CREATIVE MATH. & INF. **Online version at** `http://creative-mathematics.ubm.ro/`
17 (2008), No. 3, 339 - 345 **Print Edition: ISSN 1584 - 286X Online Edition: ISSN 1843 - 441X**

Dedicated to Professor Iulian Coroian on the occasion of his 70^{th} *anniversary*

Fixed point theory for nonself multivalued operators on a set with two metrics

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ABSTRACT. The purpose of this work is to present some fixed point results for multivalued operators on a set with two metrics. The data dependence and the well-posedness of the fixed point problem are also discussed.

1. INTRODUCTION

Throughout this paper, the standard notations and terminologies in nonlinear analysis (see [15], [16]) are used. For the convenience of the reader we recall some of them.

Let (X, d) be a metric space. By $\tilde{B}_d(x_0, r)$ we denote the closed ball centered in $x_0 \in X$ with radius $r > 0$.

We will also use the following symbols:

$P(X) := \{Y \subset X|\ Y \text{ is nonempty}\}$, $P_{cl}(X) := \{Y \in P(X)|\ Y \text{ is closed}\}$,

$P_b(X) := \{Y \in P(X)|\ Y \text{ is bounded }\}$, $P_{b,cl}(X) := P_{cl}(X) \bigcap P_b(X)$.

Let A and B be nonempty subsets of the metric space (X, d). The gap between these sets is

$$D_d(A, B) = \inf\{d(a, b)|\ a \in A,\ b \in B\}.$$

In particular, $D_d(x_0, B) = D_d(\{x_0\}, B)$ (where $x_0 \in X$) is called the distance from the point x_0 to the set B.

The Pompeiu-Hausdorff generalized distance between the nonempty closed subsets A and B of the metric space (X, d) is defined by the following formula:

$$H_d(A, B) := \max\{\sup_{a\in A} \inf_{b\in B} d(a, b), \sup_{b\in B} \inf_{a\in A} d(a, b)\}$$

If $A, B \in P_{b,cl}(X)$, then one denote

$$\delta_d(A, B) := \sup\{d(a, b)|\ a \in A,\ b \in B\}.$$

The symbol $T : X \to P(Y)$ denotes a set-valued operator. We will denote by $Graph(T) := \{(x, y) \in X \times Y | y \in T(x)\}$ the graph of T. Recall that the set-valued operator is called closed if $Graph(T)$ is a closed subset of $X \times Y$, i.e., if $x_n \subset X$ and $y_n \in T(x_n)$, for $n \in \mathbb{N}$, with $x_n \overset{d}{\to} x^*$ as $n \to \infty$ and if $y_n \overset{d}{\to} y^*$ as $n \to \infty$, then $y^* \in T(x^*)$.

Received: 28.10.2008. In revised form: 03.02.2009. Accepted: 11.05.2009.

2000 *Mathematics Subject Classification.* 47H10, 54H25.

Key words and phrases. *Set with two metrics, multivalued operator, fixed point, strict fixed point, well-posed fixed point problem, generalized contraction, data dependence.*

For $T : X \to P(X)$ the symbol $F_T := \{x \in X | \, x \in T(x)\}$ denotes the fixed point set of the set-valued operator T, while $(SF)_T := \{x \in X | \, \{x\} = T(x)\}$ is the strict fixed point set of T.

If (X, d) is a metric space, $T : X \to P_{cl}(X)$ is called a multivalued a-contraction if $a \in]0, 1[$ and $H(T(x_1), T(x_2)) \leq a \cdot d(x_1, x_2)$, for each $x_1, x_2 \in X$.

In the same setting, an operator $T : X \to P_{cl}(X)$ is a multivalued weakly Picard operator (briefly MWP operator) (see [16]) if for each $x \in X$ and each $y \in T(x)$ there exists a sequence $(x_n)_{n\in\mathbb{N}}$ in X such that:

i) $x_0 = x, \ x_1 = y$

ii) $x_{n+1} \in T(x_n)$, for all $n \in \mathbb{N}$

iii) the sequence $(x_n)_{n\in\mathbb{N}}$ is convergent and its limit is a fixed point of T.

A sequence $(x_n)_{n\in\mathbb{N}}$ in X satisfying the condition (ii) from the previous definition is called the sequence of successive approximations of T starting from $x_0 \in X$.

The purpose of this paper is to present a fixed point theory for multivalued nonself contractions defined on a ball in a space endowed with two metric structures. Our results extend some previous theorems given by A. Petruşel and I.A. Rus in [7], as well as, some other results in the literature, see [1], [2], [4], [6], [14].

