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Dedicated to Professor Iulian Coroian on the occasion of his 70^{th} *anniversary*

Assessing the ecologic-economic efficiency of an investment policy

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ABSTRACT. We present a manner of assessing the ecologic-economic efficiency of an investment project, which provides the decision maker with an inner characterization of the project, depending only on the results of the project itself. The mathematical model is based on multiple criteria binary programming. The solution of this model leads the decision maker to using an ecologic-economic efficiency index, which arises from finding a Pareto point of the multiple criteria programming problem. The possibility of gambling with the relative importance of criteria provides the decision maker with extended information on the consequences of the investment project on the environment. The stability properties of the multiple criteria decision making technique based on the ecologic-economic efficiency index is studied, determining the intervals of stability. Our method is tested on the problem of modernizing the Romanian rail from ecologic point of view.

1. INTRODUCTION AND BASIC RESULTS

1.1. **Introduction.** Ethical issues often appear in the process of selecting an investment policy taking into account both economic and environmental criteria. The need of protecting both the environment in which the project is going to develop and the employees involved in it comes often into conflict with the need of keeping the work productivity at high level, also trying to minimize the costs.
The necessity of studying the economic-ecologic efficiency of an investment policy of a company is often mentioned both in technical and in scientific literature. The relationship between industry and environment should receive considerable attention from two points of view: within the organization and between the organization and the society and nature. There are specific extra-economic possibilities of describing the efficiency of an investment policy from ecology point of view as, for example, measuring the concentration of certain substances in the soil, air, water, food, etc. A multiple criteria problem arises when we take into account the need of optimizing the cost-utility ratio and to take care both on humans and nature.
A general method of assessing this kind of efficiency is never described and, as consequence, a general tool, as an economic-ecologic efficiency index to characterize an investment project is totally absent. There are many domains, that use

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efficiency indexes to assess various types of activities, economic processes. For example, in the energy domain there are more efficiency indexes, both in terms of costs and in terms of effects, depending on the aim of the researcher (see [1], [8], [11]). The relationship between trade and environmental conditions is very important whenever countries are in the process of negotiating trade agreements. So, an environmental efficiency index for a sample of high income and low and middle income countries was developed [9] allowing to examine the role of trade on the changes in environmental efficiency. The idea of an efficiency of a legislation system was recently published ([10]).
We treat the problem of operating an investment project and keeping a healthy environment by similar methods to farmaco-economics, from mathematical point of view (see [6]), looking to an investment policy like to an operation in an unknown land that should be preceded by a vaccination to prevent a disease [5]. We elaborate a method of studying the opportunity of an investment project from environmental, social, economical, technical, a.s.o. point of view. It is an easy to apply method, not more difficult than the net present value analysis. The method of the ecologic-economic efficiency index of an investment project, which is constructed in this paper is also easy to teach to students in mathematics, computers, economics, engineering, etc. It provides the decision maker with an inner characterization of the project, depending only on the results of the project itself.

The mathematical model is based on multiple criteria binary programming. The solution of this model leads the decision maker to using an economic-ecologic efficiency index, which arises from finding a Pareto point of the multiple criteria programming problem.

1.2. **Basic results.** Let X be a nonempty set and let $f = (f_1, f_2, ..., f_n) : X \to \mathbb{R}^n$.

Definition 1.1. A point $A \in X$ is called a min-efficient point of f in X if there is no $x \in X$ such that

$$f_i(x) \leq f_i(a) \tag{1.1}$$

and in the same time

$$\sum_{j=1}^{n} f_j(x) < \sum_{j=1}^{n} f_j(a). \tag{1.2}$$

In order to solve the multiple criteria problem, denoted (PE),

$$(PE): \qquad (f_1, f_2, ..., f_n) \longrightarrow v - \min_{x \in A \subseteq X} \tag{1.3}$$

we use the weighting method, obtaining an unique synthesis function and the main theorem of Galperin, [4]

Theorem 1.1. *If $\lambda_1 > 0$, $\lambda_2 > 0$, ..., $\lambda_n > 0$ are n given real numbers, then every minimum point of the function $F : X \to \mathbb{R}$, defined by*

$$F(x) = \sum_{j=1}^{n} \lambda_j f_j(x) \tag{1.4}$$

for every $x \in X$, is a min-efficient point of the vectorial function f on X.

