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The order of convergence of some families of iterative methods for solving nonlinear equations

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ABSTRACT.
We focus on some families of iterative methods. A few particular cases of these families are: Newton's method, Halley's method, Euler's method, Osculating parabola method, Chebyshev's method and others. For each families we started with a definition, we continue with particular cases and examples. For each these families we get their convergence order. We also obtain for some polynomials the attraction basins of the studied methods.

1. INTRODUCTION

One of the most important problem in numerical analysis is solving non-linear equation. Let $f : \mathbb{C} \to \mathbb{C}$ be a differentiable or analytic function. We are interesting to approximate the root $z^* \in \mathbb{C}$ of the equation

$$f(z) = 0 \tag{1.1}$$

We suppose that f has simple roots, that is, if $f(\alpha) = 0$, then $f'(\alpha) \neq 0$. In order to obtain approximation of the roots of the equation (1.1), we shall consider another equation, of the form:

$$g(z) = z \tag{1.2}$$

where $g : D \to D$, $D = D(z^*, r) = \{z \in \mathbb{C} : |z - z^*| \leq r, r \in \mathbb{R}_+\}$.
For approximation of the roots of the equation (1.1) we shall consider an iterative method:

$$z_{s+1} = g(z_s),\ z_0 \in D, s = 0, 1, ... \tag{1.3}$$

In this paper we would analyse some families of iterative methods used for approximation of the roots of the equation (1.1). For methods of form (1.3) we are going to present the corresponding attraction basins obtained when we apply these iterative methods to the polynomials.

Definition 1.1. [1] Let z^* be an attracting fixed point of g, that is $g(z^*) = z^*$ and $|g'(z^*)| < 1$. Its basin of attraction is the set

$$A(z^*) = \{z \in \mathbb{C} : g^n(z) \to z^* \text{ as } n \to \infty\},$$

where $g^n(z) = g(g^{n-1}(z))$ is the n-fold iterate of g.

For these iterative methods we would also give the corresponding order of convergence. For proving the convergence of the methods from these families of methods, we would use the next results.

Corollary 1.1. [3] *Let $z^* \in \mathbb{C}$ be a root of equation (1.1). Let $g : D \subset \mathbb{C} \to \mathbb{C}$ be an analytic function such that $g(z^*) = z^*$ and $|g'(z^*)| = \alpha < 1$. Then there is a neighborhood U of z^* in D, such that $g|U$ is a contraction. Moreover, given $z_0 \in U$, the sequence $\{z_s\}_{s\geq 0}$ generated by (1.3) converges to z^*.*

Proof. For proof see [3]. □

Theorem 1.1. [3] *Let $g : D \subset \mathbb{C} \to \mathbb{C}$ be an analytic function and let z^* be a fixed point of g. If $g'(z^*) = g''(z^*) = ... = g^{(p-1)}(z^*) = 0$ and $g^{(p)}(z^*) \neq 0$, then the sequence $\{z_s\}_{s\geq 0}$ generated by (1.3) converges to z^* with the order of convergence p and the asymptotic error is $\frac{|g^{(p)}(z^*)|}{p!}$, that is, $\lim\limits_{s\to\infty} \frac{|z_{s+1}-z^*|}{|z_s-z^*|^p} = \frac{|g^{(p)}(z^*)|}{p!}$.*

Proof. See [3]. □

For the next theorem we are presenting a new proof.

Theorem 1.2. [5] *Let f be a polynomial function of degree n with n simple roots. Let h be an analytic function in a neighborhood of the roots of f. Then the iterative method $\varphi(z_s) = z_s - \frac{f(z_s)}{h(z_s)}$ has the order of convergence p if*
$h^{(i)}(z^*) = \frac{f^{(i+1)}(z^*)}{i+1}$ for $i = 0, ..., p-2$, where z^* is a root of $f(z) = 0$.

