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Dedicated to Professor Iulian Coroian on the occasion of his 70^{th} anniversary

Empirical study of the rate of convergence of some Newton type methods

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ABSTRACT. In this paper an empirical study of the rate of convergence of some Newton type methods is made. It is originating in the formula

$$x_{n+1} = x_n + \frac{2f(x_n)}{f'(x_n) + M_n}, \quad x_0 \in [a, b] \text{ prechosen}, n = 0, 1, 2, ...,$$

where

$$M = \sup_{x \in [a,b]} |f'(x)|, \quad M_n = M sign f'(x_n)$$

Some numerical examples to illustrate the study are also given.

1. INTRODUCTION

In the paper [2] the authors study the convergence of some Newton type methods, originating from the well-known Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, ... \quad . \tag{1.1}$$

On the other hand, in the paper [3], the authors proved a convergence theorem for a Newton type method of the form

$$x_{n+1} = x_n + \frac{2f(x_n)}{f'(x_n) + M_n}, \quad x_0 \in [a, b] \text{ prechosen}, n = 0, 1, 2, ..., \tag{1.2}$$

where

$$M = \sup_{x \in [a,b]} |f'(x)|, \quad M_n = M sign f'(x_n) \tag{1.3}$$

which has been obtained by the so called extended Newton method in [1]. In the present paper, following the lines in [2] for the Newton method, we perform an empirical study of the methods obtained from (1.2), (1.3), similarly to the case of (1.1).

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2. Newton type methods

In [2] the authors considered iterative methods of the form

$$z_n = \phi_p(x_n) \tag{2.4}$$

$$x_{n+1} = z_n - H(x_n, y_n)\frac{f(z_n)}{f'(x_n)} \tag{2.5}$$

where $H(x,y)$ represents a given two-variable function and

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{2.6}$$

Method 1 (M1)

$$z_n = \phi_3(x_n),\ \ \phi_3(x) = x - \frac{2f(x)}{f'(x) + f'(y(x))}, \tag{2.7}$$

$$H(x,y) = \frac{f'(x)}{f'(y(x))}. \tag{2.8}$$

Method 2 (M2)

$$z_n = \phi_3(x_n),\ \ \phi_3(x) = x - \frac{f(x)}{2}\left(\frac{1}{f'(x)} + \frac{1}{f'(y(x))}\right), \tag{2.9}$$

$$H(x,y) = \frac{f'(x) + f'(y)}{3f'(y) - f'(x)}. \tag{2.10}$$

Method 3 (M3)

$$z_n = \phi_3(x_n),\ \ \phi_3(x) = x - \frac{2f(x)}{f'(x) + f'(y(x))}, \tag{2.11}$$

$$H(x,y) = \frac{f'(y)}{2f'(y) - f'(x)}. \tag{2.12}$$

Method 4 (M4)

$$z_n = \phi_3(x_n),\ \ \phi_3(x) = x - \frac{f(x)}{2}\left(\frac{1}{f'(x)} + \frac{1}{f'(y(x))}\right), \tag{2.13}$$

$$H(x,y) = \frac{3f'(x) - f'(y)}{f'(x) + f\prime(y)}. \tag{2.14}$$

Method 5 (M5)

$$z_n = \phi_3(x_n),\ \ \phi_3(x) = x - \frac{2f(x)}{f'(x) + f'(y(x))}, \tag{2.15}$$

$$H(x,y) = \frac{2f'^2(y)}{f'^2(x) - 4f'(x)f'(y) + 5f'^2(y)}. \tag{2.16}$$

Method 6 (M6)

$$z_n = \phi_3(x_n),\ \ \phi_3(x) = x - \frac{f(x)}{2}\left(\frac{1}{f'(x)} + \frac{1}{f'(y(x))}\right), \tag{2.17}$$

$$H(x,y) = -\frac{2f'^2(y)}{f'^2(x) - 4f'(x)f'(y) + f'^2(y)}. \tag{2.18}$$

By considering (1.2) and (1.3) instead of (1.1) in the constructions of the methods M1 - M3 we obtain the following six new methods
NM1: For function H (2.8) and ϕ_3 defined in (2.7) we obtain

$$x_{n+1} = z_n - \frac{f'(x_n)}{f'(y_n)} \cdot \frac{2f(z_n)}{f'(x_n) + M_n}. \tag{2.19}$$

NM2: For function H from (2.10) and ϕ_3 given in (2.9) the obtained method is

$$x_{n+1} = z_n - \frac{f'(y_n) + f'(x_n)}{3f'(y_n) - f'(x_n)} \cdot \frac{2f(z_n)}{f'(x_n) + M_n}. \tag{2.20}$$

NM3: For function H from (2.12) and ϕ_3 defined in (2.11) the method looks like below

$$x_{n+1} = z_n - \frac{f'(y_n)}{2f'(y_n) - f'(x_n)} \cdot \frac{2f(z_n)}{f'(x_n) + M_n}. \tag{2.21}$$