2. Main results

Our first theorem is a fixed point result for multivalued nonself contractions on a set with two metrics.

Theorem 2.1. *Let X be a nonempty set, d and ρ two metrics on X, $x_0 \in X$, $r > 0$ and $T : \tilde{B}_\rho(x_0, r) \to P(X)$ be a multivalued operator.*

We suppose that:

(i) (X, d) is a complete metric space;

(ii) there exists $c > 0$ such that $d(x, y) \leq c\rho(x, y)$ for each $x, y \in \tilde{B}_\rho(x_0, r)$;

(iii) $T : (\tilde{B}_\rho(x_0, r), d) \to (P(X), H_d)$ is closed;

(iv) there exists $\alpha \in [0, 1[$ such that

$$H_\rho(T(x), T(y)) \leq \alpha\rho(x, y) \text{ for each } x, y \in \tilde{B}_\rho(x_0, r)$$

(v) $D_\rho(x_0, T(x_0)) < (1 - \alpha)r$.

Then:

(a) $F_T \neq \emptyset$;

(b) there exists a sequence $(x_n)_{n\in\mathbb{N}^*} \subset \tilde{B}_\rho(x_0, r)$ such that:

(1) $x_{n+1} \in T(x_n), \quad n \in \mathbb{N}$;

(2) $x_n \xrightarrow{d} x^* \in T(x^*)$ as $n \to \infty$.

Proof. Let $x_0 \in X$. From (v) we have that there exists $x_1 \in T(x_0)$ with $\rho(x_0, x_1) < (1 - \alpha)r \leq r$. From (iv) we have that:

$$H_\rho(T(x_0), T(x_1)) \leq \alpha\rho(x_0, x_1) < \alpha(1 - \alpha)r$$

On the other hand, it is known that, if $A, B \in P_{cl}(X)$ and $\epsilon > 0$, then:

$$H(A, B) < \epsilon \text{ implies that, for all } \ a \in A, \ \text{ there exists } \ b \in B \text{ such that } \rho(a, b) < \epsilon \qquad (*)$$

From $(*)$ we have that there exists $x_2 \in T(x_1)$ such that $\rho(x_1, x_2) < \alpha(1 - \alpha)r$.

We have

$$\begin{aligned}\rho(x_0,x_2) &\le \rho(x_0,x_1)+\rho(x_1,x_2)\\ &< (1-\alpha)r+\alpha(1-\alpha)r\\ &= (1-\alpha^2)r \le r\end{aligned}$$

hence $x_2 \in \tilde{B}_\rho(x_0,r)$.

Inductively we can construct a sequence $(x_n)_{n\in\mathbb{N}} \subset \tilde{B}_\rho(x_0,r)$ having the properties:

(α) $x_{n+1}\in T(x_n),\ n\in\mathbb{N}$;

(β) $\rho(x_n,x_{n+1}) < \alpha^n(1-\alpha)r,\ n\in\mathbb{N}$.

We will prove now that the sequence $(x_n)_{n\in\mathbb{N}}$ is Cauchy with respect to ρ.

We successively have:

$$\begin{aligned}\rho(x_n,x_{n+p}) &\le \rho(x_n,x_{n+1})+\rho(x_{n+1},x_{n+2})+\dots+\rho(x_{n+p-1},x_{n+p})\\ &< \alpha^n(1-\alpha)r+\alpha^{n+1}(1-\alpha)r+\dots+\alpha^{n+p-1}(1-\alpha)r\\ &\le \alpha^n(1-\alpha)r(1+\alpha+\dots+\alpha^{n+p-1}+\dots)\\ &< \alpha^n(1-\alpha)r\cdot\frac{1}{1-\alpha}=\alpha^n r.\end{aligned}$$

Letting $n\to\infty$, since $\alpha^n\to 0$, it follows that:

$$\rho(x_n,x_{n+p})\to 0 \text{ as } n\to\infty.$$

Hence $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in (X,ρ)

From (ii) it follows that the sequence $(x_n)_{n\in\mathbb{N}}$ is Cauchy in (X,d). Denote by $x^*\in X$ the limit of this sequence. From (i) and (iii) we get that $x_n \xrightarrow{d} x^* \in T(x^*)$ as $n\to\infty$. The proof is complete. □

Let us present now the notion of well-posedness for a fixed point problem.

Definition 2.1. (A. Petruşel and I. A. Rus [8]) Let (X,d) be a metric space, $Y\in P(X)$ and $T: Y\to P_{cl}(X)$ be a multivalued operator. Then the fixed point problem for T with respect to D_d is well-posed iff:

(a_1) $F_T=\{x^*\}$;

(b_1) If $x_n\in Y$, $n\in\mathbb{N}$ and $D_d(x_n,T(x_n))\to 0$ as $n\to+\infty$, then $x_n\to x^*$ as $n\to+\infty$.

