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Dedicated to Professor Iulian Coroian on the occasion of his 70^{th} anniversary

On the finiteness of the reachability set for jumping Petri nets

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ABSTRACT. In this paper we extend the decidability result concerning the finiteness of the reachability set of a net from classical Petri nets to jumping Petri nets.

1. INTRODUCTION

A Petri net ([5, 1]) is a mathematical model used for the specification and the analysis of parallel and distributed systems.

Petri nets proved to be a powerful language for system modelling and validation and they are now in widespread use for many different practical and theoretical purposes in various fields of software and hardware development.

One type of problems related to Petri nets is that of finding algorithms which take a Petri net Σ and a property π as input and answer, after a finite number of steps, whether or not Σ satisfies π. For instance, the Karp-Miller graph for Petri nets allows us to decide the boundedness problem (BP), the finiteness reachability set / tree problem (FRSP / FRTP), the quasi-liveness problem (QLP), and the coverability problem (CP) (see [4, 6, 1] for more details).

It is well-known that the behaviour of some distributed systems cannot be adequately modelled by classical Petri nets. Many extensions which increase the computational and expressive power of Petri nets have been thus introduced. One direction has led to various modifications of the firing rule of nets. One of these extension is that of jumping Petri net, introduced in [7]. A jumping Petri net is a classical net Σ equipped with a (recursive) binary relation R on the markings of Σ. The meaning of a pair $(m, m') \in R$ is that the net Σ may "spontaneously jump" from m to m' (this is similar to λ-moves in automata theory).

Previous results (see [7]) showed that the decision problems related to reachability, coverability and quasi-liveness are undecidable for general jumping nets and are decidable only for finite jumping nets, by using the techniques of Karp-Miller coverability graphs in a similar manner as for classical P / T nets ([4]).

In [9] we introduced a larger class of jumping nets than the finite jumping nets, called reduced-computable jumping nets, for which we could define finite Karp-Miller coverability graphs. Based on them, in this paper we will extend a decidability result concerning the finiteness of the reachability set of a net from classical Petri nets to jumping Petri nets.

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The paper is organized as follows. Section 2 presents the basic terminology and notation, and also previous results concerning Petri nets and jumping Petri nets. In Section 3, we use the Karp-Miller coverability structures to establish the decidability of the finiteness of the reachability set problem for reduced-computable jumping Petri nets. Finally, in Section 4 we conclude this paper and formulate some open problems.

2. PRELIMINARIES

In this section we will establish the basic terminology, notation, and results concerning Petri nets in order to give the reader the necessary prerequisites for the understanding of this paper (for details the reader is referred to ([5, 3, 1]). Mainly, we will follow [3, 7].

2.1. **Petri nets.** A *Place/Transition net*, shortly *P/T-net* or *net*, (finite, with infinite capacities), abbreviated *PTN*, is a 4-tuple $\Sigma = (S, T; F, W)$, where S and T are two finite non-empty sets (of *places* and *transitions*, resp.), $S \cap T = \emptyset$, $F \subseteq (S \times T) \cup (T \times S)$ is the *flow relation* and $W : (S \times T) \cup (T \times S) \to \mathbb{N}$ is the *weight function* of Σ satisfying $W(x, y) = 0$ iff $(x, y) \notin F$.

A *marking* of a *PTN* Σ is a function $m : S \to \mathbb{N}$; it will be sometimes identified with a $|S|$-dimensional vector. The operations and relations on vectors are defined component-wise. $\mathbb{N}^S$ denotes the set of all markings of Σ. A *marked PTN*, abbreviated *mPTN*, is a pair $\gamma = (\Sigma, m_0)$, where Σ is a *PTN* and m_0, called the *initial marking* of γ, is a marking of Σ.

In the sequel we often use the term "Petri net"(*PN*) or "net" whenever we refer to a *PTN* (*mPTN*) and it is not necessary to specify its type.

