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Developmental teaching experiment in the field of geometry

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ABSTRACT.
Our research question is how lower primary geometry teaching in Hungary, particularly the concept of symmetry is related to the levels formulated by van Hiele. Moreover to what extent are the concrete activities effectively carried out at these levels in evolving the concept of symmetry. Our hypothesis is that in the lower primary geometry teaching (classes 1-4) the first two stages of the van Hiele levels can be put into practice. By the completion of lower primary classes level 3 cannot be reached. Children do not see the logical relationship between the properties of a given shape. They cannot come to a conclusion from one property of shapes to another.
In the lower primary the basics of geometrical concepts are laid down. In this paper the development of the concept of symmetry is examined. The evolvement of several geometrical concepts - among which the concept of symmetry as well - were examined in educational development experiments conducted with fourth class students.
In our paper we present the developing teaching experiment and its observations which we support with measurement results.

1. INTRODUCTION

Teaching geometry in Hungary in the first four grades of primary school aims at laying the ground to establish the skills through which learners can prepare for gaining knowledge on their own.

The basis of learning geometry is inductive cognition based on gaining knowledge. Starting out from the concrete and gathering experience from various activities will finally lead to the formulation of general relationships. The third educational principle laid down by Farkas Bolyai also emphasizes the importance of starting with the concrete: "(The teacher)... should always start with what learners can see and touch, and not with general definitions (it is not grammar that the first utterance is based on) and he should not torture prematurely with longwinded reasoning... We should start with geometric shapes and reading... and we also should get out of the sheet... " (Dávid, 1979)

The evolvement of some geometric concepts, has been examined in a teaching experiment in grade four, and the experience gained there is described below. In this paper the evolvement of the concept of symmetry is examined. Symmetry is a phenomenon that can often be met in nature. Thus, we can introduce the concept of geometrical symmetry relying on the natural forms of symmetry. Symmetry has an important role in our daily life as well (e.g. architecture, interior design, technology etc.). This is why it has been included into the lower primary syllabus as an element of great importance. According to the frame curriculum in the field of geometry the requirements related to symmetry are as follows:

Grade 1: Gaining experience in a playful way using plane mirror.

Grade 2: Constructing reflection. Observing simple reflection and producing reflected images.

Grade 3: Producing axially reflected shapes through activities. Reflections.

Grade 4: Axial reflections.

As it can be seen, in the lower primary in terms of symmetry out of transformations we are concerned with the reflection in space related to plane and in plane we are concerned with axial reflection. We mean the symmetrical characteristics of solids related to plane and in case of plane figures we mean axial symmetry. It is the requirement of the upper primary classes to teach the rotational and centrally symmetric characteristics of shapes.

2. THEORETICAL BACKGROUND

2.1. **The levels of geometrical thinking according to van Hiele.** Young children start gaining knowledge in geometry already in kindergarten where the concept of geometrical objects (geometrical solids, plane figures) is being established by examining the shapes of the objects in the environment. Establishing the characteristics for the set of these objects implies a higher degree of gaining knowledge. A large amount of references can be found on gaining geometrical knowledge, but in this particular case we rely on van Hiele.

According to P-H van Hiele the process of gaining geometrical knowledge can be divided into five levels.

At the level of global cognition of shapes (level 1.) children perceive geometrical shapes as a whole. They easily recognize various shapes according to their forms, they remember the names of the shapes however they do not understand the relationship between the shape and their components. They do not recognize the rectangular prism in the cube, rectangle in the square, because these are totally different things for them.

At the level of analyzing shapes (level two) children break shapes down into components and then put them together. They also recognize the faces, edges and vertices of geometrical solids as well as the plane figures of geometric solids which are delineated by curves, sections and dots. At this level particular importance is attached to

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observation, measurement, folding, sticking, drawing, modelling, parquetry, and using mirrors. By means of these concrete activities children can establish and enlist the characteristics of shapes such as the parallelism, perpendicular of faces and sides, characteristics of symmetry, the presence of right angle etc, but they are not able to define and to recognize the logical relationships between the characteristics. At this level children do not perceive the relationships between shapes.

At the level of local logical arrangement (level 3) learners are able to find relationships between the characteristics of a particular shape or between various shapes. They can also make conclusions from one characteristic of shapes to the other. They understand the importance of determination, definition. However the course of logical conclusions is set by the course book or the teacher. The need to prove things is started, but it applies only to shapes.

