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Dedicated to Professor Iulian Coroian on the occasion of his 70^{th} anniversary

The stability conditions associated to the jump processes

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ABSTRACT. In this paper we study the evolution solutions of the stochastic differential equation associated to a Poisson process. As well, we present the conditions for the exponentially stability making connections between the norm of the operators which appears in the stochastic differential equation and the increase index associated to the semigroup operators.

1. INTRODUCTION

Let $(\Omega, \Im, \mathbf{P})$ be a complete probability space, H is a real Hilbert separable space with the scalar product $< \cdot, \cdot >$, the induced norm $\| \cdot \|\|$, **N** - the natural set of numbers and $I \subset \mathbf{R}_+$ an interval.

Definition 1.1. It is called a Poisson process with $\mu > 0$ parameter, the process $N : I \times \Omega \to \mathbf{N}$ with the following properties:
▷ $N(0) = 0$,
▷ $N(\cdot)$ - is a stationary process with independent increases,
▷ $\mathbf{P}\{N(h) = 1\} = \mu h + o(h), \;\; \mathbf{P}\{N(h) \geq 2\} = o(h)$.
▷ ($o(h)$ - is a positive continuous function with $\lim_{h \to 0} o(h) = 0$).

We introduce the random variables sequences $\{\tau_n\}_{n \in \mathbf{N}}$, $\{S_n\}_{n \in \mathbf{N}}$ where $\{\tau_n\}$ - is the interarrival time and $S_n = \tau_1 + \tau_2 + ... + \tau_n$ is the waiting time of the "n" event , $S_0 = 0$. For a Poisson process $\{N(t)\}_{t \geq 0}$ with $\mu > 0$ parameter the random variables $\{\tau_n\}_{n \in \mathbf{N}^*}$ are independent and identically distributed, with an exponential distribution of $\frac{1}{\mu}$ parameter and $\mathbf{P}\{S_n \leq t\} = \mathbf{P}\{N(t) \geq n\}$.

Lemma 1.1 (1). *Let $\{N(t)\}_{t \geq 0}$ be a Poisson process with $\mu > 0$ parameter. Then the next properties take place:*
i. $p_n(t) = \mathbf{P}\{N(t) = n\} = \frac{(\mu t)^n}{n!} e^{-\mu t}$;
ii. $\mathbf{E}\{N(t)\} = \mu t$ *and* $Var\{N(t)\} = \mu t$;
iii. $\mathbf{E}\{e^{ixN(t)}\} = e^{\mu t(exp(ix)-1)}$;
iv. $\mathbf{E}\{x^{N(t)}\} = e^{\mu t(x-1)}$, $(\forall) x \in \mathbf{R}, x > 0$.

If we consider a function $f \in L^2([0,T], L(H))$ and the martingale processes $m(t) = N(t) - \mu t$ then we can define an integral, associated to the function f ,of the form $\int_0^T f(s) dm(s)$, with the following properties:(more details in [2])
i. $\int_0^T f(s) dm(s) \in L^2([0,T] \times \Omega, H) \;\; and \;\; \mathbf{E}\{\int_0^T f(s) dm(s)\} = 0$

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ii. $\mathbf{E}\{\| \int_0^T f(s)dm(s)\|^2\} = \mu \int_0^T \|f(s)\|^2 ds$

$$iii.\ \mathbf{E}\{\int_0^t f(r)dm(r) \otimes \int_0^s f(r)dm(r)\} = \mu \int_0^s f(r)f^*(r)dr, 0 \leq s < t < T \quad (1.1)$$

In this way the stochastic integral can be extended to a Poisson process using the next relation:

$$\int_0^t f(s)dN(s) = \int_0^t f(s)dm(s) + \int_0^t f(s)\mu ds \quad (1.2)$$

The differential stochastic equation has this form :

$$dX(t) = A(t, X(t))dt + B(t, X(t))dN(t), \quad (1.3)$$

where $A : [0, \infty) \times H \to H$, $B : [0, \infty) \times H \to L(H)$ are bounded and continuous operators.

The equation (1.3) has an unique solution called **cádlág** (the trajectories $X(t)$ are continuous on the right side and have limit on the left side) and the value of the jumps are given by the B operator.