Let Σ be a net, $t \in T$ and $w \in T^*$. The functions $t^-, t^+ : S \to \mathbb{N}$ and $\Delta t, \Delta w : S \to \mathbb{Z}$ are defined by $t^-(s) = W(s, t)$, $t^+(s) = W(t, s)$, $\Delta t(s) = t^+(s) - t^-(s)$ and

$$\Delta w(s) = \begin{cases} 0, & \text{if } w = \lambda, \\ \sum_{i=1}^{n} \Delta t_i(s), & \text{if } w = t_1 t_2 \dots t_n \ (n \geq 1), \end{cases} \quad \text{for all } s \in S.$$

The sequential behaviour of a net Σ is given by the so-called *firing rule*:

- (ER) the *enabling rule*: a transition t is *enabled* at a marking m in Σ (or t is *fireable* from m), abbreviated $m[t\rangle_\Sigma$, iff $t^- \leq m$;
- (CR) the *computing rule*: if $m[t\rangle_\Sigma$, then t may *occur* yielding a new marking m', abbreviated $m[t\rangle_\Sigma m'$, defined by $m' = m + \Delta t$.

In fact, for any transition t of Σ we have a binary relation on $\mathbb{N}^S$, denoted by $[t\rangle_\Sigma$ and given by: $m[t\rangle_\Sigma m'$ iff $t^- \leq m$ and $m' = m + \Delta t$. If $t_1, t_2, \dots, t_n$, $n \geq 1$, are transitions of Σ, the classical product of the relations $[t_1\rangle_\Sigma, \dots, [t_n\rangle_\Sigma$ will be denoted by $[t_1 t_2 \dots t_n\rangle_\Sigma$; i.e. $[t_1 t_2 \dots t_n\rangle_\Sigma = [t_1\rangle_\Sigma \circ \dots \circ [t_n\rangle_\Sigma$. Moreover, we also consider the relation $[\lambda\rangle_\Sigma$ given by $[\lambda\rangle_\Sigma = \{(m, m) | m \in \mathbb{N}^S\}$.

Let $\gamma = (\Sigma, m_0)$ be a marked Petri net, and $m \in \mathbb{N}^S$. The word $w \in T^*$ is called a *transition sequence* from m in Σ if there exists a marking m' such that $m[w\rangle_\Sigma m'$. Moreover, the marking m' is called *reachable* from m in Σ. We denote by $RS(\Sigma, m) = [m\rangle_\Sigma = \{m' \in \mathbb{N}^S | \exists w \in T^* : m[w\rangle_\Sigma m'\}$ the set of all reachable markings from m in Σ. In the case $m = m_0$, the set $RS(\Sigma, m_0)$ is abbreviated by $RS(\gamma)$ (or $[m_0\rangle_\gamma$) and it is called *the set of all reachable markings* of γ.

We will assume to be known other notions from P / T-nets, like coverable marking, bounded place, (quasi-) live transition, pseudo-markings, etc. For more details about these notions, and about the basic decision problems for P / T-nets, the reachability structures and the Karp-Miller coverability structures for them, and also about the case of P / T-nets with infinite initial markings, the reader is referred to ([10, 3, 1]).

2.2. **Jumping Petri nets.** Jumping Petri nets ([7, 8]) are an extension of classical P / T-nets, which allows them to perform "spontaneous jumps" from one marking to another (this is similar to λ-moves in automata theory).

A *jumping P/T-net*, abbreviated *JPTN*, is a pair $\gamma = (\Sigma, R)$, where Σ is a *PTN* and R, called the *set of (spontaneous) jumps* of γ, is a binary relation on the set of markings of Σ (i.e. $R \subseteq \mathbb{N}^S \times \mathbb{N}^S$). In what follows the set R of jumps of any *JPTN* will be assumed to be *recursive*. A *marked jumping P/T-net*, abbreviated *mJPTN*, is defined similarly as a *mPTN*, by changing "Σ" into "Σ, R".