Level four (making efforts to reach complete logical set-up) and level five (axiomatic set-up) belong to the requirements of secondary and tertiary education.

In the van Hiele model each learning stage is constructed and enlarged by the thinking established by the previous stages. Transition from one level to the other happens continually and gradually, while children are acquiring the mathematical terms according to the particular levels. This process is particularly influenced by teaching, especially its content and method. For the suitable geometrical thinking none of the levels can be omitted. Every level has its own language, system of notation and logical set-up. From educational point of view it is highly relevant in the theory of van Hiele that we cannot expect from learners at a lower stage to be able to understand the instructions formulated in terms of a higher level. According to van Hiele this is probably the most frequent reasons for failures in mathematics teaching.

2.2. **Concept formation.** During the formation of a mathematical concept, the concept has to be fitted into the system of concepts established before (assimilation) but it can happen that the modification of the existing system or pattern is necessary for the fitting of the new concept. The balance of assimilation and accommodation is absolutely indispensable for the proper formation of concept. If this balance is upset i.e. assimilation is not followed by accommodation then the learners' own interpretations find their way into their mathematical knowledge, which later on may lead to misconceptions. Then the concepts formed in this way can be vague and inaccurate.

Teaching geometric concepts is as a matter of fact a long process. The principle of progressiveness should be observed, and accurate definitions should be established but not by all means. Sometimes even at lower primary classes definitions are provided in course books, although learners lack the required experience and abstraction level. In this respect, what R. Skemp the mathematician and psychologist said is:

"By means of definitions it is impossible to transmit concepts to anyone which are at a higher level than his knowledge, only by providing plenty of proper examples. Since in mathematics these examples mentioned above are almost all of them various concepts, therefore we have to make sure that the learners have already acquired there concepts. Selecting the proper examples is a lot more difficult than we suppose. The example should possess those common characteristics which make up the concepts, but they should not have any other common characteristics." (Skemp,1975)

The evolvement of scientific concepts, such as symmetry is based on education. According to his observations Vigotsky came to the conclusion that " in as much as the progress of teaching contains the proper elements of the curriculum, the development of scientific concepts will proceed the development of spontaneous concepts." In the progress of teaching the special co-operation of children and adults and the transmission of the teaching material in an order can give an account for the premature achievement of concepts. According to teaching experience it can be understood that the direct teaching of concepts is not really possible. The mere acquisition of a new word verbally covers only emptiness. In this case children acquire only words and not concepts. When children first recognize the meaning of a new word then the process of evolvement of a concept is being started. Scientific concepts are not acquired and learned 'ready-made' by children but these concepts are evolved and established through the active thinking of children. The evolvement of spontaneous and scientific concepts is closely related to each other. A basic requirement for the evolvement and acquisition of scientific concepts is the proper level of spontaneous concepts. However the evolvement of scientific concepts can also have an influence on the development of spontaneous concepts.

3. RESEARCH QUESTION

The research question raised is that what is the relationship between lower primary geometry teaching including the teaching the concept of symmetry and the geometric levels according to van Hiele. Furthermore how efficiently the concrete activities at these levels contribute to the establishment of the concept of symmetry.

4. HYPOTHESES

In lower primary (grades 1-4) geometry teaching can reach the first two stages of geometric thinking according to the van Hiele levels. It is not feasible to reach level 3 by the completion of lower primary. Although sets of concepts are established, but there is no relationship whatsoever between them. Actually children do not recognize the logical relationships between the characteristics of a shape and they are not able to draw conclusions from one characteristics of a shape to another.

5. RESEARCH BACKGROUND

It was in May-June 2006 that the teaching experiment was carried out whose content and method was devised by the author, who also was involved in the lessons. The teacher was a mathematics teacher and supervisor in class 4.c of the Practice School of József Eötvös College in Baja, whose job was assisted by the author in presenting, modeling and eventually raising supplementary questions or alternative explanations. Both of them helped the pupils in carrying out and solving the tasks in individual or pair work. Both checking and evaluation were done in cooperation. During the developmental teaching the evolvement of several geometrical concepts such as rectangles, squares, parallel, perpendicular and symmetry was examined but here we are going to present the formation of the concept of symmetry.

