

## Right simple injective FGF-ring

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### ABSTRACT.

A ring  $R$  is called right FGF-ring if every finitely generated right  $R$ -module embeds in a free (projective). A ring is called right simple-injective if  $R_R$  is simple  $R$ -injective, that is, if  $I$  is a right ideal of  $R$  and  $\gamma : I \rightarrow R$  is an  $R$ -morphism with simple image, then  $\gamma(x) = c.x$ , is left multiplication by an element  $c \in R$ . There is a conjecture due to Carl Faith which asserts that every right FGF-ring is a Quasi-Frobenius ring ( $QF$ ). In this paper we establish the conjecture in case that the ring is a simple injective ring by showing that the right simple-injective FGF ring is a right self-injective.

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