2. The stability of the jump differential equations with constant coefficients

We consider a stochastic process $X : [0, T] \times \Omega \to H$ which verifies the following differential equation:

$$dX(t) = AX(t)dt + BX(t)dN(t), \quad X(0) = x_0 \quad (2.4)$$

where $A : D(A) \subset H \to H$ is an infinitesimal generator of C_0 semigroup $\{T(t)\}_{t\geq 0}$, $B \in L(H)$, $x_0 \in H$ and $\{N(t)\}_{t\geq 0}$ is a real Poisson process with $\mu > 0$ parameter.

Lemma 2.1. *The evolution solution associated to the differential equation (2.4) is:*

$$X(t) = \prod_{i=1}^{N(t)} (I + B)T(t)X(0), \quad (2.5)$$

where

$$\prod_{i=1}^{N(t)} (I + B) = \underbrace{(I + B) \cdot (I + B) \cdot ... \cdot (I + B)}_{N(t)-ori}, \quad I \in L(H), \quad Ix = x, \quad (\forall)x \in H.$$

Proof. Let $\{Y(t)\}_{t\geq 0}$ be a process which verifies the jump equation $dY(t) = BY(t)dN(t)$, $Y(0) = x_0$ and the realisation times $\{\tau_i\}$. Then the solution of this equation can be written as: $Y(t) = x_0$ for $t \in [0, \tau_1)$; $Y(t) = Y(t) + BY(t) = (I + B)x_0$ for $t \in [\tau_1, \tau_2)$; ... ; $Y(t) = (I + B)^n x_0$ for $t \in [\tau_n, \tau_{n+1})$.

In conclusion we have

$$Y(t) = \prod_{i=1}^{N(t)} (I + B)x_0,$$

and so, it results that

$$X(t) = Y(t)T(t) = \prod_{i=1}^{N(t)} (I + B)T(t)x_0.$$

□

Remark 2.1. As an immediately result we obtain that:

$$\underbrace{\mathbf{E}\{B \cdot B \cdot ... \cdot B\}}_{N(t)-ori} = exp\{\lambda(B - I)t\}, \quad B \in L(H).$$

Theorem 2.1. *We suppose that the semigroup* $\{T(t)\}_{t\geq 0}$ *is uniformly exponentially stable* $((\exists)k > 0 \ \ and \ \ \omega_0 > 0 \ \ such \ \ that \ \ \|T(t)\| \leq ke^{-\omega_0 t}, \ \ (\forall)t \geq 0)$. *Then the following properties take place:*
i. The mean of the $\{X(t)\}_{t\geq 0}$ *process is uniformly exponentially stable if and only if* $\mu\|B\| < \omega_0$.
ii. The solution of the equation (2.4) is mean stable if the relation $\lambda\|B\| < \omega_0$ *takes place. Mutually, if the solution of the equation (2.4) is mean stable, then the semigroup* $\{T(t)\}_{t\geq 0}$ *is uniformly exponentially stable.*

Proof. i. By passing to mean in the equation (2.4) we obtain the differential equation:

$$d\mathbf{E}\{X(t)\} = (A + \mu B)\mathbf{E}\{X(t)\}, \quad \mathbf{E}\{X(0)\} = x_0 \tag{2.6}$$

From Phillips theorem [3] it results that if $B \in L(H)$, then the operator $A + \mu B$ is a generator of C_0 semigroup $\{\tilde{T}(t)\}_{n\geq 0}$ with $D(A + \mu B) = D(A)$ and verifies the property $(\exists)k, \omega \geq 0$ such that $\|T(t)\| \leq ke^{\omega t} \Rightarrow \|\tilde{T}(t)\| \leq ke^{(\omega+\mu\|B\|)t}$. If $\mathbf{E}\{X(t)\}$ is uniformly exponentially stable, then the semigroup $\tilde{T}(t)$ is uniformly exponentially stable. It results that $T(t)$ is uniformly exponentially stable. Mutually, if $T(t)$ is uniformly exponentially stable, then $\|\tilde{T}(t)\| \leq ke^{-\omega_0 t+\mu\|B\|t}$, and so $\mu\|B\| < \omega_0 \Rightarrow \|\tilde{T}(t)\|$ is uniformly exponentially stable.