An algorithm for finding the min-efficient points of a vectorial function is published in [3]. The possibility of gambling with the relative importance of criteria provides the decision maker with extended information on the consequences of the investment project on the environment.

2. THE CHARACTERIZATION OF AN INVESTMENT POLICY

The aim of this section is to obtain a practical and useful possibility of characterizing the efficiency an investment policy as completely as possible from the point of view of its consequences: cost, effectiveness, side effects on humans and nature and their seriousness, etc.

2.1. **Problem Formulation.** The consequences of an investment policy are assessed by means of n criteria, referring to the environment, security of personnel, energy consumption, etc. The benefit may come as a second level of assessment, leading to a bi-level programming problem, if needed.

Let us consider known s_{Nk}, the normal score of criterion k, $k \in \{1, 2, ..., n\}$. If no investment project is chosen then s_k is the score of criterion k, $k \in \{1, 2, ..., n\}$. If an investment project (P) is chosen then s_{Pk} is the score of criterion k, $k \in \{1, 2, ..., n\}$ after a known period of time.

The following costs are known:

c_p = the total cost of applying the investment project (P);

c_{Pk} = the total cost of treating the damage, negative reactions or side effects produced by the investment project (P) in the domain of criterion k, $k \in \{1, 2, ..., n\}$;

c_k = the total cost of treating the damage, negative reactions or side effects in the domain of criterion k, $k \in \{1, 2, ..., n\}$, if no investment project is chosen.

The main purpose is to elaborate a method of choosing, among more investment projects possible, those that brings the score of each criterion as close to its normal level as possible. A mathematical model is attached for this purpose, in terms of a multiple criteria programming problem in variables 0 and 1. These values are meant to express the preference for a type of action, meaning that two binary variables x_1 and x_2 are introduced, having the following significance:

$x_1 = 1$ means that project (P) is chosen;

$x_1 = 0$ means that project (P) is not chosen;

$x_2 = 1$ means that no project of investment is preferred;

$x_2 = 0$ means that there is a project of investment that is preferred.

Of course, $x_1 + x_2 = 1$, since an investment project can be only accepted or rejected. Let us define the following numbers:

$$p_k = \begin{cases} \frac{100(s_k - s_{Nk})}{s_{Nk}}, & \text{if } s_{Nk} \neq 0 \text{ and } s_k > s_{Nk} \\ 0, & \text{if } s_{Nk} \neq 0 \text{ and } s_k \leq s_{Nk} \\ s_k, & \text{if } s_{Nk} = 0 \end{cases}$$

which is called *the actual variation* of criterion k, $k \in \{1, 2, ..., n\}$, with respect to its normal score and

$$p_{Pk} = \begin{cases} \frac{100(s_{Pk} - s_{Nk})}{s_{Nk}}, & \text{if } s_{Nk} \neq 0 \text{ and } s_{Pk} > s_{Nk} \\ 0, & \text{if } s_{Nk} \neq 0 \text{ and } s_{Pk} \leq s_{Nk} \\ s_{Pk}, & \text{if } s_{Nk} = 0 \end{cases}$$

which is called *the variation induced by project* (P) of criterion k, $k \in \{1, 2, ..., n\}$, with respect to its normal score.

The objective functions are $f_k : \{0, 1\} \times \{0, 1\} \to \mathbb{R}$, $k \in \{1, 2, ..., n\}$ and $f_{n+1} : \{0, 1\} \times \{0, 1\} \to \mathbb{R}$, for every $(x_1, x_2) \in \{0, 1\} \times \{0, 1\}$ by:

$$f_k(x_1, x_2) = (p_{Nk} - p_{Pk})x_1 + (p_{Nk} - p_k)x_2 \qquad k \in \{1, 2, ..., n\}, \tag{2.5}$$

$$f_{n+1}(x_1, x_2) = x_1 + \frac{\alpha}{\alpha_P} x_2, \tag{2.6}$$

where the numbers α and α_P are defined in terms of costs as it follows:

$$\alpha = \sum_{k=1}^{n} c_k, \tag{2.7}$$

$$\alpha_P = c_P + \sum_{k=1}^{n} c_{Pk}. \tag{2.8}$$

Suppose that the pathology at the level of each criterion consists in increasing its value. The solution comes from finding the min-efficient points of the following vectorial programming problem:

$$((p_{N1} - p_{P1})x_1 + (p_{N1} - p_1)x_2, ..., (p_{Nn} - p_{Pn})x_1 + (p_{Nn} - p_n)x_2, x_1 + \frac{\alpha}{\alpha_P} x_2) \to v - min$$

submit to $x_1 + x2 = 1$ for $(x_1, x_2) \in \{0, 1\} \times \{0, 1\}$.

