CREATIVE MATH. & INF.
17 (2008), 88 - 97

Optimizing the finance of collective consumption using evolutionary computation

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ABSTRACT. On the market of collective goods, the state is a provider of services and the individuals are considered to be the beneficiaries. The correct resizing of public expenses for financing indivisible collective consumption is necessary mostly due to the unproductive character of these costs. The optimization of the public goods supply must take into account the individuals preferences for pure collective goods that are expressed as a collective preferences function. This function, together with the constraints resulting from budgetary equilibrium forms a maximization problem that can be solved using evolutionary algorithms.

1. INTRODUCTION

The proper sizing of public budget constitutes a real preoccupation for any public authority. The size of State's budget is a controversial problem especially for a country that had the experience of a centralized economy. This kind of economy means a substantial commitment from the part of the State. The passing to the market economy involve the reconsideration of the part of the State into the economically life.

The fact that the State's interference have a bound is an accepted idea; which is the threshold over that the commitment of the State is excessive is the remaining question. The level of the State's commitment into the economical environment can perturbs the mechanism of the market economy and becomes a problem. This problem is not usually discussed in connection with the commitment of the State as a provider of pure collective goods – national defense, public safety, justice. The proper sizing of State's expenditures with indivisible collective goods is desirable because of the unproductive nature of these expenditures.

Because the State is the provider of public services we can speak about a "market" of collective goods, therefore about the demanding of the individuals (population). The consumer of the public goods pays a masked price – the economics literature admits the income tax as a paid price for the individual's advantages after the State activity [11].

Most often we have a total management of taxes (together with the other incomes attract to the budget of the State) and it is not allowed to make a special link between a specific income and a specific expenditure of the State. Yet we make the assumption that the income tax due by the wage earners as a "price" paid for

Received: 28.05.2007. In revised form: 19.11.2007.
2000 *Mathematics Subject Classification*. 91B18, 91B16, 93B40.
Key words and phrases. *Public goods, Utility function, Computational methods.*

having access to the pure collective goods. We make the argumentation that, in Romania the pressure of taxation are supported by individuals not by the companies. Therefore, the resources made from the income taxes of natural person finance a large part of public expenditures (the income tax of the wage earners is 20.5% from the budgetary taxes, the income tax of the natural person is 22.4% and the profit tax is 17.06% - The State Budget 2004 [12]). In the meantime, we speak about non-exclusion by price that means that there are individuals who are not in debt tax, but benefit by collective goods. Non-exclusion by price and non-competition are the attributes of the pure collective goods. They make from the public authority the main provider of these goods. Each consumer uses the entire quantity of the collective good (benefits on the State's expenditure with the collective consumption) and pays the "price": the income tax paid by the society is the sum of all income taxes paid by each individual.

Comparing "the demand" and "the supply" we must have an optimal quantity of the collective goods to deliver.

Generally, the proper sizing of the supply supposes that the demand is known and in our case is also known the individuals' preferences vis-à-vis by the pure collective goods. So that, in order to optimize the collective expenditures financed through the State's budget, we follow three steps:
i) Knowledge of the real individuals' preferences vis-à-vis by the pure collective goods
ii) Determining the function of collective preference on the basis of the individuals' preferences
iii) Producing the optimal quantity of collective goods.

2. Economical problem

Let consider the utility function [9] written in the form $U_i(x, y_i)$, where x represents the consumed quantity of a pure collective good (expressed in monetary units), y_i is the annual income after taxes used for buying private goods, $i$denotes the individual and $i \in \{1, 2, ..., n\}$, n is the number of consumers from the national economy. The annual income after taxes y_i is computed assuming that the whole income is used for consumption without any savings, as follows: $y_i = z_i - t_i$, where z_i is the annual and global income before taxes and t_i represents the fiscal samplings.

The mathematical expression of the utility function can have the form:

$$U_i(x, y_i) = x^a \cdot y_i^b, a, b > 0, a + b = 1$$

where a and b show the individual's preference for a collective or private good, respectively.

Knowing the individual utilities obtained after the consumption of common and indivisible goods, a collective preference function (i.e. collective welfare function) can be deduced:

$$W = \sum_{i=1}^{n} \alpha_i \cdot U_i(x, y_i) \tag{2.1}$$

where α_i is the weight granted by the State to the satisfaction of individual i in the collective welfare function (the State is the bearer of the interests of certain groups).

