the scientific creative thinking from all the Sciences that are studying the Cosmos.

2.2. **The Newton Physics and the speed of light.** It was generally admitted that the birth of modern Physics is represented by the publication of the work *Philosophiae Naturalis Principia Mathematica* by Isaac Newton (1642-1727) in 1686. In this work, fundamental for Physics, it is presented *a mathematical model* of the physical theory that studies the motion of bodies in Physical space. This *mathematical model*, called *Newtonian dynamics*, is (for mathematicians!) *an axiomatized mathematical theory*. *The mathematical structure*, fundamental for this mathematical model, was been determined by *a defining system* formed by eight primary concepts (*absolute time, absolute space, solid body, mass, energy, force, acceleration, velocity*) and of four axioms (*Newton's three principles and the law of universal gravity*). Because of the fact that Newtonian dynamics is an axiomatized mathematical theory, and any such theory is a closed theory i.e. in such a theory everything is completely understood, it follows that Newtonian dynamics is a closed theory, where everything is understood.

After 1686, based on Newton's genial work, researches in Physics continued during the 18^{th} and 19^{th} centuries, and the theory called *Newtonian Dynamics*, as a theory of Physics, profoundly and systematically has extended. Based on the work of Newton, new concepts were defined and new laws established (achieved experimentally and by mathematical modelling!), regarding different phenomena in the physical world: rest and motion of solids, of liquids, of gases, electrical and magnetic phenomena, probabilistic and statistic behavior and others. The results obtained, through observations and experiments, developed into significant physical theories, which were modelled mathematically by important theories. We shall not try to detail this evolution here. It would force us to present a huge quantity of results from Physics and Mathematics and quote names like Euler, Lagrange, Laplace, Gauss, Faraday, Maxwell, Gibbs and other.

Certain observations are nevertheless essential:

a) *All new physical theories*, created until the end of the 19 th century, used (directly or indirectly) the fundamental concepts from Newton's 1686 work (*absolute time, absolute space, solid body, mass, energy, force, acceleration, velocity*). If we refer to this great extent of Physics, we can talk about *a Newtonian Physics*.

We refer to Heisenberg's essay "Is Physics a closed science?" ([2], p. 314-320).

b). *Specialized thinking patterns* (see the Introduction!) developed, for each of these new theories in Physics. There are *Newtonian thinking patterns*. All the specialized Newtonian thinking patterns harmoniously belonged to a system of thinking patterns, *the thinking pattern from the Newtonian Physics*.

Towards the end of the 19^{th} century, optimism for creative thinking in Newtonian Physics seemed justified. Many phenomena from the physical world were understood, and the ones that were left, were believed to follow in the future, as part of the never-ending process of knowledge. In this euphoric atmosphere, a **phenomenon that must be understood** emerged. During several experiments in 1887, which later became famous, two American physicists, A.A. Michelson and E.W. Morley, noticed that *the speed of light does not obey the velocities addition law*, which has been established by Galileo Galilee (1564-1642). The Michelson-Morley experiments proved that *the speed of light does not depend on the observer's velocity of motion* (as Galilean law of velocities addition required); *the speed of light was behaving like a physical constant*. The Dutch physicist H.A. Lorentz (1853-1928) and the French mathematician Henri Poincaré (1854-1912) tried to explain this phenomenon. They tried to give an explanation within Newtonian Physics, using the Newtonian thinking pattern. They did not succeed. The speed of light became **an incomprehensible phenomenon** in Newtonian Physics.

2.3. **The Einstein's Relativistic Dynamics and its consequences.** The failure of Lorentz and Poincaré researches required a new way (a new thinking pattern) of understanding the motion in the physical space. This new way was developed by the Jewish physicist *Albert Einstein* (1879-1955), in his work *"Zür Elektrodynamik bewegter K örper"* (On the Electrodynamics of moving bodies), published in 1905 in the Journal *Annalen der Phyzik*. The new way of understanding motion in the Physical space has determined by two great changes in the defining system of concepts and principles building the Newtonian Dynamics.