The developmental teaching experiment included 16 lessons and the aim was to put the van Hiele model of geometry into practice. In the first lesson a pre-test was done by 26 pupils of class 4 so that we could see that the transition from level 1 (the global recognition of shapes) to level 2 (the analysis of shapes) and the further development of geometric thinking is feasible. When compiling the pre-test, the syllabus of class 3 and the comments of the mathematics teacher were taken into consideration. The first lesson of the development teaching experiment was also the first lesson of the geometry topic as well.

In May-June 2008 we did the developmental teaching experiment repeatedly in class 4.b of the Practice School of the Eötvös József College, having taken into consideration the experience gained during the 2006 experiment. In the first lesson a pre-test was done again to which we added a further exercise connected to symmetry, a kind of exercise that hadn't been included in the 2006 experiment.

6. PRE-TEST

Task 4 in the pre-test is concerned with the axial symmetry of plane figures. The task for the children was to decide whether the eight plane figures are symmetrical, they checked it with a mirror, and they marked the place of mirror axis and the axis with red.

The following plane figures were examined:

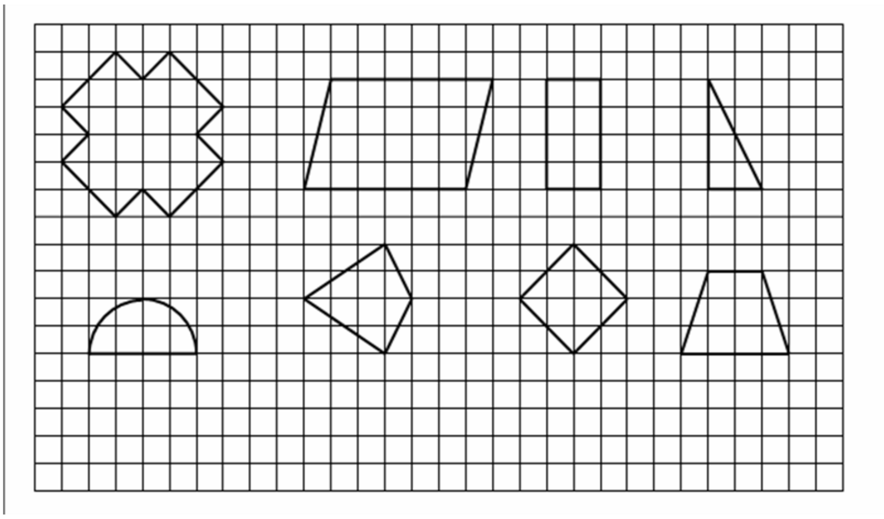


Figure 1.

The results of the task are shown in the table below:

In 2006:

All the mirror axes were drawn	26,9%
In case of plane figures with 4 mirror axes only 2 mirror axes were found but no more mistakes were made.	34,6%
Mirror axis was drawn for the general parallelogram	11,5%
Mirror axis was drawn for the triangle	7,7%
Five or more than 5 mistakes were made	34,6%

In 2008:

All the mirror axes were drawn	26,3%
In case of plane figures with 4 mirror axes only 2 mirror axes were found but no more mistakes were made.	15,8%
Mirror axis was drawn for the general parallelogram	31,6%
Mirror axis was drawn for the triangle	10,5%
Five or more than 5 mistakes were made	36,8%

As it is shown by the data gained in 2006 (73,1% committed some mistake concerning axial symmetry) and 2008 (73,7% committed some mistake concerning axial symmetry), more development of the concept is required in case of symmetry. The number of mistakes indicates that further on for the proper formation of concepts more attention should be paid to the presentation and discussion of proper examples and counter examples. By proper examples I mean the right amount of examples and counter examples on the one hand and their variety on the other (E.g. the learners should examine the symmetry of a variety of regular and non regular geometrical figures). Moreover the essential characteristics pertaining to the concept should be recognizable, and the unimportant ones should be noticeable for the children.

In 2008 in the fifth exercise of the pre-test, the pupils had to mirror 2 geometrical figures on given axes drawn on square grid. The ratio of faultless results was 68,4% in the case of the first and 73,7% in the case of the second mirroring.