The State's target is to maximize the collective welfare function W subjected to the constraint of the budget equilibrium (under the assumption that the State's functions are limited to the production of collective goods):

$$\begin{cases} W \to \max \\ x \leqslant \sum_{i=1}^{n} t_i \end{cases} \tag{2.2}$$

Contributions on the collective options problem has also Hoanţă N. [3], Lecaillon J. [5], Nemec& Wright [7] and Şerbu & Dragoş [10].

Our approach takes in consideration the wage earners from Romania (which pays or do not pay taxes) as the bearers of collective goods demands. In this aim we have made a classification of the wage earners in nine classes according to their income, their family situation (number of persons to support) and the OECD procedure [8, 13] (Table 1). Note that the nine classes considered may not cover all possible situations.

TABLE 1. The income distribution in nine classes

Class	Income (% from *APW**)	
1	33	Without persons to support
2	67	Without persons to support
3	100	Without persons to support
4	167	Without persons to support
5	67	1 child to support
6	100	1 child to support
7	67	2 children to support
8	100	2 children to support
9	100	2 children and a husband / wife to support

**APW* is the average production worker; in Romania in January 2004 *APW*=6,300,000 ROL [15, 14]

Table 2 comprises the elements used to find the annual income after taxes for each class allocated for acquiring of private goods (expressed in thousands ROL). The methodology applied for the annual income is the one that was valid in 2004. The option for this methodology is justifiable by our aim to prove the issues related to the progressive taxation system.

According to the above classification, the collective welfare function W becomes:

$$W = \sum_{i=1}^{9} \alpha_i \cdot x^a \cdot y_i^b \cdot p_i,$$

where p_i is the number of wage earners from the class i and

$$p_1 + p_2^{+} ... + p_9 = n; n = 9,283,000$$

at the end of 2003.

Supposing that the State is impartial we can consider that

$$\alpha_1 = \alpha_2 = \ldots = \alpha_9 = 1/9$$

hence the maximization problem (2.2) can be written in the form:

$$\begin{cases} W = \frac{1}{9} \cdot x^a \cdot \left(y_1^b \cdot p_1 + y_2^b \cdot p_2 + ... + y_9^b \cdot p_9\right) \to \max \\ p_1 + p_2 + ... + p_9 = n \\ a + b = 1 \\ x \leqslant p_1 \cdot AIT_1 + p_2 \cdot AIT_2 + ... + p_9 \cdot AIT_9 \end{cases} \tag{2.3}$$

The aim of this paper is to find the optimal quantity of collective goods offered by the State that best responds to individuals preferences (starting with the assumption of the uniform distribution of the wage earners in the nine classes). The results will be compared with the amounts already existent in the state budget from 2004. After we will determined $x$optimal, we will look for such distribution of wage earners which maximize the collective welfare function. For this distribution we will find a new value for x optimal.

TABLE 2. Elements used to find the annual income after taxes for each class

		1^{st} class	2^{nd} class	3^{rd} class	4^{th} class	5^{th} class	6^{th} class	7^{th} class	8^{th} class	9^{th} class
(1)	Gross income	2079	4221	6300	10521	4221	6300	4221	6300	6300
(2)	Contributions (CAS, CASS, FS)	353.43	717.57	1071	1788.57	717.57	1071	717.57	1071	1071
(3)	Personal expenses (Dpb)	2000	2000	2000	2000	2000	2000	2000	2000	2000
(4)	Professional expenses (15% Dpb)	300	300	300	300	300	300	300	300	300
(5)	Monthly net income $= (1) - (2) - (4)$	1425.57	3203.43	4929	8432.43	3203.43	4929	3203.43	4929	4929
(6)	Supplementary personal expenses (Dps)	0	0	0	0	1000	1000	2000	2000	3000
(7)	Annual and global income before tax $e = 12^*(5) - 12^*((3) + (6))$	negative	14441.16	35148	77189.16	2441.16	23148	negative	11148	negative
(8)	Annual income tax (AIT)	0	2599.408	6644.040	16692.965	439.408	4166.64	0	2006.64	0
(9)	Annual income after taxes y=12*(2.1)-12*(2.2)-(8)	20706.84	39441.752	56103.96	88096.195	41601.751	58581.36	42041.16	60741.36	62748

3. Evolutionary algorithms

During the last years there has been a growing interest in problem solving techniques based on the principles of evolution and hereditary. A lot of computational models have been proposed and studied. The domain covering all these techniques is called Evolutionary Computation (EC) and it is now considered to be a stable sphere of Artificial Intelligence. The techniques that model the evolutionary process are called evolutionary algorithms.