2.2. **Solution to problem (PE).** In order to solve problem (PE), we use the pounds $\lambda_k > 0$, for $k \in \{1, 2, ..., n\}$ and $\lambda_{n+1} \geq 0$ to introduce a synthesis function $F : \{0, 1\} \times \{0, 1\} \to \mathbb{R}$, getting

$$F(x_1, x_2) = \sum_{k=1}^{n+1} \lambda_k f_k(x_1, x_2). \tag{2.9}$$

With this function, problem (PE) turns into the following problem denoted by (PU):

$$F(x_1, x_2) = \sum_{k=1}^{n} \lambda_k [(p_{Nk} - p_{Pk})x_1 + (p_{Nk} - p_k)x_2] + \lambda_{n+1}(x_1 + \frac{\alpha}{\alpha_P} x_2) \to min,$$

submit to $x_1 + x2 = 1$ for $(x_1, x_2) \in \{0, 1\} \times \{0, 1\}$. By elementary calculus one gets

$$F(0, 1) = \sum_{k=1}^{n} \lambda_k (p_{Nk} - p_k) + \lambda_{n+1} \frac{\alpha}{\alpha_P},$$

$$F(1,0)=\sum_{k=1}^{n}\lambda_k(p_{Nk}-p_{Pk})+\lambda_{n+1},$$

and, as consequence,

$$F(1,0)-F(0,1)=\sum_{k=1}^{n}\lambda_k(p_k-p_{Pk})+\lambda_{n+1}(1-\frac{\alpha}{\alpha_P}). \tag{2.10}$$

This is the stage of decision making. If

$$F(1,0)-F(0,1)\geq 0 \tag{2.11}$$

then one can decide that the investment project (P) is acceptable since it is supposed to bring the set all the scores of criteria closer to normal than the actual set.

Remark 2.1. An investment project (P1) is better than another one, (P2) if $(F(1,0)-F(0,1))(P1)>(F(1,0)-F(0,1))(P2)$.

This is the reason of using the difference $F(1,0)-F(0,1)$ as a method of decision making, when a choice of an investment policy is under debate.

2.3. **The ecologic-economic efficiency index of an investment project.** Due to the previous remark we decided to introduce the following tool to characterize an investment project.

Definition 2.2. The ecologic-economic efficiency index of an investment project (P) is the number

$$EEef(P)=\sum_{k=1}^{n}\lambda_k(p_k-p_{Pk})+\lambda_{n+1}(1-\frac{\alpha}{\alpha_P}).$$

Remark 2.2. As one can see, $EEef(P)=(F(1,0)-F(0,1))(P)$.

It is now necessary to study the monotony properties of $EEef(P)$, in order to understand the manner in which it characterizes the effect of the investment project in the directions represented by the set of criteria.

Definition 2.3. An investment project (B) is said to be strongly dominated by an investment project (A) if $s_{Nk}<s_{Ak}<s_{Bk}$ for any $k\in\{1,2,...,n\}$ and if $1-\frac{\alpha}{\alpha_A}<1-\frac{\alpha}{\alpha_B}$.

Definition 2.4. An investment project (B) is said to be dominated with respect to the set of criteria by an investment project(A) if $s_{Nk}<s_{Ak}<s_{Bk}$ for any $k\in\{1,2,...,n\}$.

The following remarks, which contain properties of the ecologic-economic effectiveness index, are easy to prove.

Remark 2.3. $EEef(P)>0$ when $s_{Pk}<s_k$ for any $k\in\{1,2,...,n\}$ and $\lambda_{n+1}=0$.

Remark 2.4. If project B is strongly dominated by project A then $EEef(B)<EEef(A)$.

Remark 2.5. If project B is dominated with respect to the set of criteria by project A and $\lambda_{n+1} = 0$ then $EEef(B) < EEef(A)$.

