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Serviceability analysis of the ambulance car fleet and examination of the transport security of an Emergency Aid Centre

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ABSTRACT. The paper examines the serviceability of the vehicles and the transport security of the EAC in Ruse. The aim of the article is to determine the number of vehicles in working order at a certain rate of breakdowns and if these vehicles can manage to respond to all incoming emergency calls. A graph shows the possible states of the vehicles. A system of Kolmogorov's equations is made up to describe the process. The paper comes to definite conclusions about the state of the ambulance car fleet.

1. INTRODUCTION

The successful performance of the Emergency Aid Centre (EAC) in the town of Ruse is the result of the strict organization and coordination of its medical and transport service teams. This highly sophisticated process comprises two equally important components - well- trained medical specialists and well maintained means of transport, so that they can get to the suffering patient as soon as possible. The EAC has its own car fleet consisting of specially equipped ambulance cars and motorcars for specific needs.

At present the EAC in Ruse has twenty vehicles but it does not have its own service station. The vehicles with major technical problems are directed to the Ford and VW company service stations. A mechanic and two technicians are in charge of the daily maintenance of the vehicles and they also deal with minor faults. One electrician and one radio mechanic take care of the specific technical equipment.

2. MODEL PROBLEM

Each of the vehicles available may (independently of the rest) break down. Let's mark the rate of breakdowns with λ. The aim of this analysis is to find out how many of the vehicles will be in working order at this rate of breakdowns and if they will manage to respond to all incoming patient calls. Any vehicle which has broken down remains in the garage and waits to be serviced. The time spent on waiting for the repair work to begin is distributed according to an exponential low with a parameter γ. We assume that the time spent on the repair work itself is distributed according to an exponential law with a parameter μ.

We will define the probability of the states of the vehicle on the condition that at the initial moment it was in working order.

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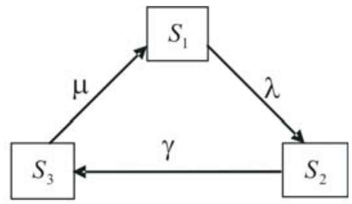

FIGURE 1. Graph of the states of the vehicle

Each of the vehicles can be in any of the following states:
S_1- the vehicle is in working order,
S_2- the vehicle is awaiting repairs,
S_3- the repair work on the vehicle is in progress.
The graph of the states of the vehicle is shown in Figure 1.

In accordance with the graph in Figure 1 and the Kolmogorov's equations [2] which describe the process in the system we are examining, we make up the system of differential equations about the probabilities of states of any of the vehicles of this ambulance car fleet:

$$\begin{aligned} \dot{p}_1(t) &= \mu p_3(t) - \lambda p_1(t) \\ \dot{p}_2(t) &= \lambda p_1(t) - \gamma p_2(t) \\ \dot{p}_3(t) &= \gamma p_2(t) - \mu p_3(t) \end{aligned} \tag{2.1}$$

Instead of the first differential equation in (2.1) the normalizing condition (2.2) can be used:

$$p_1(t) + p_2(t) + p_3(t) = 1. \tag{2.2}$$

Since we have assumed above that at the initial moment the vehicle is in working order, then the initial conditions in this case will be the following:

$$p_1(0) = 1,\ p_2(0) = p_3(0) = 0. \tag{2.3}$$

The system (2.1) in which the first equation is substituted with (2.2), can be presented in an operating way as follows:

$$\begin{aligned} xF_2(x) &= \lambda F_1(x) - \gamma F_2(x) \\ xF_3(x) &= \gamma F_2(x) - \mu F_3(x) \\ F_1(x) &+ F_2(x) + F_3(x) = \frac{1}{x}. \end{aligned} \tag{2.4}$$

When the system of algebraic equations (2.4) is solved, we get:

$$\begin{aligned} F_2(x) &= \tfrac{1}{x}\tfrac{\lambda(x+\mu)}{x^2+x(\lambda+\mu+\gamma)+\lambda\mu+\lambda\gamma+\gamma\mu}, \\ F_3(x) &= \tfrac{\gamma F_2(x)}{x+\mu} = \tfrac{1}{x}\tfrac{\gamma\lambda}{x^2+x(\lambda+\mu+\gamma)+\lambda\mu+\lambda\gamma+\gamma\mu}. \end{aligned} \tag{2.5}$$

We introduce the designation:

$$p(x) = x^2 + x(\lambda+\mu+\gamma) + \lambda\mu + \lambda\gamma + \gamma\mu = x^2 + C_1 x + C_0, \tag{2.6}$$

where

$$C_1 = \lambda + \mu + \gamma, \;\; C_0 = \lambda\mu + \lambda\gamma + \gamma\mu$$

The discriminant of the equation

$$p(x) = 0 \tag{2.7}$$

is $D = C_1^2 - 4C_0$. The parameters λ, μ and γ are positive, but the discriminant D can be positive, negative or null.