The most employed classes of evolutionary algorithms are: Genetic algorithms (Holland, 1975 [4, 6]), Evolutionary programming (Fogel, Owens and Walsh, 1966), Evolution strategies (Rechenberg, 1973), Genetic programming (Koza, 1992) [2, 1].

The common idea of all evolutionary algorithms is to evolve a population of candidate solutions to the problem using search operations inspired by biology, like recombination, mutation and selection. The evolution process takes place until a termination condition is met. The final population is expected to contain the best solutions for the given problem.

3.1. **Genetic algorithms.** Genetic algorithms were developed by John Holland (University of Michigan in Ann Arbor) in the early 1960s. Genetic algorithms evolve a population of chromosomes (candidate solutions). Here they are encoded as binary strings. The search operators are typically crossover, mutation and sometimes inversion. Chromosomes are evaluated using a fitness function.

Suppose we have a maximization problem: $f(x) \to \max$, $x \in \Omega$ where f is a real-valued function$f : \Omega \to R, \Omega \subset R^n$.

Genetic algorithms evolve a set of chromosomes that are encoded as binary strings in order to detect the maxima of the function. A fitness function is used in order to evaluate the chromosomes. The evolution process takes place for a number of generations until a termination condition is fulfilled. At each generation t a new population of chromosomes is created using genetic operators like selection, recombination and mutation.

The structure of the canonical genetic algorithm is presented here:

$t := 0$;
Generate randomly population $P(t)$;
Evaluate $P(t)$ by using a fitness function;
While not *termination_condition* **do**
begin
Select from $P(t)$ the individuals that will undergo changes using variation operators (recombination, mutation). Let P' be the set of selected individuals. Choose individuals from P' to enter the mating pool *(MP);*
Recombine chromosomes in *MP* forming population P'';
Mutate chromosomes in P'' forming P''';
Select for replacement from P''' and $P(t)$ forming $P(t+1)$;
$t := t + 1$;
end

Remarks:

The size of the population used is a parameter of the algorithm.

The termination_condition in Step 4 can be a maximum number of generation to be achieved, or some other condition on population $P(t)$.

Another decision that has to be made is which types of genetic operators to use for selection, recombination and mutation.

Genetic algorithms are also very useful for providing a practical insight into the problem. Often they are used to simulate certain real situations in order to better understand the dynamic of the problem. In this particular case we have used a genetic algorithm to simulate and to study the behavior of the collective welfare function.

4. Results

The distribution of the population into the nine classes can not be controlled by the state. This fact makes difficult the estimation of the function of the collective welfare. For theoretical reasons, in order to solve the problem, we started with the assumption that each class has an equal number of individuals

$$p_1 = p_2 = \ldots = p_9 = p.$$

Then the function W becomes

$$W = \frac{1}{9} \cdot p \cdot x^a \cdot (y_1^b + y_2^b + \ldots + y_9^b) \tag{4.4}$$

Consider also $a = 0.5$ and $b = 0.5$. If

$$p_1 = p_2 = \ldots = p_9 = p$$

and

$$p_1 + p_2 + ... + p_9 = 9283000$$

(the situation at the end of 2003), then

$$p = 9283000/9 \Rightarrow p = 1\,031\,444$$

By replacing p,$y_1, y_2, ..., y_9$ in (4.4), we have:

$$W(x) = 7\,335\,827\,736.12 \cdot x^a \tag{4.5}$$

subjected to

$$x \leqslant p_1 \cdot AIT_1 + p_2 \cdot AIT_2 + ... + p_9 \cdot AIT_9$$

The function (4.5) reaches the maximum value for x_{max} =33,572,590,429,555.60 and the maximum value is W_{max} =4.25051E+16.