7. TEACHING EXPERIMENT

When compiling the teaching material the principle of gradualness was observed and the problems were made more and more difficult. During the first lessons we focused on the characteristics of rectangular solids and cubes. Children were introduced to symmetry during the third lesson of development teaching. They produced the mirror image of solids made from match boxes then among others they determined the place of symmetry planes of various solids. First in the plane the mirror image of a stain was examined after folding a coloured sheet of paper. Then the children cut symmetrical shapes from paper and tried to find the symmetry axes of plane figures by means of folding and mirrors and on square grid they mirrored the given shape on a given axis.

The detailed description of the lessons can be found below:

Lesson 1: Pre-test

Lesson 2 : Naming and describing rectangular objects, such as matchboxes, cupboards etc, the number of vertexes, edges and sides, comparing the length of edges, the shape and the size of the sides, understanding what opposite and neighboring sides are and their position. Naming and describing cubic objects: the number of vertexes, edges and sides, comparing the length of the edges, the shape and size of sides, understanding what opposite and neighboring sides are and their position.

Lesson 3: Giving a list of the characteristics of rectangular prisms and cubes by means of models. Making up various rectangular prisms using four matchboxes. Producing the reflections of the solid made from matchboxes. Finding objects in symmetrical arrangement in the classroom. Listing symmetrical objects. Defining the position of the planes of symmetry in case of various solids.

Lesson 4: Defining the position of the planes of symmetry in rectangular prisms and cubes. By using a model, learners studied the parallel and perpendicular position of the opposite and the neighboring sides of rectangular solids, regular pentagon prisms, quadrilateral pyramids. Spreading rectangular prisms and cubes, examining the shape and size of the sides. Cutting squares from rectangles.

Lesson 5: The various grids of cubes. Studying the rectangles. The number of vertexes, opposite and neighboring vertexes, diagonal. Cutting the rectangle into two along the diagonal. Producing other plane figures by fitting the triangles gained in this way, and naming them. Gathering experience on plane figures and describing them. Further study of the rectangles: the number of sides, comparison of their length, determining the opposite and neighboring sides, the parallel position of the opposite sides, the perpendicular position of the neighboring sides. Measuring the sizes of angles by means of folded right angles.

Lesson 6: Studying the characteristics of plane figures made from two congruent right-angled triangles during the previous lesson: the number of vertexes and sides, defining the opposite and neighboring vertexes and sides, comparing the length of the sides, studying the parallel and perpendicular position of the opposite and the neighboring sides, the size of the angles. Comparing the characteristics of rectangles and parallelograms and highlighting their differences. Studying squares: the number of vertexes, opposite and neighboring vertexes, the diagonal. Cutting the square into two parts along the diagonal. Producing plane figures from the two right-angled isosceles triangles. Further study of squares: the number of sides, comparing their length, opposite and neighboring sides, the parallel and perpendicular position of the opposite and neighboring sides, the size of the angles.

Lesson 7: Demonstrating parallel and perpendicular pairs of straight lines as well as straight lines which are not parallel and perpendicular. Producing plane figures cut out from paper without restriction, and describing their characteristics. Listing the characteristics of rectangles and squares. Producing planes figures from the 2, 3, 4 and

6 regular triangles from the set of logics, which consists of 48 various plane figures, which can be red, yellow, blue or green. Their sizes are, small or large, their shape can be circle, square or triangle, their surface can be smooth or there is a hole in them. Making observations on parallel pairs of sides. Producing rectangles of different length and identical height from strips of paper.

Lesson 8: Producing various plane figures from paper strips by one cut. Naming them and describing their characteristics and shared characteristics. Cutting general rhombus from rectangle, its characteristics. Cutting general deltoid from rectangle, and its characteristics. Making rectangles and then the "frame" of a general parallelogram from six match sticks. Making squares then general rhombus from four match sticks. Comparing the characteristics of squares and rhombuses.

Lesson 9: Comparing the characteristics of squares and rectangles. Making 2 rectangles, a pentagon and a triangle, a triangle and a quadrangle, 2 quadrangles and 2 triangles from a rectangle by one cut. Drawing squares on square grid.

Lesson 10: Drawing various quadrangles on square grid. Drawing various triangles on square grid. Drawing parallel and perpendicular pairs of straight lines.