1. Two of the eight fundamental concepts had been replaced. They are exactly those that seemed the least prone to change. They seemed completely understood even in Antiquity. *The absolute time does not depend on observer* and *its mathematical model is the set of real numbers with its properties*. *The absolute space does not depend on observer* and *its mathematical model is the 3-dimensional Euclidean space with its geometry*. These two concepts had been replaced by a new fundamental concept, *the Space of events*, *which is dependent on the observer and whose geometry is the 4-dimensional Minkowski geometry*.

2. After the Newton's four principles, two new principles have been added (using the notion of *inertial reference frame* i.e. a reference frame on which no force acts, so this reference frame moves linearly and uniformly in the absolute space as well as in the event space):

Principle of special relativity. *The laws of Physics are the same for all inertial reference frames.*

Principle of constant speed of light. *In vacuum, with respect to two inertial reference frames that move relative to one another at a speed v, the light moves at the same speed, denoted by c.*

The new Dynamics based on a new system of fundamental concepts and principles called *Relativistic Dynamics*, amazed physicists. Due to the elimination of absolute time and absolute space and to the acceptance of the constant speed of light, several conclusions have been reached. They are the followings: *the highest limit speed* in Physical space is c (speed of light), so $v < c$; *body mass increases with speed*; *weight turns into energy*; for events studied by two observers, moving linearly and uniformly with respect to one another with a speed v, the concepts of *"being in the same place"* and *"simultaneity"* are eliminated; the phenomena of *"length contraction"* and *"time dilatation"* emerge. Remark: when the speed v is smaller compared to the speed of light (with $c/v > 10^5$ **ratio) the results of *Newtonian Dynamics* and *Relativistic Dynamics*, at an acceptable approximation, are the same.**

The study of *Relativistic Dynamics* required physicists' creative thinking *to be able to understand in a different manner what seemed to be completely understood*. The understanding of new phenomena requires efforts, not at all easy efforts, which are involved in the creation and daily usage of *new thinking patterns* - called here **relativistic thinking patterns**.

At that time, in 1905, after the change of the Newtonian thinking patterns with the relativistic thinking patterns, the maximum liberation of creative thinking was finally achieved within Physics. This consists of **the freedom of creative thinking to understand in a different way what seemed completely understood, when some aspect, which cannot be understood, appeared**.

What followed is a known fact. A few years after 1905, as an attempt to understand gravitational field the *General Theory of Relativity* was set up, in which the event space was replaced by a 4-dimensional Lorentzian manifold, the special principle of relativity was replaced by the general principle of relativity and the law of gravitation was eliminated and replaced by the hypothesis (verified experimentally during the total solar eclipse of 1919) that matter existence changes the space curvature (it changes the metric of the modelling manifold and, implicitly, the manifold curvature). These changes brought to fundamental concepts and principles in Newton's Philosophiae Naturalis Principia Mathematica and progressively led to the creation of a new Physics, *the Relativistic Physics*, with a fundamentally changed thinking pattern.

Fundamental progresses in Physics due to the total liberation of creative thinking did not stop there. During the same period, in the first decade of the 20^{th} century, any attempts to explain the phenomenon called blackbody radiation within Newtonian Physics failed, thus leading to the birth of the concept of *"quantum"* and the elaboration of *"the Quantum Theory"*. Two of the main creators of this theory were the German physicists Max Planck (1858-1947) and later Werner Heisenberg (1901-1976). Within a few decades a new type of physics was born, *the Quantum Physics*, which required new changes in thinking patterns.

Around the end of the 20^{th} century, due to the liberation of creative thinking, except for Physics (also called Classical physics) there were developed *Relativistic physics, Quantum physics, Physics of elementary particles* and other. These new domains of Physics came together, partially overlapping, building the most formidable tool for the knowledge of the Physical world and a basis for great changes in all other sciences.

2.4. **Relations between Physics and Mathematics during the 20^{th} century.** a) We have seen that the effective usage of maximum freedom of creative thinking within Physics, during the 20^{th} century, manifested through changes in thinking patterns. This freedom also required certain *epistemological changes regarding the usage of Mathematics as a knowledge tool*. Let us look at this.