Using these results we took different values for the parameters a and b and using a genetic algorithm we estimated the optimal distribution of individuals in classes (as we can see in Table 4). The parameters used to run the genetic algorithm are presented in Table 3.

TABLE 3. Parameters used to run the genetic algorithm

Nr. Crt.	Parameter	Value
	Number of generations	3,000
	Population size	200
	Crossover probability	0.5
	Mutation rate	0.05
	q-tournament selection	30

For each pair (a, b) the algorithm was run two times.

TABLE 4. Results

		p1	p2	p3	p4	p5	p6	p7	p8	p9	$\sum(p)$	x_{max}
a	0.25											
b	0.75											
VAL W	3.09E+15	36	20	9,244	6,188,700	1,475	50,925	2920	1998600	1031100	9283020	107,592,526,963,695
VAL W	2.78E+15	21	114,850	75,350	8,008,200	18,103	533,400	26877	63505	442700	9283006	136,837,644,806,127
a	0.33											
b	0.67											
VAL W	7.66E+15	1	1	4,052	7,194,600	1	415,820	114950	547560	1006000	9282985	122,957,458,721,097
VAL W	8.20E+15	1	631	14,044	8,251,600	1,260	97,606	1104	114600	802200	9283046	138,475,822,781,388
a	0.50											
b	0.50											
VAL W	1.12E+17	22	3,028	101,970	8,251,200	6	42,817	12747	300260	570950	9283000	139,203,275,961,958
VAL W	1.12E+17	1	1	114,680	8.251,200	4	28,369	158	89994	798270	9282977	138,802,732,532,064
a	0.66											
b	0.33											
VAL W	9.21E+17	1	1	94,420	8,251,600	12,720	107,430	27335	114610	674900	9283017	139,054,195,278,288
VAL W	8.40E+17	1	5	15,857	7,220,100	2	213,490	13	916840	916680	9282988	123,259,548,805,838
a	0.75											
b	0.25											
VAL W	3.62E+18	1	18	22,821	7,220,100	3	228,140	5663	774960	1031300	9283006	123,182,191,324,911
VAL W	4.02E+18	1	2	111,640	8,251,600	2	114,570	3023	458400	343800	9283038	139,882,632,418,034

For the cases: $(a,b) \in \{(0.25, 0.75), (0.5, 0.5), (0.75, 0.25)\}$ the estimated distribution of individuals in classes was represented in Figures 1-3. In the last column of Table 4 we presented the corresponding optimum value of x for each distribution of individuals in classes.

5. CONCLUSIONS

As we can see, if we have an uniform distribution of the wage earners into the nine classes, the collective welfare is maximum for an expenditure of collective goods about 33,572.590 billion ROL. On the other hand we can say that the consumption of a quantity of collective goods about 33,000 billions ROL brings to the wage earners the maximum satisfaction. In fact, this is that part of the public expenditures that they accept to finance through the income taxes. Also they understand the position of the State as a provider of the goods. Beyond this bound we speak about "a compulsion" as a modality of bringing income taxes to the budget for finance the respective goods. The sums from the State's Budget in 2004 are bigger then that optimal value: the expenditure for defense, public order and generally public services are about 99,183.8 billions (increasing: in 2002 around 62,000 billions, in 2003 around 84,000 billions). The comparison leads us to the conclusion of an overrating of the collective goods supply. There is a supply that has no answer in the demand; it does not maximize the collective satisfaction. We are tempted to say that it is a waste of public money but our analysis begins with a hypothetical situation that we have a uniform distribution of the wage earners in the nine classes. In the absence of real data of the distribution we looked for the optimum one in order to have the maximum of the collective welfare function, in the conditions given by the preferences of the wage earners for a public good or a private one (the a and b parameters). For each optimum distribution we found the new optimal values for x and we remark that the values are greater then those assigned in the budget. Even when the parameters a and b take different values (we have three situations: $a < b, a > b \ and \ a = b)$, the optimal value of x is very big and the optimal distribution indicates the predominance of the individuals with high income and without persons to support (4^{th} class). Analyzing the results we conclude that the collective welfare reaches a maximum in 2004 in Romania if 77-88% from the wage earners would have incomes about 167%*APW* (belonging to the 4^{th} class). We mention that our analysis also is not considering social aspects like birth rate, death rate, unemployment, retirement, etc. In addition, our analysis allows us to say that public services offer - at the actual dimensions - could find a sociological legitimacy only under the condition that the incomes of individuals increase and the fiscal pressure decrease. A first step was already done by the fiscal relaxation beginning with 2005, which involves the increasing of the available income after taxation and implicit the welfare of the individuals. Replacing the progressive taxation with the proportional one is a good thing, also because the progressive system was not adapted for the population structure on income cluster.