Lesson 11: Coloring the parallel pairs of sides of the quadrangles drawn on grid and designating the right angles. In triangles coloring the sides perpendicular to each other. Drawing quadrangles according to given requirements. Studying the structure of the edges of rectangular prism and cubes. Observing the parallel and perpendicular edges.

Lesson 12 : Producing reflection on plane through activity : folding a painted sheet of paper, on a black photographic paper folded into two making a pattern by running a pin through it, then unfolding it holding it in the direction of light. Cutting a given pattern from a sheet of paper folded into two parts. Observing reflections. On grid reflecting given figures on given axis. Producing figures symmetrical on axis by clipping.

Lesson 13: Finding the symmetry axes of plane figures cut out from paper by means of folding and mirror. Formulating experiences and observations. Drawing plane figures which have no symmetry axis, and which have exactly 1, 2, 3 and 4 symmetry axes.

Lesson 14: Producing figures symmetrical on axis on square grid. Selecting plane figures according to given characteristics. Formulating statements "every" and "there is such…"

Lesson 15: Selecting plane figures according to given characteristics. Establishing the logical validity of statements. Drawing plane figures according to given requirements. Twenty questions.

Lesson 16: Post test

When designing the lessons what we considered of utmost importance was that children could discover geometrical concepts first through concrete experience in real games and activities, later at visual level (drawing) then at an abstract level.

7.1. **Activities with concrete objects.**

a) Producing the reflection of solids made from match boxes.

b) Building symmetrical solids.

c) Determining the place symmetry planes of rectangular solids, cubes, spheres and cylinders.

d) On folded black photographic paper running a pin into contour of a 'moon' drawn earlier, then holding the paper turned out against the light to discover the reflection.

e) Producing axial symmetric shapes by means of clipping.

f) In case of plane figures finding the reflection axis by means of folding and mirrors etc.

During every lesson minutes were taken and the children's responses were also put down. When doing task a) children worked in pairs. A pencil laid in the middle of the desk signified the place of the imaginary mirror. One of them built something from three matchboxes and the other child had to produce its reflection. The following conversation took place between the author and the child called Martin:

- Let's check with a mirror whether you have really built the reflection. Look into the mirror! Is the reflection as far as the one that you have built?
- No, the reflection is nearer.
- What should we change?
- To push it (what I built) nearer to the mirror.

When defining the symmetry planes of rectangular prisms the teacher provided the following instructions:

"Hold the matchbox in your hands and draw a mark on the cut where we could cut the matchbox and we could put one half of it on the mirror along the cut and we could actually see the whole matchbox. If you find more put a mark to all of them."

There were four or five children who intended to cut along the diagonal but after checking it with a mirror they realized that it was not a good idea. In order to clarify things the teacher cut a rectangular prism shaped sponge along one diagonal then she put one of the pieces on the mirror where it was cut. Looking into the mirror the children were able to realize that what they saw was not the shape of the real sponge.

In task f) 14 plane figures were examined: regular triangles, general triangles, isosceles triangles, general trapezoids, symmetrical trapezoids, general parallelograms, rhombuses, rectangles, squares, convex deltoids, concave deltoids, general pentagons, regular hexagons and circles. The children carried out the task on their own and checked it together. After finding the two reflection axes of the general rhombus checking it with a mirror and folding, we dealt with the 2 reflection axes of the rectangle and the 4 reflection axes of the square. To the question raised by the teacher as to what plane figures the square is related to they came up with the rectangle and rhombus as well. At this point on child called Szabolcs came to the following conclusion: ”The square got it from the rhombus that the diagonals are reflection axes and the other two was inherited from the rectangle.” Then the reflection axes of the isosceles, equilateral and general triangles were examined, and it was again Szabolcs who responded to the equilateral triangle like this: “The square has four vertices, four sides and four reflection axes, and this triangle has three vertices, three sides and three reflection axes.” The teacher responded like this: “This triangle has three reflection axes but the former one (she pointed to the equilateral triangle) has only one. Why? “ Szabolcs answered the question like this: “This one has three – and he pointed to the equilateral triangle – because all its three sides are of equal length, whereas the other one has only two sides of equal length.” When examining the general triangle through several folding attempts the children came to the conclusion that it has no reflection axis. The teacher asked Szabolcs: “Why is it that we could not find reflection axis in this triangle?” Then Szabolcs explained: “Because all its sides are of different length.” When examining the general and symmetrical trapezoids, the teacher again asked Szabolcs: ”Why is not there a reflection axis in this quadrangle, - she pointed to the general trapezoid - and why is there in the other one – she pointed to the symmetric trapezoid?” Szabolcs pointed to the two sides of the general trapezoid and said: “These are of different length and in the other one they are of equal length .” When looking for the symmetry axes of the general pentagon Szabolcs said without being asked: ”We do not even have to fold it, it is so much visible that all its sides are of various length and therefore it has no reflection axis.” When looking for the reflection axis of a circular sheet, both children and the teacher produced several reflection axes by means of folding, which they marked with red pencil. When the teacher showed her own paper one of the children immediately said that it was possible to draw on it more. The teacher asked: ”Where could I draw ?” The child said: “Where it is still white. The whole circular sheet should have been coloured red”.