Scientific research in Physics (as well as in any other science) has followed an "algorithm" which was decoded for the first time by the English philosopher Francis Bacon (1521-1626), in his famous paper *Novum Organum Scientiarum*, published in 1620. The modern form of this algorithm is the following: the scientific research of any phenomenon in the real world usually consists of three steps. The first step is to develop, through observations and experiments, a theory (system of results) on that phenomenon. The second step is to model this theory by a mathematical theory, which will be studied by mathematicians. The third step is to interpret and check in the real world the mathematical results obtained in the second step. There are two clear examples where the scientific researches followed the Bacon's algorithm: the study of the inert Physical space (time was not taken into account) and the study of the movement in the Physical space. In these two examples the Mathematics was used in the second step of Bacon's algorithm and there the Euclidean space and, respectively, the Newtonian dynamics were elaborated.

Starting with 1905 and during the 20^{th} century, Francis Bacon's algorithm of how scientific research should be guided, it was sometimes disregarded in Physics. *Sometime, the scientific research has been put face to face with a new phenomenon in the Physical world, which must necessarily be understood. Usually, it is not known too much about this phenomena and it is difficult, if not impossible (frequently due to costs!) to develop an empirical theory towards its understanding. So, the first step in Bacon's algorithm cannot be accomplished. In this case, the second step follows directly. A mathematical model of a physical yet-inexistent theory is proposed in order to serve the understanding of that phenomenon. In the three step of Bacon's algorithm, the results obtained through mathematical studies of the mathematical model are verified by experiments, thus establishing whether the virtual mathematically-modelled physics theory is "good" or not*. Such situations often took place in researches done during the 20^{th} century in Physics (Relativistic physics, Quantum physics, Physics of elementary particles and others). In such researches, Mathematics played a primordial role. We can conclude that, in the 20^{th} century, *Mathematics has become* (*sometimes*) *for physicists, the main research tool of certain phenomena from the Physical world*.

b). *How did the freedom of mathematical creative thinking influence the liberation of physical creative thinking?* Such an influence existed but it was not essential. It was materialized in two ways. First, the liberation of creative thinking in Mathematics from the all-powerful principle, considered by everyone as "sine qua non" (the case of the Aristotelian principle!) but which became an obstacle for creation, was a good example to be followed towards the liberation of creative thinking in Physics. Physicists understood that they had the right to disregard certain principles that were considered untouchable but which were obstacles for creation.

We start describing the second way by a remark: *Mathematics has offered "mathematical tools" already finished, necessary to the development of some physical theories that agreed with the complete liberation of creative thinking in Physics*. The usage in Physics of some of the new mathematical structures which are setup without involvement of the Physical world, was an amazing phenomenon for physicists, and it somehow seemed unexplainable.

Here is what Albert Einstein (1879-1955) said:

"The most amazing thing is that the most abstract mathematical reasoning (without any reference to the real world, N/A) end up improving our knowledge about the world" ([3]; p. 512).

The Chinese physicist Chen Nang Yang (Nobel Prize in Physics in 1957) was also amazed by this phenomenon. In 1974, he said ([3]; p. 516):

"The fact that these non-Abelian gauge fields are conceptually identical to the ideas contained in the beautiful theory of fibre bundles (produced by mathematicians around 1950, N/A), which were developed by mathematicians without any reference to the physical world, seems to me a great wonder."

The English physicist Paul Adrien Maurice Dirac (1902-1984) tried to talk about this aspect ([3], p.393). Influenced by his own experience, he wrote:

"The great progress in Physics requires for its theoretical formulation a more and more advanced mathematics. This is only natural and to be expected. What was not expected by the scientific researchers of the last century was the special form taken by mathematicians' progress; it was expected that mathematics would become more and more complex but would still have its unchanged basis of axioms and definitions, while developments in modern Physics would demand a mathematics that would continuously change its basis and become even more abstract. Non-Euclidean geometry and non-commutative algebra, which were considered until not long ago pure fiction, ways to pass time for logicians, proved to be extremely necessary in the description of general facts in the physical world. It looks like this process of increased abstraction will continue in the future and that progress in Physics is associated with a continuous change and generalization of axioms fundamental to Mathematics, rather than a logical development of certain mathematical constructions with a set basis."