NM4: For function H given in (2.14) and ϕ_3 defined in (2.13) the method becomes

$$x_{n+1} = z_n - \frac{3f'(x_n) - f'(y_n)}{f'(y_n) + f'(x_n)} \cdot \frac{2f(z_n)}{f'(x_n) + M_n}. \tag{2.22}$$

NM5: For function H given in (2.16) and ϕ_3 defined in (2.15) the method becomes

$$x_{n+1} = z_n - \frac{2f'^2(y)}{f'^2(x) - 4f'(x)f'(y) + 5f'^2(y)} \cdot \frac{2f(z_n)}{f'(x_n) + M_n}. \tag{2.23}$$

NM6: For function H given in (2.18) and ϕ_3 defined in (2.17) the method becomes

$$x_{n+1} = z_n + \frac{2f'^2(y)}{f'^2(x) - 4f'(x)f'(y) + f'^2(y)} \cdot \frac{2f(z_n)}{f'(x_n) + M_n}, \tag{2.24}$$

where M_n is given in (1.3).

All computations were done in MAPLE using 256 digit floating point arithmetics. We set $\varepsilon = 2^{-255}$ as iteration tolerance number. We used the following test functions and display the approximate zeros x_* up to the 31st decimal place.

$f_1(x) = x^3 + 4x^2 - 10, \quad x_* = 1.3652300134140968457608068289817$
$f_2(x) = \sin^2 x - x^2 + 1, \quad x_* = 1.4044916482153412260350868177869$
$f_3(x) = x^2 - e^x - 3x + 2, \quad x_* = 0.2575302854398607604553673049371$
$f_4(x) = \cos x - x, \quad x_* = 0.7390851332151606416553120876739$
$f_5(x) = x \cdot e^x - \sin^2 x + 3\cos x + 5, \quad x_* = -1.0942870722082091204228027872276$
$f_6(x) = (x^3 + 4x^2 - 10)^2, \quad x_* = 1.3652300134140968457608068289817$
$f_7(x) = (x-1)^2 e^x, \quad x_* = 1$

$f(x)$	NM1 (M1)	NM2 (M2)	NM3 (M3)	NM4 (M4)	NM5 (M5)	NM6 (M6)
$f_1(x)$ $x_0 = 0.8$	6 (5)	5 (4)	5 (5)	6(4)	6 (5)	6 (5)
$f_1(x)$ $x_0 = 1$	5 (4)	5 (4)	5 (4)	5 (4)	6 (5)	5 (5)
$f_2(x)$ $x_0 = 2.3$	6 (5)	5 (4)	6 (5)	5 (4)	6 (6)	6 (5)
$f_2(x)$ $x_0 = 1$	6 (5)	6 (4)	6 (5)	6 (5)	6 (5)	6 (5)
$f_3(x)$ $x_0 = 1$	5 (4)	5 (4)	5 (4)	5 (4)	5 (4)	5 (5)
$f_3(x)$ $x_0 = 0$	5 (4)	5 (4)	5 (4)	5 (4)	5 (4)	5 (4)
$f_4(x)$ $x_0 = 1.7$	5 (4)	5 (4)	5 (4)	5 (4)	5 (5)	5 (5)
$f_4(x)$ $x_0 = 0$	5 (4)	5 (4)	5 (4)	6 (4)	5 (5)	5 (6)
$f_5(x)$ $x_0 = -1$	6 (4)	6 (4)	6 (4)	6 (4)	6 (5)	6 (5)
$f_5(x)$ $x_0 = -0.5$	div (div)	div (div)	7 (23)	div (div)	8 (8)	div (div)
$f_6(x)$ $x_0 = 1.4$	531 (388)	421 (315)	div (div)	421 (361)	532 (618)	422 (409)
$f_6(x)$ $x_0 = 0.8$	533 (389)	422 (315)	div (div)	422 (361)	533 (620)	423 (409)
$f_7(x)$ $x_0 = 1.1$	532 (389)	422 (315)	div (div)	422 (361)	533 (619)	422 (410)
$f_7(x)$ $x_0 = 0.9$	532 (389)	422 (315)	div (div)	422 (361)	533 (619)	422 (410)

3. CONCLUSIONS

Displayed in the table above are the number of iterations required such that $|f(x_n)| < \varepsilon$. In the table 'div' means that the sequence of approximative zeros produced from the corresponding method does not converge within the maximum iteration number. The numbers in parentheses represent the values obtained by use of the methods described in [2].

During the study we also tried to replace $\dfrac{f(x)}{f'(x)}$ with $\dfrac{2f(x)}{f(x)+M_n}$ in z_n and x_{n+1}. But as the study went along we noticed that the results were better when we made the replacements in only one place. The number of iteration decreased at least by two.

We also noticed that the new methods we have studied empirically in the paper can be considered as good as the Newton like method presented in [1].

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