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Proof. By applying Theorem 1.1 for the iterative function $\varphi(z)$ we obtain that the method $\varphi(z_s) = z_s - \frac{f(z_s)}{h(z_s)}, s \geq 0$, has the order of convergence p if

$\varphi'(z^*) = \varphi''(z^*) = \varphi'''(z^*) = ... = \varphi^{(p-1)}(z^*) = 0$ and $\varphi^{(p)}(z^*) \neq 0$, where z^* is a root of $f(z) = 0$.

Now, $\varphi'(z^*) = 0 \Leftrightarrow 1 - \frac{f'(z^*)h(z^*) - f(z^*)h'(z^*)}{[h(z^*)]^2} = 0 \Leftrightarrow$

$\Leftrightarrow [h(z^*)]^2 - f'(z^*)h(z^*) = 0 \Leftrightarrow h(z^*) = f'(z^*) = \frac{f'(z^*)}{1}$,

$\varphi''(z^*) = 0 \Leftrightarrow -\frac{2f(z^*)[h'(z^*)]^2}{[h(z^*)]^3} + \frac{2h'(z^*)f'(z^*)}{[h(z^*)]^2} + \frac{f(z^*)h''(z^*)}{[h(z^*)]^2} - \frac{f''(z^*)}{h(z^*)} = 0 \Leftrightarrow$

$\Leftrightarrow \frac{2h'(z^*)f'(z^*)}{[h(z^*)]^2} - \frac{f''(z^*)}{h(z^*)} = 0 \Leftrightarrow h'(z^*) = \frac{f''(z^*)}{2}$.

Continuing in this way we obtain

$\varphi^{(p-1)}(z^*) = 0 \Leftrightarrow h^{(p-2)}(z^*) = \frac{f^{(p-1)}(z^*)}{p-1}$ and

$\varphi^{(p)}(z^*) \neq 0 \Leftrightarrow h^{(p-1)}(z^*) \neq \frac{f^{(p)}(z^*)}{p}$. □

Inspired by the Theorem 1.2 we have the next result.

Theorem 1.3. *If $q(z^*) = 0$ and $q'(z^*) = 1$, then the iterative method*

$$\psi(z_s) = z_s - \frac{q(z_s)}{2}(3 - q'(z_s)), s \geq 0, \tag{1.4}$$

has the order of convergence 3.

Proof. Define $h(z) = \frac{2}{3-q'(z)}$, then we get the iterative method

$\psi(z_s) = z_s - \frac{q(z_s)}{2}(3 - q'(z_s)), s \geq 0$. Now, applying Theorem 1.2, we have the order of convergence 3, since $h(z^*) = \frac{2}{3-q'(z^*)} = q'(z^*) \Leftrightarrow q'(z^*) = 1$ and

$h'(z^*) = \frac{2q''(z^*)}{[3-q'(z^*)]^2} = \frac{2q''(z^*)}{4} = \frac{q''(z^*)}{2}$ □

2. Some families of iterative methods

2.1. **Schröder's family of iterative methods [1], [4].** Let f be an analytic function, and let σ be a positive integer greater than or equal to two. For each $\sigma \geq 2$, define Schröder's iterative maps of order $\sigma = 2, 3, ...$ associated to f by:

$$S_{\sigma,f}(z) = z + \sum_{k=1}^{\sigma-1}[\frac{1}{k!}(\frac{1}{f'(z)}\frac{d}{dz})^{(k-1)} \cdot \frac{1}{f'(z)}](-f(z))^k \tag{2.1}$$

where $S_{\sigma,f}$ is an algorithm of order σ.

Particular cases:

• for $\sigma = 2$, $S_{2,f}$ is the Newton's iterative map of order 2;

The general form of this method is:

$$z_{s+1} = z_s - \frac{f(z_s)}{f'(z_s)}, s \geq 0 \tag{2.2}$$

For more details about this method see [7], [9].