Except from the article Quantized singularities in the electromagnetic field, Proc. Soc. Roy. London, A133, 60-72, (1931):

Certain explanations concerning quote from Dirac are in order. We can observe a change of roles. Namely, not the "developments in modern Physics (that) required mathematics to continuously change its foundations and become more and more abstract" (Dirac), but Mathematics itself decided to do this when in the third decade of the 19th century it was eliminated within mathematical creativity the constraint to take into account the Physical world. Non-Euclidean geometries and non-commutative algebras (quoted by Dirac) were produced due to the liberation of creative thinking in Mathematics from its compulsory obeying the "physical world" and not due to requirements in Physics.

The phenomenon that we are trying to explain is the following: *mathematical structures, created without any reference to the real world, "proved extremely necessary for the description of general facts within the physical world"* (quotation from Dirac). Attempts, made to explain this phenomenon, were regarded by Einstein and Yang as "amazing". There was presented certain erudite, but not too convincing, explanations. They involved, in general, the experimental origin of the rational thinking. Perhaps the things are much simpler.

Due to the freedom of creation, mathematicians turned the independent Mathematical Universe into a "hyper-shop", which contained, even at the beginning of the 20th century, many essentially distinct mathematical structures and even more numerous mathematical theories, developed during the study of these structures. Many of these structures and theories *developed without any reference to the Physical world* and they were, obviously, irrelevant for this world. The physicist, which needs a certain mathematical structure or theory in his research, comes into this hyper-shop and searches for it. There is a high probability that, *surprisingly*, among the great variety of "mathematical tools" he finds exactly the needed "tool" already realized. There are many examples of such situation:

– Einstein found Minkowski space for the Special theory of relativity and the pseudo-Riemannian manifold, i.e. Lorentzian manifold, for the General theory of relativity;

– Yang found Theory of fibre bundles and the differentiable connections for his Gauge theory;

– the creators of Quantum mechanics found realized Hilbert space; and so on. If the physicist does not find the necessary tool, there should be no worries. He orders it and mathematicians, *with enthusiasm and delight*, will honour the request. In the latter case, the Aristotelian principle of connecting Mathematics to the real world proves again its everlasting value, and thus, the "hyper-shop" (called the Mathematical Universe) with mathematical tools, becomes richer with a new and interesting mathematical structure and new mathematical theories based on this structure. We present an example, taken exactly from Dirac's activity as a physicist:

Dirac successfully used a real function with a real variable in his research, which seriously disobeyed fundamental mathematical results (Dirac's delta function). Mathematicians protested, but Dirac disregarded them. He could not give up his "flawed" function. Mathematicians (especially, Laurent Schwartz, b. 1915 and S. L. Sobolev, b. 1908) understood how things really were and, between 1950(?)-1960, they defined a new mathematical structure and produced a new mathematical theory (*the structure of distributions* and *the theory of distributions*) in which Dirac's function had a definition with full mathematical legitimacy. It is obvious that in this case, such as many others, Mathematics enriched again and again on the basis of the Aristotelian principle.

3. The achievement to a limitless freedom by the European artistic creative thinking during the 20Th

3.1. The limitless freedom of artistic creative thinking during the 20th century. From Greek antiquity until now, the *artistic creative thinking*, through its creations, called *works of art*, contributed to a better understanding of the first element of the unit-pair Human-Cosmos, i.e. to the understanding of the Human as thinking and emotional being. The European culture of 19th century and of the first half of 20th century reached, in the field of Arts, a level high that it can be compared with the level of Pericles' epoch or that of Italian Renaissance. This assertion could be easily argued by means of any history of arts.

Towards the end of the 19th century the evolution of the European arts was more and more fast, the change of styles was more and more unexpected in form and significance. "A revolt" against the Academic styles in Art began to be perceptible. The domain of European arts was apparently "threatened" by a dichotomy. It became discernible the existence of "an Academic art" (with works of art of great value) and the appearance of "a Nonconformist art" (whose works of art were often received with perplexity). Before the Second World War, the existence and the success of the Nonconformist art (nonconformist with regard to the Academic art!) became effective. For clearing up, some of the important styles of the European arts, throughout the 19^{th} century until the middle of the 20^{th} century, are: *new-classicism, realism, naturalism, symbolism, impressionism, the group Nabi, fauvism, cubism, futurism, metaphysical art, expressionism, the group "Die Brücke", the group "Der Blaue Reiter", naive art, dadaism, superrealism, abstract art* and many other styles, which are either *"combinations"* or *"reaction against the… "* of these styles. The majority styles from this list belong to the Nonconformist arts.