All the above mentioned monotony properties of the ecologic-economic efficiency index $EEef$ of an investment project show us that it is able to characterize the project. This is an inner characterization of the project, being able to take into account its effect on all the directions described by the set of criteria. It is possible to replace the function f_{n+1} by other criteria arising from the cost / utility analysis, combining them with the environmental ones such as the above described reasoning stands. A new formula of the same class of efficiency indexes may result.

3. EXAMPLE: THE ECOLOGICAL MODERNIZATION OF THE RAILWAY TRANSPORT SYSTEM

In this section we intend to study, using the ecologic-economic efficiency index $EEef$, more projects to ecologically modernize the railway transport system on the technological level. We intend to compare all the possibilities of ecologically improving the carriages from technical point of view. Three projects are possible:

Project P1) Ecological modernization of the box of the railway vehicle in order to improve the travellers' comfort, basically on high speed ($V > 160$km / h);

Project P2) Optimization, from constructive point of view, of the Lifting structure of the railway vehicle, especially of the bogie frame and tread apparatus, for avoiding shocks and transversal or longitudinal vibrations, which are source of major travellers discomfort;

Project P3) Modernization of braking systems of railway vehicles, in order to produce as soft as possible breaking, with deceleration allowed by the human body ($af < 0,8m/s^2$), in established braking way.

One of the most important goals of the International Union of Railways (UIC) is to enable the railway companies to measure the impact of their activity on the environment (see [13]). Environment indicators in the domain of railway transport are defined under UIC and the project RAVEL, funded by the European Union (RAVEL Sustainable Mobility Railway in the future Projects, see[13]). The Working Group UIC on environment presented the *Guide to establish the indicators of environment for the railways*, [13], that is updated each year and is included in the technical portfolio of the Committee C6 - UIC, dealing with economy, finance and environment protection. Four criteria are taken into account, according to this guide, in assessing the impact of the railway transport on the environment. The data are presented according to the measurements made by the railway station of Arad and its depot. The costs are established by the Timişoara central unit of Romanian Railways, CFR.

Criterion 1) Concentration of CO_2 in the air:
Normal: 0,03% from the atmosphere;

Actual: 1,68 g / km;
Estimated after improvement: 1 g / km;

Criterion 2) Level of noise:
Normal: 50 dB(A) (STAS 10009-88 and STAS 6161 / 1-79);
Actual: 125-130 dB(A) (Noise produced by wagons);
Estimated after improvement: 60-70 dB(A).

Criterion 3) Energy consumption:
Normal: 4,5 t / day over 1 million tones $\times$ km;
Actual: 4,05 t / day over 1 million tones $\times$ km;
Estimated after improvement: 3,85 t / day over 1 million tones $\times$ km.

Criterion 4) Annual number of accidents:
Normal: 0
Actual: 2900 dead / year;
Estimated after improvement: max. 1500 dead / year

Project (P1) is estimated to reduce the annual number of accidents by 3%, project (P2) by 2% and project (P3) by 35%. The following table contains the differences $p_j - p_{Pkj}$ between the actual variations and the variations induced by project (Pk) on criterion j, for $k \in \{1, 2, 3\}$ and $j \in \{1, 2, 3, 4\}$. The costs, expressed in Romanian currency, are after the last evaluations.

Project	CO_2	Noise	Energy	Accidents	Cost
1	0	0	7.5	87	1.5 million lei / wagon
2	0	40	17	58	500000 lei / bogie
3	44	30	3	1015	250000 lei / br. sys

Since the costs of treating the damages are not supported by CFR directly ([12]), this company paying only occasionally to each County's Authority for Environment various amounts of money for damages, we take $\lambda_5 = 0$. As consequence, the symbol $<$ is used to describe the order of preference according to the four criteria only. In [7] the authors discuss about the significance of handling effects of reinforced preference and counter-veto in credibility of outranking. Various experiments of decision aid are described in the following table. The first column contains the values given to pounds, presented as a vector $\lambda = (\lambda_1; \lambda_2; \lambda_3; \lambda_4)$.