ii. By passing to norm in the relation (2.5) we obtain :

$$\|X(t)\| \leq \|T(t)\| \cdot \| \prod_{i=1}^{N(t)} (B + I)x_0\| \leq \|T(t)\| \cdot (\|B\| + 1)^{N(t)}\|x_0\|$$

$$\Rightarrow \mathbf{E}\{\|X(t)\|\} \leq \|T(t)\|e^{\mu\|B\|t}.$$

If $\{T(t)\}_{n\geq 0}$ is uniformly exponentially stable, then $\mathbf{E}\{\|X(t)\|\} \leq ke^{-\omega_0 t+\mu\|B\|t} \Rightarrow \mu\|B\| < \omega_0$, and the conclusion is obtained immediately.

Mutually, the proof is obtained immediately using the stability definition.

□

Lemma 2.2. *If B is a normal operator, then the covariance operator associated to the solution of the equation (2.4), denoted by* $\tilde{Q} \in L(H)$, *is given by the relation :*

$$\langle\tilde{Q}(t)y, z\rangle = \langle\tilde{T}(t)(e^{\mu BB^* t} - I)(\tilde{T}(t))^* y, z\rangle, (\forall)y, z \in H \tag{2.7}$$

Proof. Using the processes $m(t) = N(t) - \mu(t)$ we obtain the following equation :

$$\begin{cases} d\,X(t) &= (A + \mu B)X(t)dt + BX(t)dm(t) \\ X(0) &= x_0 \end{cases}, \tag{2.8}$$

which is equivalent with the equation (2.4) .

We consider the stochastic processes $\{Y(t)\}_{t\in I}$ which verifies the differential equation :

$$\begin{cases} d\,Y(t) &= BY(t)dm(t) \\ X(0) &= x_0 \end{cases}, \tag{2.9}$$

with the next integral form: $Y(t) = x_0 + \int_0^t BY(s)dm(s)$ *and* $\mathbf{E}\{Y(t)\} = x_0$. Because $\mathbf{E}\{Y(t) \otimes Y(t)\} - x_0 \otimes x_0 = \int_0^t \mu B\mathbf{E}\{Y(s) \otimes Y(s)\}B^* ds$ and the fact that B is a normal operator it results that: $\mathbf{E}\{Y(t) \otimes Y(t)\} = e^{\mu BB^* t} x_0 \otimes x_0, \ \ (\forall) \ \ t \geq 0$ and
$\langle covY(t)y, z\rangle = \langle (e^{\mu BB^* t} - I)x_0 \otimes x_0 y, z\rangle, \ \ (\forall) \ \ y, z \in H$.

Using the lemma 2.2 , we obtain:

$$\langle \tilde{Q}(t)y, z\rangle = \langle \tilde{T}(t)covY(t)\tilde{T}(t)^* y, z\rangle$$

□

Theorem 2.2. *The solution of the equation (2.4) is uniformly exponentially stable if the semigroup $\{T(t)\}_{n\geq 0}$ is uniformly exponentially stable and the exponential increase index verifies the relation:*

$$2\mu\|B\|^2 < \omega_0 \tag{2.10}$$

Proof. From Philips theorem we consider that the operator $A + \mu B$ is a generator of C_0 semigroup $\{\tilde{T}(t)\}_{n\geq 0}$ with the following property $\|\tilde{T}(t)\| \leq ke^{-\omega_0 t + \mu\|B\|t}$, where $\|T(t)\| \leq ke^{-\omega_0 t}, k, \omega_0 > 0$. By passing to the integral form in the equation (2.8) it results : $X(t) = \tilde{T}(t)x_0 + \int_0^t \tilde{T}(t-s)BX(s)dm(s)$

$\Rightarrow \mathbf{E}\{\|X(t)\|^2\} \leq 2\|\tilde{T}(t)\|^2\|x_0\|^2 + 2\mu\int_0^t \|\tilde{T}(t-s)\|^2\|B\|^2\mathbf{E}\{\|X(s)\|^2\}ds$

$\Rightarrow \|\tilde{T}(t)\|^{-2}\mathbf{E}\{\|X(t)\|^2\} \leq 2\|x_0\|^2 + 2\mu\int_0^t \|\tilde{T}(s)\|^{-2}\|B\|^2\mathbf{E}\{\|X(s)\|^2\}ds.$

From Gronwall lemma applied to the next function $f(t) = \|\tilde{T}(t)\|^{-2}\mathbf{E}\{\|X(t)\|^2\}$ we obtain that $\mathbf{E}\{\|X(t)\|^2\} \leq 2\|x_0\|^2k^2e^{(-2\omega_0+4\mu\|B\|^2)t}$.