It is important to point out that the division "Academic arts" and "Nonconformist arts" involves especially the works of art and not the artists. The same artist, by his works of art, can belong to more styles.

The first great success of the Nonconformist art was *the total liberation of the artistic creative thinking of European arts from the 20th century*. It is a limitless freedom. Indeed, at the half of 20^{th}century, *the European artistic worker claimed the right*

– *to express emotions through his creation,* **no matter what kind of emotions**, *born by various aspects of the unit-pair Human – Cosmos,* **no matter what kind of aspects**,

– *to use "ad libitum" as means of bringing to life his emotions words, sounds, colors, spatial shapes, movements, various materials, various geometrical elements, any technical machinery and so on,*

– *to break (without to be blamed!) any rule or canon from the Academic arts throughout his process of bringing his emotions to life,*

but all these in order to reach **the purpose of any artistic creation**: *the transmission of the artist's emotions to another person and to assure the deep perception of these emotions by this another person.*

Obviously all these rights assure *the existence of a freedom without limits of artistic creative thinking*, i.e. the existence of Chaos in the Arts field. *However, we must not forget the existence of two essential, two "sine qua non" conditions that a work to be* **a work of art**. *The first is to reach the purpose of any artistic creation above indicated. The second condition asks as the result of the creator's activities to be* **a creation**, *i.e.* **the result to be new** *(to be original) and* **to have value**, *i.e. to contribute by means of emotions to a deeper understanding the Human as an emotional and thinking being*. When the total free artistic thinking followed these two extremely restrictive conditions, then will be created everlasting works of art, like those of **Pablo Ruiz Picasso** (1881-1973; Les Demoiselle d'Avignon, Guernica), **Constantin Brâncuşi** (1876-1957; the Kiss, Mlle Pogany, Bird in Space), *Igor Feodorovich Stravinski* (1882-1949; The Firebird, Petrouchka, The Rite of Spring), **Arthur Rimbaud** (1854-1891; Drunken Boat, Illuminations), **James Joyce** (1882-1941; Ulysses), **Eugène Ionesco** (1909-1994; The

---

This list was made by means of the work Histoire illustrée de la peinture de l'art rupestre à l'art abstrait; Fernand Hazer Éditeur, Paris, 1991; Rumanian edition, Editura Meridiane, Bucureşti, 1968.

Bald Soprano, The Chairs, Rhinoceros), **Chearles-Edourd Jeanneret-Gris Le Corbusier** (1887-1965; Notre-Dame-du Haut) (just a few examples) and many others.

3.2. **Some considerations around the arts of the 20^{th} century.** .

a) *Some characteristics of Arts of the 20^{th} century. Throughout the 20th* century, Academic arts, which continued the currents of the 19^{th} century at a superior level, produce works of high emotional value. Due to the freedom of creative thinking in the European Arts, considered by many to be a total liberation, new artistic currents emerged, some of these breaking any rule or canon of Academic Art. It follows that for the Arts of the 20^{th} century, the following characteristics are visible:

– *In the process of creation of works of art* **everything is admissible**, **everything is allowed**. The professionalism in Arts of the 20^{th} century is not at all an imposed quality.

– *There is a never-end search for new means of expression*. By this, it is often building a gap (even fundamental) between some nonconformist styles and the Academic Art. In order to reach deeper into the emotions of the modern Human being, there are used, by any nonconformist styles, sometimes stubbornly, challenging means of expression, going directly in implication of immediate, real emotion-arousing aspects of the unit-pair Human – Cosmos. In opposite to these, there are nonconformist styles that visibly distanced themselves from the Physical world, thus justifying, in some cases, the title of Abstract arts. It is worth to mention that in some Nonconformist Arts, beside the classical beauty (mainly searched by the Academic Arts in order to express the emotionality) emerged the first time in Arts, expressing special emotions, sometimes even disturbing ones, the sordid and the trivial.