Definition 2.2. (A. Petruşel and I. A. Rus [8]) Let (X,d) be a metric space, $Y\in P(X)$ and $T: Y\to P_{cl}(X)$ be a multivalued operator. Then the fixed point problem for T with respect to H_d is well-posed iff:

(a_2) $(SF)_T=\{x^*\}$;

(b_2) If $x_n\in Y$, $n\in\mathbb{N}$ and $H_d(x_n,T(x_n))\to 0$ as $n\to+\infty$, then $x_n\to x^*$ as $n\to+\infty$.

For other details and results on this topic see A. Petruşel, I. A. Rus and J.-C. Yao [9].

Next, we will prove a strict fixed point theorem for a multivalued nonself contraction.

Theorem 2.2. *Let X be a nonempty set, d and ρ two metrics on X, $x_0 \in X$, $r > 0$ and $T : \tilde{B}_\rho(x_0, r) \to P(X)$ be a multivalued operator. We suppose that:*

(i) (X, d) is a complete metric space;

(ii) there exists $c > 0$ such that $d(x, y) \leq c\rho(x, y)$, for each $x, y \in \tilde{B}_\rho(x_0, r)$;

(iii) $T : (\tilde{B}_\rho(x_0, r), d) \to (P(X), H_d)$ is closed;

(iv) there exists $\alpha \in [0, 1[$ such that

$$H_\rho(T(x), T(y)) \leq \alpha\rho(x, y), \text{ for each } x, y \in \tilde{B}_\rho(x_0, r);$$

(v) $D_\rho(x_0, T(x_0)) < (1 - \alpha)r$;

(vi) $(SF)_T \neq \emptyset$.

Then we have:

(a) $F_T = (SF)_T = \{x^*\}$;

(b) $\rho(x, x^*) \leq \dfrac{1}{1 - \alpha} H_\rho(x, T(x))$ for each $x \in \tilde{B}_\rho(x_0, r)$;

(c) the fixed point problem is well-posed for T with respect to D_ρ.

Proof. (a) From Theorem 1 it results $F_T \neq \emptyset$. From (vi) we have that there exists $x^* \in (SF)_T$. It is obvious that $(SF)_T \subset F_T$. We will prove that $F_T \subset (SF)_T$.

Let $y \in F_T$. We will prove that $y = x^*$.

By putting $x := x^*$ in (iv) we have

$$H_\rho(T(x^*), T(y)) \leq \alpha\rho(x^*, y).$$

We have that:

$$\rho(x^*, y) = D_\rho(T(x^*), y) \leq H_\rho(T(x^*), T(y)) \leq \alpha\rho(x^*, y).$$

Thus, $y = x^*$ and so $F_T = (SF)_T = \{x^*\}$.

(b) We successively have:

$$\begin{aligned} \rho(x, x^*) &\leq H_\rho(x, T(x)) + H_\rho(T(x), T(x^*)) \\ &\leq H_\rho(x, T(x)) + \alpha\rho(x, x^*). \end{aligned}$$

Hence

$$\rho(x, x^*) \leq \frac{1}{1 - \alpha} H_\rho(x, T(x)).$$

(c) Let $x_n \in \tilde{B}_\rho(x_0, r)$, $n \in \mathbb{N}$ be such that $D_\rho(x_n, T(x_n)) \to 0$ as $n \to +\infty$. We will prove that $\rho(x_n, x^*) \to 0$ as $n \to +\infty$. We have:

$\rho(x_n, x^*) \leq D_\rho(x_n, T(x_n)) + H_\rho(T(x_n), T(x^*)) \leq D_\rho(x_n, T(x_n)) + \alpha\rho(x_n, x^*)$.
Thus, $\rho(x_n, x^*) \leq \frac{1}{1-\alpha} D_\rho(x_n, T(x_n)) \to 0$ as $n \to +\infty$. The proof is complete. □

A data dependence result is the following theorem.

