

## Monotone semilinear equations in Hilbert spaces and applications

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### ABSTRACT.

Consider a abstract semilinear equation of the form  $Au + F(u) = 0$ , where  $A$  is a maximal monotone map acting into a real Hilbert space  $H$ , and  $F$  is a Lipschitz strongly monotone map on  $H$ . Such equations were studied by H. Amann (1982), T. Bartsch (1988), C. Mortici and S. Sburlan (2005, 2006), D. Teodorescu (2005). By standard arguments we can reformulate the problem as a fixed point equation and prove easier some existence results. Based on these abstract results some applications to partial differential equations are also appended. The method can be adapted for teaching PDE in Technical Universities.

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