• for $\sigma = 3$, $S_{3,f}$ is the Chebyshev's method, with the order of convergence 3, the general form of this method is:

$$z_{s+1} = z_s - \frac{f(z_s)}{f'(z_s)}\left(1 + \frac{1}{2}\frac{f(z_s)f''(z_s)}{[f'(z_s)^2]}\right), s \geq 0 \tag{2.3}$$

Regarding to the Theorem 1.3, we have the next result for the Chebyshev's method (2.3).

Theorem 2.4. *Let $q(z) = \frac{f(z)}{f'(z)}$, then the iterative (1.4) becomes the Chebyshev method $\psi(z_s) = z_s - \frac{f(z_s)}{f'(z_s)}(1 + \frac{1}{2}\frac{f(z_s)f''(z_s)}{[f'(z_s)]^2}), s \geq 0$, which is a third-order convergence iterative method.*

Proof. It is easy to see that the conditions of the Theorem 1.3 are satisfied, since:

$q(z^*) = \frac{f(z^*)}{f'(z^*)} = 0$ and $q'(z^*) = 1 - \frac{f(z^*)f''(z^*)}{[f'(z^*)]^2} = 1$, thus the order of convergence of the Chebyshev's method (2.3) is 3. □

Example 2.1. In Figure 2.1 (a) and Figure 2.1 (b) we show the fractal pictures that appear when we apply the Newton's iterative method and the Chebyshev's iterative method to find the roots of the polynomial $p(z) = z^3 - z^2 + z - 1$. The roots of p are 1, $-i$ and i. We take a rectangle $R = [-1.5; 1.5] \times [-2.5; 2.5]$, 150 points, a limit of 10 iterations and we assign a gray level to each point $(x_0, y_0) \in R$ according to the root at which the iterative methods starting from $z_0 = x_0 + y_0 \cdot i$ converge to the roots. We make the gray level lighter or darker according to the number of iterations needed to reach the root with tolerance 10^{-3} in a maximum of 10 iterations. We also mark with Black the points $(x_0, y_0) \in R$ for which the corresponding iterative methods starting in $z_0 = x_0 + y_0 \cdot i$ do not reach any root with tolerance 10^{-3} in a maximum of 10 iterations, these are the points for which the methods do not converge. In this way, we may distinguish the attraction basins by their colors. The region $A(\alpha)$ constitutes the basin of attraction of the root α ($\alpha = 1, i, -i$). See Annex.

2.2. **König's family of iterative methods [2].** Let f be an analytic function, and let σ be a positive integer greater than or equal to two. For each $\sigma \geq 2$, define König's iterative maps of order $\sigma = 2, 3, ...$ associated to f by:

$$K_{\sigma,f}(z) = z + (\sigma - 1)\frac{(\frac{1}{f(z)})^{(\sigma-2)}}{(\frac{1}{f(z)})^{(\sigma-1)}}, \tag{2.4}$$

where $K_{\sigma,f}$ is an algorithm of order σ.

Particular cases:

• for $\sigma = 2$, $K_{2,f}$ is the Newton's iterative map of order 2;

• for $\sigma = 3$, $K_{3,f}$ is the Halley's method;

The general form of this method is:

$$z_{s+1} = H_f(z_s) = z_s - \frac{1}{\frac{f'(z_s)}{f(z_s)} - \frac{f''(z_s)}{2f'(z_s)}}, s \geq 0 \tag{2.5}$$

Based on the Theorem 1.2, we have the next result for the method (2.5).