In conclusion , $X(t)$ is uniformly exponentially stable if $2\mu\|B\|^2 < \omega_0$. □

Lemma 2.3. *Let $\{X(t)\}_{t\geq 0}$ be a stable stochastic uniformly exponentially process associated to the equation (2.4). Then the Lyapunov associated equation is written like this:*

$$\begin{cases} \dot{P}(t) + (A + \mu B)P(t) + P(t)(A + \mu B)^* + \mu BP(t)B^* &= 0 \\ P(0) &= x_0 \otimes x_0 \end{cases}$$

and it has an unique solution in the space of bounded and positive operators given by the relation $P(t) + \tilde{Q}(t) = -\tilde{T}(t) \cdot \tilde{T}(t)^*$.

3. THE STABILITY OF THE JUMP DIFFERENTIAL EQUATIONS IN THE CASE OF TIME DEPENDENT OPERATORS

Let $B : [0,T] \to L(H)$ be an integral bounded function having operators as values and the stochastic differential equation:

$$dY(t) = B(t)Y(t)dN(t), \quad Y(0) = x_0 \in H \tag{3.11}$$

Lemma 3.1. *The evolution solution of the equation (3.11) is given by the following formula:*

$$Y(t) = \prod_{i=1}^{N(t)} (I + B(\tau_i))x_0$$

and has the next properties:
i. $\mathbf{E}\{Y(t)\} = exp\{\mu \int_0^t B(s)ds\}x_0$;
ii. *If* $B(t)$ *is a normal operator,* $(\forall)0 \le t \le T$*, then:*

$$covY(t) = exp(\mu \int_0^t B(s)ds)(exp\{\mu \int_0^t B(s)B^*(s)ds\} - I)(exp(\mu \int_0^t B(s)ds))^* \cdot x_0 \otimes x_0;$$

iii. $\mathbf{E}\{\|Y(t)\|^2\} \le e^{\mu t |||B|||^2} \cdot \|x_0\|^2$*, where* $|||B||| = \sup_{s\in[0,T]} \|B(s)\|$ *is the supremum norm.*

Proof. For us to find the evolution solution we will proceed in the same way as in the proof of the theorem 2.1.
i. Passing to mean in the relation (3.11) , we obtain: $d\mathbf{E}\{Y(t)\} = \mu B(t)\mathbf{E}\{Y(t)\}dt$, from where we have the conclusion. An immediate result is:

$$\mathbf{E}\{\prod_{i=1}^{N(t)} (I + B(\tau_i)\} = exp\{\mu \int_0^t B(s)ds\}.$$

ii. The solving idea remains the same like in the lemma 2.3. □

Lemma 3.2. *Let* $A : D(A) \subset H \to H$ *be a generator of strong continuous semigroup* $\{T(t)\}$, $B : [0,T] \to L(H)$ *a bounded integrable family with operators as values and the equation*

$$\begin{cases} dX(t) &= AX(t)dt + B(t)X(t)dN(t) \\ X(s) &= x_s, \ \ x_s \in H, \ \ 0 \le s < t < T \end{cases} \tag{3.12}$$

The equation (3.12) has an unique solution :

$$X(t) = \prod_{i=N(s)+1}^{N(t)} (I + B(\tau_i))T(t)x_s \tag{3.13}$$

with the next properties :
i. $\mathbf{E}\{X(t)\} = U(t,s)$ *is an evolution operator of which generator family is*

$$\{A + \mu B(t)\}_{t \ge 0}.$$

So the mean of the process $\{X(t)\}_{n\geq 0}$ is the perturbation of the semigroup $\{T(t)\}_{t\geq 0}$ using the operators $\{B(t)\}_{t\geq 0}$. The integral equation associated to the family of evolution operators is

$$U(t,s)x = T(t-s)x + \mu \int_s^t T(t-r)B(r)U(r,s)xdr;$$

ii. In the hypothesis of the lemma 3.5 (ii) we obtain that :