As a result of the research done at the EAC in Ruse it has been found out that $\lambda = 5$, $\mu = 1$, $\gamma = 1$. For the stated values of λ, μ and γ the discriminant $D > 0$ and then there are two different negative real roots corresponding to equation (2.7)

$$\alpha_1 = \frac{-C_1 + \sqrt{C_1^2 - 4C_0}}{2}, \;\; \alpha_2 = \frac{-C_1 - \sqrt{C_1^2 - 4C_0}}{2}, \tag{2.8}$$

i.e.

$$p(x) = x^2 + C_1 x + C_0 = (x - \alpha_1)(x - \alpha_2), \tag{2.9}$$

$$\alpha_1\alpha_2 = C_0 > 0, \;\; \alpha_1 + \alpha_2 = -C_1 < 0. \tag{2.10}$$

When we take into account (2.6), then $p_1(t)$ will be presented as:

$$F_3(x) = \frac{\gamma\lambda}{xp(x)},$$

and

$$F_2(x) = \frac{\lambda(x+\mu)}{xp(x)} = \frac{\lambda}{p(x)} + \frac{\lambda\mu}{xp(x)}.$$

When the inverse transformation of Laplace is taken into consideration we get:

$$p_3(t) = \frac{\gamma\lambda}{\alpha_1\alpha_2} + \left(\frac{e^{\alpha_1 t}}{\alpha_1(\alpha_1 - \alpha_2)} + \frac{e^{\alpha_2 t}}{\alpha_2(\alpha_2 - \alpha_1)}\right)\gamma\lambda$$

and

$$p_2(t) = \lambda\left(\frac{e^{\alpha_1 t} - e^{\alpha_2 t}}{\alpha_1 - \alpha_2}\right) + \frac{\lambda\mu}{\alpha_1\alpha_2} + \lambda\mu\left(\frac{e^{\alpha_1 t}}{\alpha_1(\alpha_1 - \alpha_2)} + \frac{e^{\alpha_2 t}}{\alpha_2(\alpha_2 - \alpha_1)}\right).$$

We find the probability $p_1(t)$ from the normalizing condition $p_1(t) = 1 - p_2(t) - p_3(t)$.

The Maple [1] programme product has been used to draw the graphs of the functions.

The graphs of the functions $p_1(t)$, $p_2(t)$, $p_3(t)$ for the stated values of λ, μ and γ are shown in Figure 2. When $t \to \infty$ the system switches to stationary mode, and then the probabilities p_1, p_2, p_3 do not depend on time any longer and they are respectively equal to:

$$\begin{aligned} p_1 &= \lim_{t\to\infty} p_1(t) = \frac{1}{11} \\ p_2 &= \lim_{t\to\infty} p_2(t) = \frac{5}{11} \\ p_3 &= \lim_{t\to\infty} p_3(t) = \frac{5}{11}. \end{aligned} \tag{2.11}$$

In stationary mode the vehicles will change their state S_1, S_2 or S_3, but the probabilities of these states will not depend on time any more. We can refer to

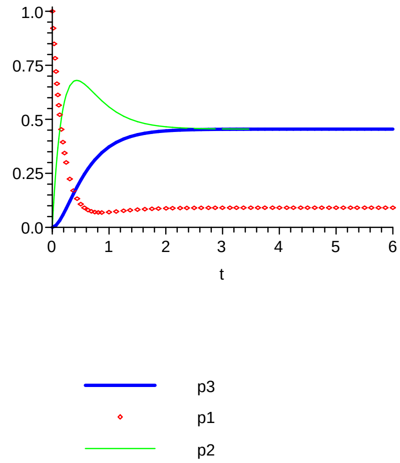

FIGURE 2. Probabilities of the states of the vehicles

them as average relative time of stay of the vehicle in the respective states S_1, S_2 or S_3 [3].

As the ambulance car fleet is quite old, the rate of breakdowns is high, and in stationary mode the probability a given vehicle to be in working order will be $p_1 = \dfrac{1}{11}$, the probability a given vehicle to be awaiting repair is $p_2 = \dfrac{5}{11}$ and to be in the process of repair is $p_3 = \dfrac{5}{11}$. So it is necessary to renew the ambulance car fleet in order to decrease the rate of breakdowns λ.

3. CONCLUSION

The average number of vehicles in working order $\overline{k}$ will be equal to the number of vehicles in the system (the ambulance fleet of the EAC - Ruse) multiplied by the probability of any single vehicle to be in order. This probability for the boundary mode (when $t \to \infty$) is p_1, hence
$\overline{k} = n.p_1 = 20.\frac{1}{11} \approx 2.$

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