Theorem 2.5. *If f is twice continuously differentiable function with only simple roots and z^* is a root of the equation (1.1), then the Halley's iterative method (2.5) has the order of convergence 3 and the asymptotic error is*

$$\left|\frac{1}{4}\left[\frac{f''(z^*)}{f'(z^*)}\right]^2 - \frac{1}{6}\frac{f'''(z^*)}{f'(z^*)}\right|$$

Proof. It is easy to see that $H_f(z^*) = z^*$, and $H'_f(z^*) = H''_f(z^*) = 0$. Now to see that $H'''_f(z^*) \neq 0$, in general, we have that function $h(z) = f'(z)\left(1 - \frac{1}{2}\frac{f''(z)f(z)}{[f'(z)]^2}\right)$ satisfies the relations: $h(z^*) = f'(z^*)$, $h'(z^*) = \frac{f''(z^*)}{2}$ and $h''(z^*)=\frac{1}{2}\frac{[f''(z^*)]^2}{f'(z^*)} \neq \frac{f'''(z^*)}{3}$, then by the Theorem 1.2 we obtain that the order of convergence for the Halley's method is 3. The asymptotic error is follow from the Theorem 1.1 by considered the function $\varphi(z) = z - \frac{f(z)}{h(z)}$, that is $\left|\frac{\varphi'''(z^*)}{3!}\right| = \left|\frac{1}{4}\left[\frac{f''(z^*)}{f'(z^*)}\right]^2 - \frac{1}{6}\frac{f'''(z^*)}{f'(z^*)}\right|$. □

Example 2.2. In Figure 2.2 (a) and Figure 2.2 (b) we show the fractal pictures that appear when we apply the Newton's iterative method and the Halley's iterative method to find the roots of the polynomial $p(z) = z^3 - 8z^2 + z - 8$. The roots of p are 8, $-i$ and i. We take a rectangle $R = [-9.5; 15.5] \times [-9.5; 15.5]$, 212 points, a limit of 15 iterations and we assign a gray level to each point $(x_0, y_0) \in R$ according to the root at which the iterative methods starting from $z_0 = x_0 + y_0 \cdot i$ converge to the roots. We make the gray level lighter or darker according to the number of iterations needed to reach the root with tolerance 10^{-3} in a maximum of 15 iterations. We also mark with Black the points $(x_0, y_0) \in R$ for which the corresponding iterative methods starting in $z_0 = x_0 + y_0 \cdot i$ do not reach any root with tolerance 10^{-3} in a maximum of 15 iterations, these are the points for which the methods do not converge. In this way, we may distinguish the attraction basins by their colors. The region $A(\alpha)$ constitutes the basin of attraction of the root α ($\alpha = 8, i, -i$). See Annex.

2.3. **Laguerre's family of iterative methods [6].** Let f be an analytic function in some complex domain D with a simple or multiple zero z^* . For each υ ($\neq 0, 1$) define Laguerre's iterative maps associated to f by:

$$L_{\upsilon,f}(z) = z - \frac{\upsilon f(z)}{f'(z) \pm \sqrt{(\upsilon-1)^2[f'(z)]^2 - \upsilon(\upsilon-1)f(z)f''(z)}} \tag{2.6}$$

We can obtain some well-known methods by a suitable choice of the parameter υ:

• for $\upsilon = 0$, $L_{0,f}(z)$ is the Halley's iterative map;

• for $\upsilon = 1$, $L_{1,f}(z)$ is the Newton's iterative map;

• for $\upsilon = 2$, $L_{2,f}(z)$ is the Euler's iterative map, which has third order;

The general form of the Euler's method is:

$$z_{s+1} = z_s - \frac{2f(z_s)}{f'(z_s) + \sqrt{[f'(z_s)]^2 - 2f(z_s)f''(z_s)}}, \ s \geq 0. \tag{2.7}$$

For approximation of the roots of the equation (1.1) we shall also consider the iterative Laguerre method:

$$z_{s+1} = L_{n,f}(z_s) = z_s - \frac{n[f(z_s)/f'(z_s)]}{1 + (n-1)\sqrt{1 - \frac{n}{n-1}L_f(z_s)}}, L_f(z_s) = \frac{f(z_s)f''(z_s)}{[f'(z_s)]^2}, \tag{2.8}$$

$\upsilon = n = deg(f), n \geq 2$, generated by the Laguerre iteration function $L_{n,f}$.