Due to the permanent searching for new ways of expression, the Nonconformist arts are sometimes blamed (and often not without reason!) of attempting to gain a challenging originality at any cost. What is even more serious is that in this search for originality the fundamental purpose of any work of art (*the transmission of the artist's emotions to another person and to assure the deep perception of these emotions by this another person*) is sometimes not reached.

From the theoretical point of view, there are critics of art arguing that, the full liberty in the domain of means of expression allows a more complete understanding of emotions of the Human of the 20^{th} century.

b). *How this total freedom of creative thinking did emerge in the European Arts of the 20th century?*

There are probably many answers. We shall mention here two of them. The first one, which was often discussed, confirms the existence of a connection between the freedom of creative thinking in Arts and the maximum liberation of creation thinking in sciences. This answer was closely analyzed by Werner Heisenberg in his essay "*Tendencies towards the abstract in modern art and sciences*", published in Salzburg, 1969 ([2], pp. 269-281). Heisenberg draws a parallel between the evolution of European Arts and the evolution of sciences during first half of the 20^{th} century (actually until 1969, when he wrote his essay).

Modern art from the title of Heisenberg essay is equivalent with the title Nonconformist Arts from this essay.

There are two big differences between those two evolutions. They lie within the ratios between form and contents. **First**, Heisenberg said that in *the modern sciences* the contents dictate the form, and in t*he European Arts* the form searches for its own contents. **Second**, there lie within the attitude towards the accomplishments from the past: for *modern sciences* the results of classical sciences are of everlasting value in understanding certain phenomena in the physical world. For *the Nonconformist art*, the role of Academic Art towards the understanding of modern Human's emotion does not apply anymore.

In Heisenberg's view, there is only one resemblance between the two phenomena. In the 20^{th} century, *the evolution of sciences* and *the evolution of Nonconformist art*, at their beginnings, each went through a troubled period of searches. For sciences, this period was short (the first decades of the 20**th** century), followed by great enlightenment and an amazing progress. The troubled period of the Nonconformist arts was not finished in 1969 as Heisenberg considered. Did the troubled period from Nonconformist art end in 2006? If not, will it ever end?

The second answer was given by William Fleming in the introduction and conclusion paragraphs of the last two chapters of his very interesting work [1], entitled "*Styles in the* 20^{th} *century*" and "*Artistic styles after* 1945" (vol. 2, pp. 275-378). In William Fleming's opinion, what is directly responsible for the emergence and development of Nonconformist art is the material and social life of the Human being from the 20^{th} European century. Indeed, we must admit that Academic Arts cannot totally express the emotionality of the European human being, what lived the nightmares of the 20^{th} century. Other kind of Arts is necessary. We think that the beginning was made by the Spanish painter Goya y Lucentes (1746-1828) in his Caprices and the Disaster of War. Obviously, the styles of these new Arts belong to the European Nonconformist Arts of the 20^{th} century.

c). *Nothing about the contents of the European Arts of the 20th century*

We shall not discuss here the contents of the European Arts of the 20*th* century. We are not competent and it is almost impossible to sketch in a few words the essential of a complex phenomenon. It is a phenomenon in a permanent transformation, from one decade to another, even from one year to another. We can only recommend William Fleming's book "*Arts and Ideas*" ([1]), especially its last two chapters.

d). *How and who can establish the value of creations of an art, produced under a limitless freedom of the creative thinking*

Culture (philosophy, sciences and arts) is created by an elite that has *a strong education* and *a huge talent*, ensuring the progress of any civilization. It follows that anyone belonging to these elite will certainly achieve success and fame in the social life. Obviously, many would desire and dream to enter to these elite. The total freedom of creativity in Nonconformist art (literature, poetry, music, painting, sculpture, architecture, cinematography, and others) makes possible the adherence of many amateurs and impostors, with pseudo artistic creations, into the high world of any contemporary human society, *the world of the cultural creators*. How can this wave of unwelcome people, compromising for Nonconformist art, be sorted? By art critics? Taking into account how transient, diverse, and numerous artistic currents are, can these critics really establish clear value criteria, which can be used? This is unlikely. What then?