$$cov\{X(t)\} = U(t,0)covY(t)U(t,s)^*.$$

Proof. We apply the anterior lemma and the lemma 2.2 . □

Theorem 3.1. *Let $\{X(t)\}_{n\geq 0}$ be the solution of the equation (3.12) in the case where $s=0$. We suppose that the semigroup $\{T(t)\}_{t\geq 0}$ is uniformly exponentially stable.*
The following implications take place :
i. If $\mu\|\|B\|\| < \omega_0$, then the mean of the process $\{X(t)\}_{n\geq 0}$ is uniformly exponentially stable;
ii. If $\mu\|\|B\|\| < \omega_0$, then the process $\{X(t)\}_{n\geq 0}$ is mean stable ;
iii. If $2\mu\|\|B\|\| < \omega_0$, then the process $\{X(t)\}_{n\geq 0}$ is uniformly exponentially stable.

Proof. Is similar with the proof of the theorems 2.1 and 2.2.

The reciprocal of the affirmations (i), (ii) and (iii) are true. Namely, the mean stability and the uniformly exponentially stability of the process $\{X(t)\}_{n\geq 0}$ involve the uniformly exponentially stability of the semigroup $\{T(t)\}_{t\geq 0}$.

In the hypothesis in which $B \in B_\infty(0,T;L(H))$, for the equation (3.12), we obtain the following result . The mean of the process $\{X(t)\}_{t\geq 0}$ is well determined by the evolution operator $S(t,s)$ given by the differential equation :

$$S(t,s)x_0 = T(t-s)x_0 + \int_s^t \mu T(t-r)B(r)S(r,s)x_0 dr, \quad 0 \leq s \leq r \leq t \leq T. \tag{3.14}$$

This equation has an unique evolution solution with the property that:

$$\|S(t,s)\| \leq \|T(t-s)\| + \int_s^t \mu\|T(t-r)\| \cdot \|B\|_\infty \cdot \|U(r,s)dr\|, \tag{3.15}$$
$$\|B\|_\infty = ess \sup_{0\leq t\leq T} \|B(t)\|_{L(H)}$$

(details in [4]). □

Theorem 3.2. *If the semigroup $\{T(t)\}_{t\geq 0}$ is uniformly exponentially stable and verifies the relation $k\cdot\mu\cdot\|B\|_\infty < \omega_0$, then the mean of the process is uniformly exponentially stable.*

Proof. In the relation (3.15) we consider $s=0$ and we apply Gronwall's lemma to the next function:

$$f(t) = e^{\omega_0 t}\|S(t,0)\|, \quad 0 \leq t \leq T.$$

In this way we obtain the relation

$$\|S(t,0)\| \leq ke^{-(\omega_0 - \mu\cdot k\cdot\|B\|_\infty)t}$$

and so the conclusion is verified. □

For determining the covariance operator associated to the $\{X(t)\}_{t\geq 0}$ process we use the evolution operator (denoted by $\tilde{S}(t,s),\ \ 0 \leq s \leq t$) associated to the generator family $\{\mu B(t)B^*(t)\}_{t\geq 0}$, in the case where $B(t)$ is a normal operator $(\forall) 0 \leq t \leq T$. So

$$covX(t) = U(t,0)(\tilde{S}(t,0) - I)U^*(t,0)x_0 \otimes x_0,\ \ 0 \leq t \leq T.$$

Remark 3.1. In the study of the stability of $\{X(t)\}_{n\geq 0}$ solution we obtain the same results as in the theorem 3.3, mentioning that the supremum norm of the operator $B(\cdot)$ has been changed with the essential supremum norm $\|B\|_\infty$.

Lemma 3.3. *The Lyapunov equation has the next form :*

$$\left\{ \begin{array}{rcl} \dot{P}(t) + (A + \mu B(t))P(t) + P(t)(A + \mu B(t))^* + \mu B(t)P(t)B^*(t) & = & 0 \\ P(0) & = & x_0 \otimes x_0 \end{array} \right. \tag{3.16}$$