Because the Laguerre method doesn't have "the problem of the initial value" the method converges no matter what initial value we have chosen. That is with exception of the roots of f'. This fact makes the method to be the most recommended method for determination of the approximation roots of the function f.

Based on the Theorem 1.2, in our next result we would analyse the convergence of the sequence $\{z_s\}_{s\geq 0}$ given by (2.8).

Theorem 2.6. *Let z^* be a root of a polynomial f of degree n with simple roots. Let $l(z) = \frac{f(z)}{f'(z)}$ and $h(z) = \frac{1+(n-1)\sqrt{1-\frac{n}{n-1}(1-l'(z))}}{n}$. Then the iterative Laguerre method $L_{n,l}(z_s) = z_s - \frac{l(z_s)}{\frac{1+(n-1)\sqrt{1-\frac{n}{n-1}(1-l'(z_s))}}{n}} = z_s - \frac{l(z_s)}{h(z_s)}$ has the order of convergence 3.*

Proof. Let z^* be a root of f. We have: $h(z^*) = 1 = \frac{l'(z^*)}{1}$, because $l'(z^*) = 1 - \frac{f(z^*)f''(z^*)}{[f'(z^*)]^2} = 1$; $h'(z^*) = \frac{l''(z^*)}{2\sqrt{1-\frac{n}{n-1}(1-l'(z^*))}} = \frac{l''(z^*)}{2}$ and $h''(z^*) = -\frac{n[l''(z^*)]^2}{4(n-1)[1-\frac{n}{n-1}(1-l'(z^*))]^{\frac{3}{2}}} + \frac{l'''(z^*)}{2\sqrt{1-\frac{n}{n-1}(1-l'(z^*))}} \neq \frac{l'''(z^*)}{3}$.
Thus the function h satisfies the condition of the Theorem 1.2, and because of that the convergence order of Laguerre method is three. □

Example 2.3. In Figure 2.3, we show the fractal picture that appear when we apply the $L_{3,p}$ iterative method to find the roots of the cubic polynomial
$p(z) = z^3 - z^2 + z - 1$. The roots of p are 1, $-i$ and i.
We take a rectangle $R = [-1.5; 1.5] \times [-2.5; 2.5]$, 150 points, a limit of 10 iterations and we assign a gray level to each point $(x_0, y_0) \in R$ according to the root at which the $L_{3,p}$ iterative method starting from $z_0 = x_0 + y_0 \cdot i$ converges to the roots. We make the gray level lighter or darker according to the number of iterations needed to reach the root with tolerance 10^{-3} in a maximum of 10 iterations. We also mark with Black the points $(x_0, y_0) \in R$ for which the corresponding iterative method starting in $z_0 = x_0 + y_0 \cdot i$ does not reach any root with tolerance 10^{-3} in a maximum of 10 iterations, these are the points for which the method does not converge. In this way, we may distinguish the attraction basins by their colors. The region $A(\alpha)$ constitutes the basin of attraction of the root α ($\alpha = 1, i, -i$). See Annex.

2.4. **Hansen-Patrick's family of iterative methods [8].** This family can be obtain quite easily from Laguerre's method by a choice of the parameter v, substituting $v = \frac{1}{\alpha} + 1$. Let f be an analytic function in some complex domain, and let α be a complex parameter. For each α ($\neq -1$) define Hansen-Patrick's iterative maps of order 3 associated to f by:

$$HP_{\alpha,f}(z) = z - \frac{(\alpha+1)f(z)}{\alpha f'(z) \pm \sqrt{[f'\ (z)]^2 - (\alpha+1)f(z)f''(z)}} \tag{2.9}$$

Particular cases:
• for $\alpha = 0$, $HP_{0,f}(z)$ is the Ostrowski iterative map;
The general form of the Ostrowski method is:

$$z_{s+1} = z_s - \frac{1}{\sqrt{1 - \frac{f''(z_s)f(z_s)}{[f'(z_s)]^2}}} \cdot \frac{f(z_s)}{f'(z_s)},\ s \geq 0 \tag{2.10}$$

Regarding to the Theorem 1.2, we have the next result for the Ostrowski method (2.10).