Weights	$EEef(P1)$	$EEef(P2)$	$EEef(P3)$	Order	Best
$(100; 100; 1; 1)$	94.5	4075	8418	$P1 < P2 < P3$	P3
$(5; 5; 1; 10)$	877.5	815	10523	$P2 < P1 < P3$	P3
$(10; 10; 100; 1)$	837	2158	2055	$P1 < P3 < P2$	P2
$(1; 10; 1; 1)$	94.5	475	1362	$P1 < P2 < P3$	P3

The weights are used to emphasize either the environment point of view or the humanitarian point of view or the energy consumption direction. In the first case we find that if the environment criteria (CO_2 emission and noise) are of great importance, then project P3 seems to be the most suitable. The same result is obtained if the number of accidents is considered of greater importance than some other criteria. But project P2 seems to be the most preferred from the point of view of energy consumption. It is easy to see how such a gambling with weights procedure is able to provide the decision maker with a great amount of information on the consequences of each decision.

4. Stability analysis with respect to the pounds of criteria

4.1. **The concepts of stability of a multiple criteria decision method.** The results from the previous example show us that there are values of pounds for which one gets the same order of projects. Therefore, a stability research of the decision method based on the ecologic-economic efficiency index is needed.
In this section we identify an investment project with a point $s = (s_1, ..., s_n) \in \mathbb{R}^n$ obtained by taking into account its scores on the set of criteria. Also, every system of pounds is supposed to be a point $\lambda = (\lambda_1, ..., \lambda_n) \in \mathbb{R}^n$. Therefore, for every $\lambda \in \mathbb{R}^n$, the number $EEef(s)$ is a filter depending on the point $\lambda \in \mathbb{R}^n$. In the sequel we intend to study the stability with respect to the pounds of the decision making method using the ecologic-economic efficiency index $EEef$. Following the way described in [2], we define:

Definition 4.5. A multiple criteria method is called additive if it satisfies the following conditions:
(i) The method allows the construction of a (D, I)-type preference relation on the set A of actions, where D is preference and I is indifference;
(ii) There is a function $V : A \to \mathbb{R}$ defined by

$$V(a) = \sum_{j=1}^{n} \lambda_j V_j(f_j(a), f_j(A)), \tag{4.12}$$

where $\lambda_j > 0$, $j \in \{1, 2, ..., n\}$ are the pounds associated to the criteria.
(iii) The preference relation (D, I) is defined by:

$$\begin{cases} aDb, & \text{if } V(a) \geq V(b) \text{ (at least once } >\text{)}, \\ aIb, & \text{if } V(a) = V(b). \end{cases} \tag{4.13}$$

Let us remark that the relation (D, I) is a complete preorder. The pounds are usually considered to be normed, i.e. $\sum_{j=1}^{n} \lambda_j = 1$. Otherwise, we denote them by w_j and we norm them by putting

$$\lambda_j = \frac{w_j}{\sum_{k=1}^{n} w_k}.$$

Definition 4.6. There is a stability of the preference relation (D, I) on $M \subseteq A \times A$ for the pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$ if

$$(D, I)_M = (D', I')_M, \tag{4.14}$$

where (D', I') denotes the preference relation corresponding to pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$.

Definition 4.7. There is a stability of an additive multiple criteria decision method on a set $M \subseteq A \times A$ of actions for the pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$ if there is a stability on M of the preference relation (D, I), constructed by the multiple criteria decision method, for the pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$.

4.2. **Stability of the $EEef$ decision method.**

Theorem 4.2. *The decision technique using $EEef$ is an additive multiple criteria method.*

Proof. First we remark that if the action $N \in A$ brings the scores of all the criteria to their normal value then

$$0 = max\{EEef(a) | a \in A\} = EEef(N).$$

Let us denote this value by $EEef(A)$. We define the function $V : A \to \mathbb{R}$ by $V(a) = EEef(a)$ and the following preference relation (D, I):

$$\begin{cases} aDb, & \text{if } EEef(a) \geq EEef(b) \text{ (at least once } >), \\ aIb, & \text{if } EEef(a) = EEef(b). \end{cases} \tag{4.15}$$

This preference relation satisfies the above defined conditions of an additive multiple criteria method. □

In the sequel (D, I) will be referred as the preference relation induced by $EEef$.