Theorem 2.3. *Let X be a nonempty set, d and ρ two metrics on X and $T, S : \tilde{B}_\rho(x_0, r) \to P(X)$ be two multivalued operators. We suppose that:*

(i) (X, d) is a complete metric space;

(ii) there exists $c > 0$ such that $d(x, y) \leq c\rho(x, y)$, for each $x, y \in \tilde{B}_\rho(x_0, r)$;

(iii) $T : (\tilde{B}_\rho(x_0, r), d) \to (P(X), H_d)$ is closed;

(iv) there exists $\alpha \in [0, 1[$ such that $H_\rho(T(x), T(y)) \leq \alpha\rho(x, y)$, for each $x, y \in \tilde{B}_\rho(x_0, r)$;

(v) $D_\rho(x_0, T(x_0)) < (1 - \alpha)r$;

(vi) $(SF)_T \neq \emptyset$*;*

(vii) $F_S \neq \emptyset$*;*

(viii) there exists $\eta > 0$ *such that* $H_\rho(T(x), S(x)) \leq \eta$*, for each* $x \in \tilde{B}_\rho(x_0, r)$*.*

Then $H_\rho(F_T, F_S) \leq \frac{\eta}{1-\alpha}$*.*

Proof. By Theorem 2 we have $F_T = (SF)_T = \{x^*\}$. Let $y^* \in F_S$. Then: $\rho(y^*, x^*) \leq H_\rho(S(y^*), x^*) \leq H_\rho(S(y^*), T(y^*)) + H_\rho(T(y^*), T(x^*)) \leq \eta + \alpha\rho(y^*, x^*)$. Hence:

$$\rho(y^*, x^*) \leq \frac{\eta}{1-\alpha}.$$

Hence $H_\rho(F_T, F_S) = \sup\limits_{y^* \in F_S} \rho(y^*, x^*) \leq \dfrac{\eta}{1-\alpha}$. The proof is complete. □

We will present now a strict fixed point theorem for the so-called Reich δ-contractions, on a set endowed with two metrics.

Theorem 2.4. *Let* $X \neq \emptyset$ *be a nonempty set and* d*,* ρ *two metrics on* X*. Let* $T : X \to P_b(X)$ *be a multivalued operator.*

We suppose that:

(i) (X, d) *is a complete metric space;*

(ii) there exists $c > 0$ *such that* $d(x, y) \leq c\rho(x, y)$*, for all* $x, y \in X$*;*

(iii) there exist $a, b, c \in \mathbb{R}_+$ *with* $a + b + c < 1$ *such that*

$$\delta_\rho(T(x), T(y)) \leq a\rho(x, y) + b\delta_\rho(x, T(x)) + c\delta_\rho(y, T(y)), \text{ for all } x, y \in X.$$

Then:

(a) $(SF)_T = \{x^*\}$*;*

(b) for each $x \in X$ *there exists* $(x_n)_{n\in\mathbb{N}}$ *such that:*

(b_1) $x_0 = x,\ x_1 = y$*;*

(b_2) $x_{n+1} \in T(x_n),\ n \in \mathbb{N}$*;*

(b_3) $x_n \overset{d}{\to} x^* \in (SF)_T$*,* $n \to \infty$*.*

(c) the fixed point problem is well-posed for T *with respect to* H_ρ*.*

Proof. (a) and (b) Let $q > 1$ and $x_0 \in X$ be arbitrarily chosen. Then, there exists $x_1 \in T(x_0)$ such that

$$\delta_\rho(x_0, T(x_0)) \leq q\rho(x_0, x_1).$$

We have:

$$\delta_\rho(x_1, T(x_1)) \leq \delta_\rho(T(x_0), T(x_1)) \leq a\rho(x_0, x_1) + b\delta_\rho(x_0, T(x_0))$$
$$+c\delta_\rho(x_1, T(x_1)) \leq (a + bq)\rho(x_0, x_1) + c\delta_\rho(x_1, T(x_1)).$$

It follows that

$$\delta_\rho(x_1, T(x_1)) \leq \frac{a + bq}{1-c}\rho(x_0, x_1)$$

For $x_1 \in T(x_0)$, there exists $x_2 \in T(x_1)$ such that

$$\delta_\rho(x_1, T(x_1)) \leq q\rho(x_1, x_2).$$

We have:

$$\delta_\rho(x_2, T(x_2)) \leq \delta_\rho(T(x_1), T(x_2))$$
$$\leq a\rho(x_1, x_2) + b\delta_\rho(x_1, T(x_1)) + c\delta_\rho(x_2, T(x_2))$$

$$\leq (a+bq)\rho(x_1,x_2) + c\delta_\rho(x_2,T(x_2)).$$

It follows that

$$\delta_\rho(x_2,T(x_2)) \leq \frac{a+bq}{1-c}\rho(x_1,x_2)$$
$$\leq \frac{a+bq}{1-c}\delta_\rho(x_1,T(x_1)) \leq \left(\frac{a+bq}{1-c}\right)^2 \rho(x_0,x_1).$$

We construct inductively the sequence $(x_n)_{n\in\mathbb{N}}$ with the properties:
(α) $x_n \in T(x_{n-1}),\ n \in \mathbb{N}^*$;
(β) $\rho(x_n,x_{n+1}) \leq \delta_\rho(x_n,T(x_n)) \leq \left(\frac{a+bq}{1-c}\right)^n \rho(x_0,x_1)$.