Theorem 2.7. *Let f be a polynomial of degree n with n simple roots. Let $l(z) = \frac{f(z)}{f'(z)}$ and $h(z) = \sqrt{l'(z)}$. Then the Ostrowski iterative method $\varphi(z_s) = z_s - \frac{l(z_s)}{h(z_s)}$ has the order of convergence 3.*

Proof. Let z^* be a root of f, we have:
$h(z^*) = 1 = \frac{l'(z^*)}{1}$, since $l'(z^*) = 1 - \frac{f(z^*)f''(z^*)}{[f'(z^*)]^2} = 1$;
$h'(z^*) = \left(\sqrt{1 - \frac{f(z^*)f''(z^*)}{[f'(z^*)]^2}}\right)' = \frac{-\frac{f''(z^*)}{f'(z^*)} + \frac{2f(z^*)[f''(z^*)]^2}{[f'(z^*)]^3} - \frac{f(z^*)f'''(z^*)}{[f'(z^*)]^2}}{2\sqrt{1-\frac{f(z^*)f''(z^*)}{[f'(z^*)]^2}}} = \frac{l''(z^*)}{2}$ and
$h''(z^*) = \frac{5}{4}\left[\frac{f''(z^*)}{f'(z^*)}\right] - \frac{f'''(z^*)}{f'(z^*)} \neq \frac{l'''(z^*)}{3}$. Thus function h satisfies the condition of the Theorem 1.2, so the convergence order of Ostrowski method is three. □

• for $\alpha = 1$, $HP_{1,f}(z)$ is the Osculating parabola method of which the order of convergence is three.
The general form of the Osculating parabola method is:

$$z_{s+1} = z_s - \frac{2}{1 + \sqrt{1 - 2\frac{f''(z_s)f(z_s)}{[f'\ (z_s)]^2}}} \cdot \frac{f(z_s)}{f'(z_s)},\ s \geq 0 \tag{2.11}$$

• for $\alpha = \infty$, we obtain the Newton's iterative map;

Example 2.4. In Figure 2.4 (a) and Figure 2.4 (b) we show the fractal pictures that appear when we apply the Ostrowski iterative method and the Osculating parabola iterative method to find the roots of the polynomial $p(z) = z^3 - 3.5z^2 + 5z - 3$. The roots of p are $1 - i$, $1 + i$ and 1.5. We take a rectangle $R = [-2.5; 2.5] \times [-2.5; 2.5]$, 200 points, a limit of 10 iterations and we assign a gray level to each point $(x_0, y_0) \in R$ according to the root at which the iterative methods starting from $z_0 = x_0 + y_0 \cdot i$ converge to the roots. We make the gray level lighter or darker according to the number of iterations needed to reach the root with tolerance 10^{-3} in a maximum of 10 iterations. We also mark with Black the points $(x_0, y_0) \in R$ for which the corresponding iterative methods starting in $z_0 = x_0 + y_0 \cdot i$ do not reach any root with tolerance 10^{-3} in a maximum of 10 iterations, these are the points for which the methods do not converge. In this way, we distinguish the attraction basins by their colors. The region $A(\alpha)$ constitutes the basin of attraction of the root α ($\alpha = 1.5, 1 + i, 1 - i$). See Annex.

3. Annex

In this section we explain how the Figures of this paper were generated. First, we define function f and its derivatives. We use a procedure that identifies which root of f has been approximated with a tolerance of 10^{-3}, if any. We define the iterative methods, that is, the different $z_{s+1} = g(z_s),\ s \geq 0$. We define an algorithm to see if a root is reached in a maximum of limit iterations. We use a limit of 10-15 iterations and the complex rectangle R. Finally, we define the procedure to paint the figures according the strategy described in the each example. A gray level is used to identify the attraction basins of each root. To assign the intensity of a gray level of a point, we take into account the number of iterations used to reach the root when the iterative method starts at that point. The points for which the iterative method does not reach any root (with the desired tolerance in the maximum of iterations) are pictured as black.