7.2. **Tasks at visual level.**
a) Drawing plane figures, which have no symmetry axes, and which have 1, 2, 3 and 4 symmetry axes.
b) Drawing the reflection of plane figures
c) Selection of plane figures according to given characteristics
d) Correction of symmetry axes of plane figures drawn in the wrong way etc.

7.3. **Abstract level.** After gaining experience at the previous two levels the characteristics of various geometrical shapes were summarized at an abstract level:

-in case of solids, especially cubes and rectangle prisms counting the number of faces, edges and vertices repeatedly, determining the length of edges, the parallel and perpendicularity of faces and edges, and the number of symmetry planes.

-in case of polygons, especially squares and rectangles counting the sides and vertices repeatedly, examining the length and parallel and perpendicularity, determining the number of symmetry axes, and the size of angles produced by the neighbouring sides. Obviously the geometric characteristics were studied through models or the visual representation of the given shape.

Twenty questions is one of the favourite games among children, which is also suitable for practicing the characteristics of solids and plane figures. During a game what the children had to guess was the symmetrical trapezoid. These are the questions and answers of a game:

-Is it a quadrangle?
-Yes, it is.
-Are the opposite sides parallel?
-No, they aren’t.
At this point the teacher realized that child need some help.
-We can as the question in another way: Are all the opposite pairs of sides parallel? And I said no, they aren’t.
-Does it have parallel sides?
-Yes, it does.
-Does it have a right angle?
-No, it doesn’t
-Are its sides of the same form?
-We can also put the question like this, said the teacher. Are all the sides equal? No, they aren’t.
-Does it have sides of equal length?
-Yes, it does. Anyone, who already knows it, can draw it. Those who don’t, keep on asking.
-Is it symmetrical?
-Yes, it is.
-Does it have one reflection axis?
-Yes, it does.
Then we drew the plane figure on the board.

During the game of twenty questions we wanted the children to realize that instead of just guessing it is a good strategy to limit the options. We had to convince them that they should not be afraid of asking questions and they should also see that some questions are more purposeful than others and 'no' to a good question is just as good as a 'yes'. Moreover it is no use asking a question when they already know the answer.

8. THE TASKS OF THE POST-TEST AND EVALUATION

In the 2006 developmental teaching experience was completed by an evaluation worksheet, which was filled in by 25 learners in class 4.c, 23 learners in class 4.a and 24 learners in class 4.b respectively. In these latter two classes the mathematics teacher – supervisor was another teacher. In 2008 19 learners belonging to the experimental group, and 22 learners (from 4.a) and respectively 15 learners (from 4.c) belonging to the control groups, did the evaluation worksheet.

In one of the tasks as it happened in the pre-test, children had to decide that out of the given plane figures which were symmetrical and they marked the reflection axis with red pencil.