Theorem 4.3. *For every system of pounds $\lambda = (\lambda_1, ..., \lambda_n) \in \mathbb{R}^n$ there is a set of pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$ and a subset $M \subseteq A \times A$ such that the preference relation (D, I) induced by $EEef$ has a stability on M for the pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$.*

Proof. Let us suppose that $\lambda = (\lambda_1, ..., \lambda_n) \in \mathbb{R}^n$ and $\lambda^{'} = (\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'}) \in \mathbb{R}^n$. For $a \in A$ and $b \in A$ we denote by $EEef(a)$ the ecologic-economic index of action a obtained using λ and by $EEef'(a)$ the value obtained for $\lambda^{'}$.
If aDb then $EEef(a) \geq EEef(b)$ and the stability condition becomes, according to [2], condition (5.15) on pg.124,

$$EEef'(a) - EEef'(b) \geq 0. \tag{4.16}$$

If aDb then $EEef(a) = EEef(b)$ and the stability condition becomes, according to [2], condition (5.17) on pg.124,

$$EEef'(a) - EEef'(b) = 0. \tag{4.17}$$

Let us define

$$M^* = \{(a, b) | EEef(a) \geq EEef(b)\}. \tag{4.18}$$

Let us consider the following partition of the set of criteria $J = \{1, 2, ..., n\}$ into h nonempty subsets $J_1, J_2, ..., J_h$ such as: $J_u \neq \oslash$, $J_u \subset J_v$,$v \in \{1, 2, ..., h\}$ and $J_u \cap J_v = \oslash$, whenever $u \neq v$. Then it is possible to associate to every subset J_v an overall pound ω_v, defined by:

$$\omega_v = \sum_{j \in J_v} \lambda_j.$$

If we denote by $d_j(a) = p_j - p_{aj}$, then

$$EEef(a) = \sum_{j=1}^{n} \lambda_j d_j(a) = \sum_{v=1}^{h} \omega_v g_v(a), \tag{4.19}$$

where $g_v(a)$ are defined by:

$$g_v(a) = \frac{1}{\omega_v} \sum_{j \in J_v} \lambda_j d_j(a).$$

In order to obtain the stability intervals let us consider $h = 2$. The pounds modified become

$$\omega_1^{'} = \alpha_1 \times \omega_1; \omega_2^{'} = \alpha_2 \times \omega_2.$$

Here, taking into account the nonnegativity conditions and the normalization of pounds, we have:

$$0 \leq \alpha_1 \leq \frac{1}{\omega_1}; \alpha_2 = \frac{1 - \alpha_1 \omega_1}{1 - \omega_1}.$$

If we denote $D_j(a, b) = g_j(a) - gj(b)$, then

$$\omega'[D_1(a, b) - D_2(a, b)] \geq -D_2(a, b), \tag{4.20}$$

for any $(a, b) \in M^*$. Let us introduce the following sets:

$$M_+^* = \{(a, b)|(a, b) \in M^*, D_1(a, b) > D_2(a, b)\}, \tag{4.21}$$

$$M_-^* = \{(a, b)|(a, b) \in M^*, D_1(a, b) < D_2(a, b)\}. \tag{4.22}$$

Then the previous inequality becomes

$$\max_{(a,b) \in M_+^*} \frac{-D_2(a, b)}{D_1(a, b) - D_2(a, b)} \leq \omega_1^{'} \leq \min_{(a,b) \in M_-^*} \frac{-D_2(a, b)}{D_1(a, b) - D_2(a, b)}. \tag{4.23}$$

The stability interval corresponding to pound ω_1 is obtained taking into account the nonnegativity conditions:

$$\omega_1^- \leq \omega_1^{'} \leq \omega_1^+, \tag{4.24}$$

where the bounds are:

$$\omega_1^- = \max\{0; \max_{(a,b) \in M_+^*} \frac{-D_2(a, b)}{D_1(a, b) - D_2(a, b)}\}, \tag{4.25}$$

$$\omega_1^+ = \min\{0; \min_{(a,b) \in M_-^*} \frac{-D_2(a, b)}{D_1(a, b) - D_2(a, b)}\}. \tag{4.26}$$

□

Remark 4.6. For every system of pounds $\lambda = (\lambda_1, ..., \lambda_n) \in \mathbb{R}^n$ there is a set of pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$ and a subset $M \subseteq A \times A$ such that the multiple criteria decision method based on $EEef$ has a stability on M for the pounds $(\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_n^{'})$.

As one can see from the properties in this section, the multiple criteria decision method based on the ecologic-economic efficiency index is a outranking method of the same type as PROMETHEUS II. They are additive and generate the same type of preference relations on the given set of actions. The $EEef$ method is not accompanied by a software giving a geometric representation that shows its stability domains, but it is very easy to apply, using easy and popular software as, for example, EXCEL.

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