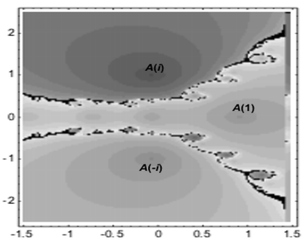

Figure 2.1 (a) Newton's method

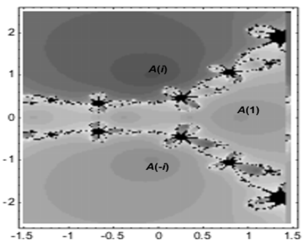

Figure 2.1 (b) Chebyshev's method

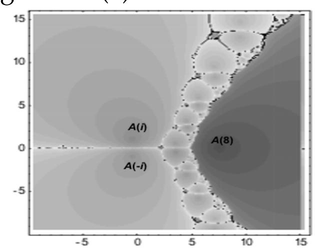

Figure 2.2 (a) Newton's method

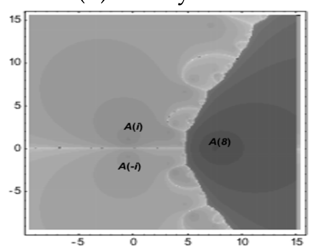

Figure 2.2 (b) Halley's method

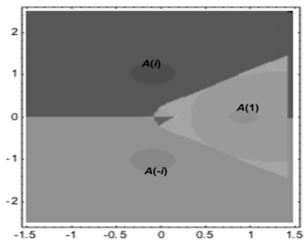

Figure 2.3 $L_{3,p}$ method

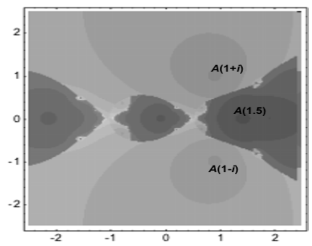

Figure 2.4 (a) Ostrowski method

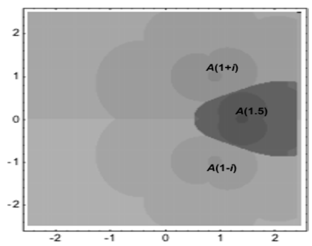

Figure 2.4 (b) Osculating parabola method

References

[1] Amat, S., Busquier, S. and Plaza, S., *Review of some iterative root-finding methods from a dynamical point of view*, Scientia, Series A: Mathematical Sciences, Vol. 10 (2004), 3-35

[2] Buff, X. and Henriksen, C., *On the König root-finding algorithms*, Nonlinearity 16 (2003), 989-1015
[3] Cira, O., *Metode numerice pentru rezolvarea ecuaţiilor algebrice*, Academia Română, Bucureşti, 2005
[4] Drakopoulos, V., *Schröder iteration functions associated with a one-parameter family of biquadratic polynomials*, Chaos, Solitons and Fractals, 13 (2002), pp.233-243
[5] Hasan, M. A., *On the derivation of higher order root-finding methods*, Proceedings of the 2007 American Control Conference Marriot Marquis Hotel at Times Square, New York City, USA, July 11-13, 2007
[6] Osada, N., *A one parameter family of locally quartically convergent zero-finding methods*, Journal of Computational and Applied Mathematics, 2007, 205 (1), 116-128
[7] Păvăloiu, I. and Pop, N., *Interpolare şi aplicaţii*, Editura Risoprint, Cluj-Napoca, 2005
[8] Petković, L. D., Petković, M. S. and Živiković, D., *Hansen-Patrick's family is of Laguerre's type*, Novi Sad J. Math., Vol. 33, No. 1, 2003, 109-115
[9] Pomian, R. A., *Metoda lui Newton*, Didactica Matematicii, Vol. 24 (2006), 161-186

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