Finally, the mathematical and computational devices proposed in this paper are desired to be a helpful tool to be used in order to improve the economic decisions.

As further research, our aim is to identify the interfered mutation at the level of general welfare according to the last changes at the level of taxation.

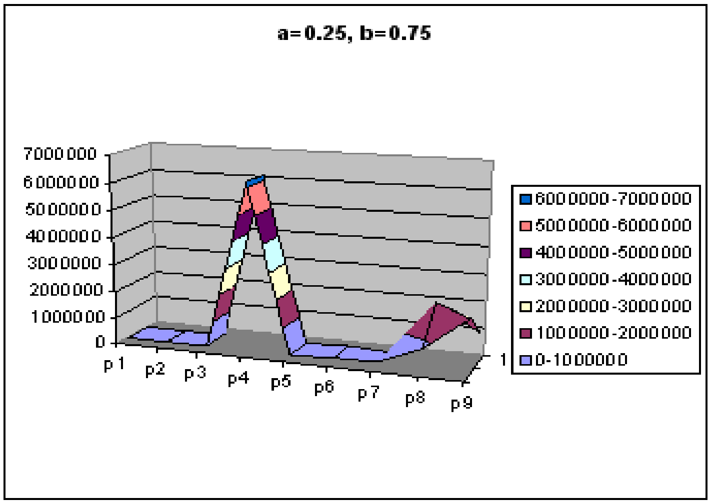

FIGURE 1

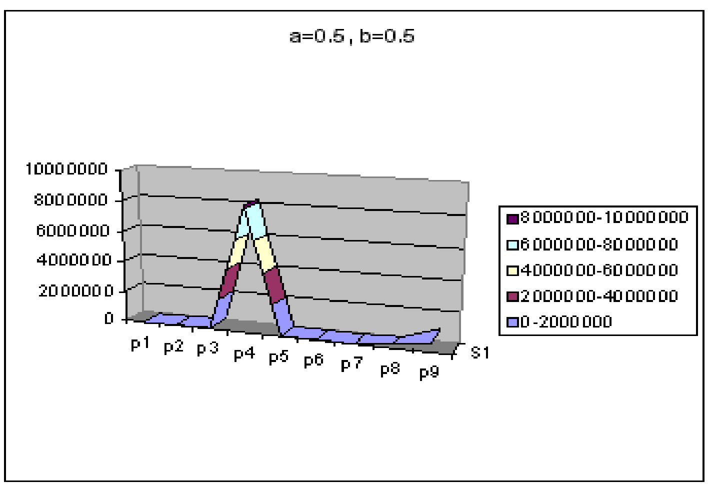

FIGURE 2

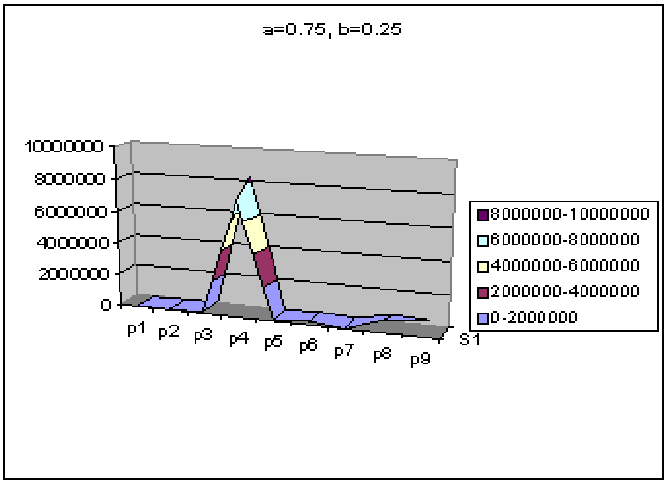

FIGURE 3

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