Denote by $\alpha := \dfrac{a+bq}{1-c}$.

We prove now that (x_n) is a Cauchy sequence with respect to ρ.

We have:

$$\rho(x_n,x_{n+p}) \leq \rho(x_n,x_{n+1}) + \rho(x_{n+1},x_{n+2}) + \cdots + \rho(x_{n+p-1},x_{n+p})$$
$$\leq (\alpha^n + \alpha^{n+1} + \cdots + \alpha^{n+p-1})\rho(x_0,x_1).$$

It follows

$$\rho(x_n,x_{n+p}) \leq \alpha^n(1+\alpha+\cdots+\alpha^{p-1})\rho(x_0,x_1)$$
$$= \alpha^n\frac{\alpha^p-1}{\alpha-1}\rho(x_0,x_1).$$

If we choose $q < \dfrac{1-a-c}{b}$, then we have that $\alpha < 1$. Hence, $\rho(x_n,x_{n+p}) \to 0,\ n\to\infty$ and thus $(x_n)_{n\in\mathbb{N}}$ is Cauchy sequence in (X,ρ).

From (ii) we get that there exists $c>0$ such that

$$d(x_n,x_{n+p}) \leq c\rho(x_n,x_{n+p}).$$

Hence, we get $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in (X,d).

By the completeness of the space (X,d), it follows that there exists $x^* \in X$ such that $x_n \overset{d}{\to} x^*,\ n\to\infty$.

We prove now that $x^* \in (SF)_T$. We have:

$$\delta_\rho(x^*,T(x^*)) \leq \rho(x^*,x_n) + \delta_\rho(x_n,T(x_n)) + \delta_\rho(T(x_n),T(x^*))$$
$$\leq \rho(x^*,x_n) + \delta_\rho(x_n,T(x_n)) + a\rho(x_n,x^*) + b\rho(x_n,T(x_n)) + c\delta_\rho(x^*,T(x^*))$$

We have:

$$\delta_\rho(x^*,T(x^*)) \leq \frac{1+a}{1-c}\rho(x^*,x_n) + \frac{1+b}{1-c}\delta_\rho(x_n,T(x_n)) \to 0,\ \text{as } n\to\infty.$$

Notice that, in the above relation, we have used the fact that $\delta_\rho(x_n,T(x_n)) \leq \alpha^n\rho(x_0,x_1)$.

Hence, $\delta_\rho(x^*,T(x^*)) = 0$ and thus $x^* \in (SF)_T$.

We will prove now the uniqueness of the strict fixed point. Suppose that there exist $x^*,y^* \in (SF)_T$. Then:

$$\delta_\rho(x^*,T(x^*)) = 0, \quad \delta_\rho(y^*,T(y^*)) = 0.$$

We have:

$$\rho(x^*,y^*) = \delta_\rho(T(x^*),T(y^*)) \leq a\rho(x^*,y^*) + b\delta_\rho(x^*,T(x^*)) + c\delta_\rho(y^*,T(y^*)).$$

If $\rho(x^*,y^*)>0,$ then $a\geq 1$, which is contradiction with the hypothesis.

If $\rho(x^*,y^*)=0$, then $x^*=y^*$ and thus $(SF)_T=\{x^*\}$.

(c) Let $(x_n)_{n\in\mathbb{N}}$ be a sequence in X such that $H_\rho(x_n,T(x_n))\to 0$ as $n\to+\infty$. We will prove that $\rho(x_n,x^*)\to 0$ as $n\to+\infty$. We have:

$\rho(x_n,x^*)\leq\rho(x_n,x_{n+1})+\rho(x_{n+1},x^*)\leq H_\rho(x_n,T(x_n))+\rho(x_{n+1},T(x^*))\leq H_\rho(x_n,T(x_n))+\delta_\rho(T(x_n),T(x^*))\leq H_\rho(x_n,T(x_n))+a\rho(x_n,x^*)+b\delta_\rho(x_n,T(x_n))+c\delta_\rho(x^*,T(x^*))=(1+b)H_\rho(x_n,T(x_n))+a\rho(x_n,x^*)$. Thus

$$\rho(x_n,x^*)\leq\frac{1+b}{1-a}\cdot H_\rho(x_n,T(x_n))\to 0 \text{ as } n\to+\infty.$$

The proof is now complete. □

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