If $\{X(t)\}$ is uniformly exponentially stable, then the equation (3.16) has an unique solution which verifies the relation

$$P(t) + Q(t) = -U(t,0)U^*(t,0).$$

For passing from the semigroup $\{T(t)\}_{t\geq 0}$ operator case to the evolution operator case we are using the Kato-Tanabe hypothesis ([3], [4]) associated to the operator family on H, $\{A(t)\}_{t\geq 0}$. In this case the equation (2.4) is rewritten like this:

$$\left\{ \begin{array}{rcl} dX(t) & = & A(t)X(t)dt + B(t)X(t)dN(t) \\ X(s) & = & x_s,\ \ x_s \in H,\ \ 0 \leq s < t < T \end{array} \right. \tag{3.17}$$

Lemma 3.4. *i. Let $\{U(t,s)\},\ \ 0 \leq s \leq t \leq T$ be the evolution operator associated to the $\{A(t)\}_{t\geq 0}$ family. Then the mean of the process $\{X(t)\}_{t\geq 0}$ is given by the relation:*

$$\mathbf{E}\{X(t)\} = \tilde{U}(t,s),$$

where $\tilde{U}(t,s)x = U(t,s)x + \mu \int_s^t U(t,r)B(r)\tilde{U}(r,s)xdr$, $(\forall) x \in H$, *with the property that*

$$\|\tilde{U}(t,s)\| \leq p(t-s)e^{(t-s)\mu \int_s^t \|B(r)\|dr},\ \ 0 \leq s < t < T,$$

having $p : \mathbf{R}_+ \to \mathbf{R}_+$ an increase function [4] ;
ii. The mean of the process $\{X(t)\}_{n\geq 0}$ is uniformly exponentially stable if and only if the evolution operator $\{U(t,s)\}$ is uniformly exponentially stable and $\mu \cdot \|B\|_{L^1} < \nu$, where $\|B\|_{L^1} = \int_0^T \|B(r)\|dr$.

Proof. i. Can be found in [4];
ii. $\{U(t,s)\}$ is uniformly exponentially stable if $(\exists) k > 0, \nu > 0$ such that

$$\|U(t,s)\| \leq ke^{-\nu(t-s)},\ \ 0 \leq s < t < T \Rightarrow \|\tilde{U}(t,s)\| \leq ke^{(-\nu + \mu \int_s^t \|B(r)\|dr)(t-s)}.$$

So $\mu\|B\|_{L^1} < \nu \ \ \Rightarrow \ \ \|\tilde{U}(t,s)\|$ is uniformly exponentially stable.
Mutually the result is obvious.

□

Theorem 3.3. *It can be seen very easily from the previous theorems that we can obtain the next results :*
i. $X(t,0) = \prod_{i=0}^{N(t)}(I + B(\tau_i))U(t,0)X(0)$ *is the evolution solution associated to the equation (3.17) ;*
ii. $\mathbf{E}\{X(t,s)\} = \tilde{U}(t,s), \quad 0 \leq s < t < T$ *;*
iii. $cov X(t,s) = \tilde{U}(t,s)(S(t,s) - I)\tilde{U}(t,s)^*, \quad 0 \leq s < t < T$ *;*
iv. $\{X(t,0)\}$ *is uniformly exponentially stable if and only if the evolution operator is uniformly exponentially stable and* $2\mu\|B\|_\infty \leq \nu$*.*

REFERENCES

[1] Capasso, V. and Bakstein, D.,*An Introduction to Continuous - Time Stochastic Processes, Theory, Models and Applications to Finance, Biology and Medicine*, Birkhauser, Boston, (2005)

[2] Curtain, R. F., *Estimation Theory for Abstract Evolution Equations Excited by General White Noise Processes*, SIAM J. Control, 14, 1976, 124-150

[3] Curtain, R. F. and Zwart, H., *An Introduction To Infinite Dimensional Linear System Theory*, Springer Verlag, (1995)

[4] Megan, M., *Propriétès Qualitatives Des Systèmes Linéaires Contrôlés Dans Les Espaces De Dimension Infinie, Monographies Mathématiques*, Timişoara, 1998

[5] Moleriu, R., *The Stability for the Solutions of Stochastic Differential Equations*, Analele Universităţii de Vest din Timişoara, Seria Matematică Informatică, Vol. XLV, Fasc. 2, 2007, 67-74

[6] Sennewald, K., and Walde, K., *Îto's lema and Bellman equations for Poisson processes: An Applied View*, Journal of Economics, Springer 2006, 1-36

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