Let us turn back to the past and see how this issue was dealt with. Until the beginning of the 20^{th} century, art was generally targeted at a thin social stratum: the aristocracy, rich people, people educated in some of the greatest art centres, people that had the rare possibility to meet some of the greatest creations of human genius. This social stratum reached the point where it itself had the power to decide which creation was or was not valuable, and so the role of critics was no longer essential. Amateurism and imposture had small chances of success in an art made for those people who were themselves, at least to some degree, knowledgeable.

In the 20^{th} century, things completely changed. Due to the creation of important and systematic means of sharing the arts (printing offices, museums, philharmonics, even in smaller urban centres), but especially due to the technical accomplishments within the field of the high quality reproductions of art works and also due to tourism, great numbers of people come into contact with the creations of the past and present arts. Unfortunately, when encountering the creation of Nonconformist arts, many people are puzzled. Most often, they feel confused, regarding their own inability to understand *the value* of many creations of Nonconformist art as an intellectual handicap. Some of these creations do not look like they required a great competency or talent to be accomplished. *Who helps these people, eager for culture, to understand and correctly appreciate abstract and modern creations*? Art critics? Most often, in their desire to prove their own professionalism, they are not very clear for those that need their help. Thus, critics are most often not convincing. What can be done?

It appears that there is a solution for this difficult, but fundamental issue of art in general and Nonconformist art in particular: *a general education of people done systematically and professionally in schools, by means of presentation and explanation of past and present artistic accomplishments, for the purpose of their correct understanding and appreciation*. We will not give arguments here that such an education, humanist and not technical, is absolutely necessary and possible. Again, we will not give arguments here that such an education is absolutely necessary, as it fundamentally contributes (fact not understood by some people!) to the resolving of many serious social problems (xenophobia, racism, chauvinism, demographic pressures, ecological threats, unemployment, and others) present in the human civilization at the beginning of the third millennium. Ostentatiously using the commercial terminology of our consumption society, we will say that through this kind of education, a large set of "consumers" of humanist culture will be created, who, due to their education, will have high quality standards for to appreciate the cultural "merchandise" offered to be "bought". Explanations of Nonconformist art and a set of criteria of values will emerging in time, the amateurism will be in danger. Actually, such a humanist and not technical education is more than just a way to reach a certain level of competency in art. It is a first essential step towards the creation

---

See the essay "Proposals regarding the reform of the Romanian educational system during high school", published, like of a first form of this essay, on the Internet http://www.math.uvt.ro/papuc of ”a society based on culture” and not a ”consume-based society”. The first is the only society able to ensure the existence and progress of human civilization.

At the end ...

During the two centuries, the 19^{th} and 20^{th}, the European Human being reached to claim *the right to choose, the right to change, the right to create in a total freedom*. All these represent a *revolution* that influenced the sciences and the arts of the entire human civilization of the 20^{th} century.

At the beginning of the third millennium, we all hope that the European Human being was purified in the hells of two horrible world wars and of two criminal ideologies, from the 20^{th} century and from the Europe. We all hope that this Human being is ready to become a well-informed European Citizen.

REFERENCES

[1] Fleming William, *Arts and Ideas*, 1974, Holt, Rinehart and Winston, Inc.; Editura Meridiane, Bucuresti, 1983

[2] Heisenberg Werner, *Schritte über Grenzen*. Gesammelte Reden und Aufstze (Across the Frontiers); C.R. Piper & Co Verlag, Munchen, 1971; Editura Politica, Colecţia Idei Contemporane, Bucuresti, 1977

[3] Dan I. Papuc, *The Mathematical Universe of Human Civilization*, Editura Marineasa, Timişoara, 2003 (in Romanian)

[4] Dan I. Papuc, www.math.uvt.ro/papuc

WEST UNIVERSITY OF TIMIŞOARA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE
4, VASILE PÂRVAN BLVD.
300223 TIMIŞOARA
ROMANIA
E-mail address: danpapuc@math.uvt.ro