The following plane figures were examined:

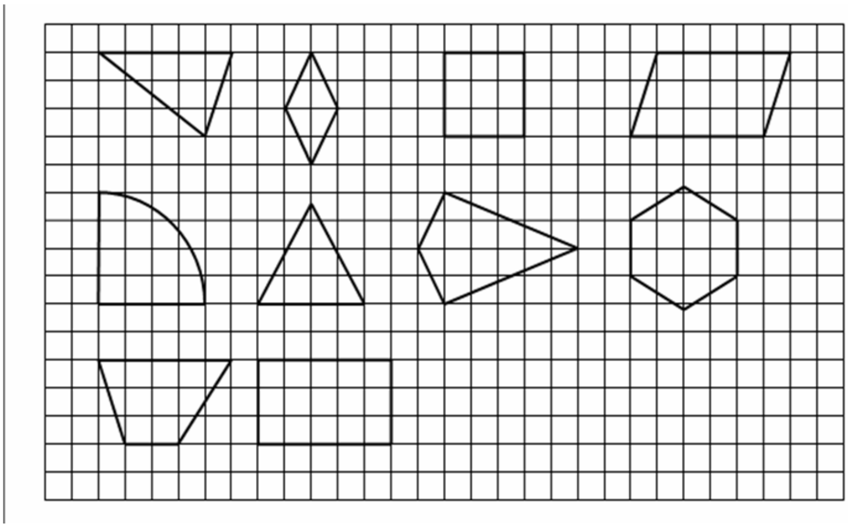


Figure 2.

The results of the tasks are shown in the following table:

In 2006:

	4.c	4.a	4.b
Correct solutions	8%	0%	0%
A mistake in one plane figure	16%	0%	0%
The reflection axis was not marked in the isosceles triangle	68%	95,7%	54,2%
Two or fewer symmetry axes were marked in the square	8%	30,4%	25%
Only one or no symmetry axis was drawn in the general rhomboid	0%	21,7%	4,2%
Reflection axis was drawn in the general parallelogram	0%	0%	0%
One symmetry axis or none was drawn in the equilateral triangle	68%	100%	87,5%
No reflection axis was drawn in the quarter circular sheet	44%	52,2%	29,2%
Not all the reflection axes were found in the regular hexagon	80%	95,7%	91,7%
The diagonals in the rectangles were believed to be symmetry axis	0%	0%	4,2%

This task proved to be a lot more difficult than the pre-test. Children had to draw a lot more symmetry axes, thus they had to pay much more attention to the task. Quite a lot of children made mistakes in case of the isosceles triangle and the reason for this might have been that one side of the triangle was not on the grid and therefore they were not able to discover that this side was of the same length as the one fitting on the grid. In case of the regular triangles and hexagons the typical mistake they made was that they discovered that reflective character of the figures but they were not able to find all the symmetry axes. We consider it a rather significant achievement that in the experimental group no one made a mistake with parallelograms and rectangles. In case of squares only two children (8%) were not able to draw all the four reflection axes. In the control groups this rate was somewhat higher (30,4% and 25%) respectively.

In the 2008 worksheet the square grid cannot be seen in the case of plane figures. We considered that the recognition of symmetry would be easier this way. There is another difference as well compared to the previous worksheet: instead of the first triangle we have drawn a general triangle.

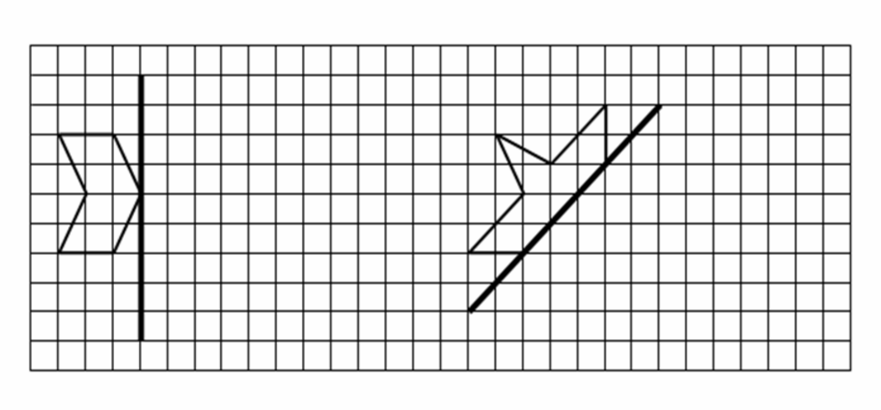
The results of the tasks are shown in the following table:

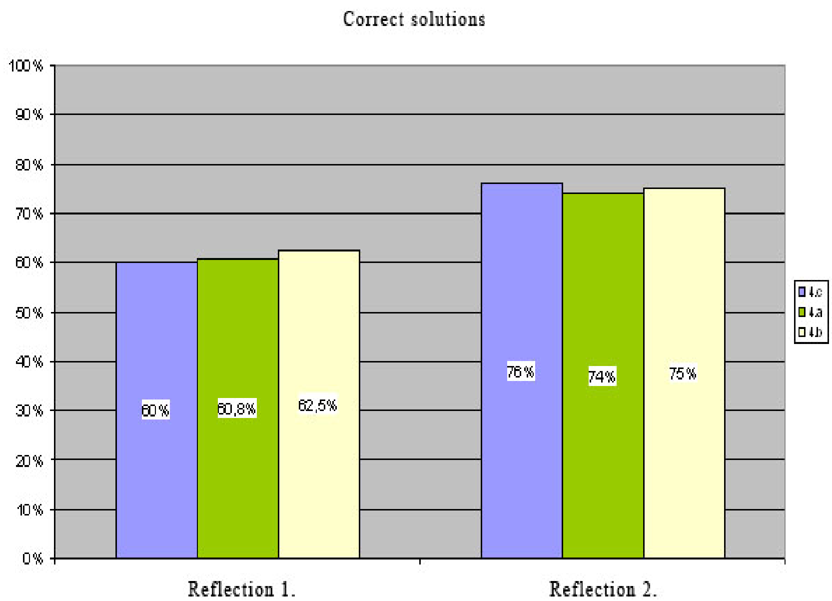
In 2008:

	4.b	4.a	4.c
Correct solutions	47,7%	9,1%	0%
A mistake in one plane figure	31,6%	27,3%	20%
A mistake in two plane figures	15,8%	22,7%	13,3%
Two or fewer symmetry axes were marked in the square	0%	27,3%	26,7%
Only one or no symmetry axis was drawn in the general rhomboid	0%	9,1%	13,3%
Reflection axis was drawn in the general parallelogram	0%	4,5%	20%
One symmetry axis or none was drawn in the equilateral triangle	36,8%	86,4%	100%
No reflection axis was drawn in the quarter circular sheet	10,5%	27,3%	60%
Not all the reflection axes were found in the regular hexagon	36,8%	59,1%	73,3%
The diagonals in the rectangles were believed to be symmetry axis	0%	4,5%	6,7%

In another task of the worksheet the children had to draw on a given axis the reflection of two figures on a grid.

Figure 3.





The results of the task are shown in the following table:

In the task of these reflections all the three classes did fairly well and their results were rather similar. In the case of data gained in 2008 the difference between the experimental and control groups is more significant, as the ratio of faultless results in the experimental group was 73,7% and in the control groups 68,2% and 46,7% at the first mirroring. The data referring to the second mirroring are as follows: 84,2% (4.b), 77,3% (4.a) and 53,3% (4.c)

The mistakes in the two tasks indicate that the concept of symmetry is not stable enough. During the developmental teaching experiment a considerable progress was made but the concept needs to be further developed.

9. CONCLUSION

The developing teaching experiment guided by the author efficiently contributed to the establishment of the concept of symmetry. We can also state that the effectiveness was considerably better during the 2008 experiment. This can be accounted for the fact that we have been able to rely on our former experience, and that we have dealt with exercises connected to symmetry in detail and depth. The comparison of the 2008 pre-test and evaluation worksheet with the ones done in 2006 shows better results. The efficiency is also shown by the fact that the achievement in the experimental group was better, occasionally much better than in the two other control groups, as it can be seen in the graph. Our findings are related to only the samples examined, which are not representative, and therefore no statistical trials have been carried out.

The data measured, the interviews and the games support the hypothesis that it is not possible to reach level 3 of geometrical thinking according to van Hiele by the completion of lower primary (the first four classes of primary education), only reaching the first two levels is feasible. Children are not really able to make conclusions from one characteristic of the figures to the others. They cannot find the relationships between the characteristics of a given figure.

The cognition of children of 6-10 years olds is highly attached to real life, which is why during the formation of concepts only starting out from concrete, manual activities and examples taken from their immediate experience is it possible to reach the level of abstraction. A large number of examples and counter example and making the concept concrete several times and modelling are the preconditions that make it possible that children could recognize the essential characteristics of a concept and they could reach the level of abstraction.

As György Pólya said: "We should not pass up anything that could bring mathematics closer to students. Mathematics is a very abstract science and this is why it has to be presented in a very concrete way."

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