Let $\gamma = (\Sigma, R)$ be a *JPTN*. The pairs $(m, m') \in R$ are referred to as *jumps* of γ. If γ has finitely many jumps (i.e. R is finite) then we say that γ is a *finite jumping net*, abbreviated *FJPTN*.

We shall use the term "*jumping net*" (*JN*) ("*finite jumping net*" (*FJN*), resp.) to denote a *JPTN* or a *mJPTN* (a *FJPTN* or a *mFJPTN*, resp.) whenever it is not necessary to specify its type. Pictorially, a jumping Petri net will be represented as a classical net and, moreover, the relation R will be separately listed.

The behaviour of a jumping net γ is given by the *j-firing rule*, which consists of

- (jER) the *j-enabling rule*: a transition t is *j-enabled* at a marking m (in γ), abbreviated $m[t\rangle_{\gamma,j}$, iff there exists a marking m_1 such that $mR^*m_1[t\rangle_\Sigma$ (Σ being the underlying net of γ and R^* the reflexive and transitive closure of R);
- (jCR) the *j-computing rule*: if $m[t\rangle_{\gamma,j}$, then the marking m' is *j-produced* by occurring t at the marking m, abbreviated $m[t\rangle_{\gamma,j}m'$, iff there exists markings m_1, m_2 such that $mR^*m_1[t\rangle_\Sigma m_2R^*m'$.

The notions of *transition j-sequence* and *j-reachable marking* are defined similarly as for Petri nets (the relation $[\lambda\rangle_{\gamma,j}$ is defined by $[\lambda\rangle_{\gamma,j} = \{(m, m') | m, m' \in \mathbb{N}^S, mR^*m'\}$). The *set of all j-reachable markings* of a marked jumping net γ is denoted by $RS(\gamma)$ (or by $[m_0\rangle_{\gamma,j}$).

All other notions from P / T-nets (i.e. coverable marking, bounded place, pseudo-markings, etc.) are defined for jumping Petri nets similarly as for Petri nets, by considering the notion of *j-reachability* instead of *reachability* from P / T-nets. Also, all the decision problems from P / T-nets, like (RP), (CP), (BP) and (FRSP), are defined for jumping Petri nets similarly as for P / T-nets.

Some jumps of a marked jumping net may be never used. Thus we say that a marked jumping net $\gamma = (\Sigma, R, m_0)$ is *R-reduced* ([7]) if for any jump $(m, m') \in R$ of γ we have $m \neq m'$ and $m \in [m_0\rangle_{\gamma,j}$. The *reduction problem* (RedP) is: Given γ a *JPTN*, is γ R-reduced?

Remark 2.1. The following decidability results were proved in [7, 3]: i) the problems (RP), (CP), (BP) are undecidable for *mJPTN*; ii) the problems (RP), (RedP), (CP), (BP) are decidable for *mFJPTN*.

Coverability structures for jumping Petri nets. The previous positive decidability results from [7] were based on defining Karp-Miller coverability trees only for the subclass of *finite* jumping Petri nets. Therefore, we were interested in extending the class of jumping Petri nets for which we can define finite Karp-Miller coverability structures. Having such a larger class of nets, afterwards we can solve the above decidability problems for it based on these finite coverability structures.

In [9] we succeeded to introduce a larger class of jumping nets than the finite jumping nets, called *reduced-computable jumping nets*, for which we could define finite Karp-Miller coverability structures (trees and graphs), and also minimal coverability structures; moreover, we extended the results about the minimal coverability structures for P / T-nets from [2] to this class of jumping nets.

Let us recall from [9] the definition of reduced-computable jumping nets.

Let $\gamma = (\Sigma, R)$ be an arbitrary jumping net. We associated to γ a finite subset of jumps $R_{\omega-max}$ (which is maximal in a sense specified below, and which can be used instead of the whole set of jumps R to construct the coverability graphs) as follows. We denoted by "ω-jumps" the set

$R_\omega = \Big\{ r \in \mathbb{N}_\omega^{2|S|} - \mathbb{N}^{2|S|} \,;\, \exists \{r_n\}_{n \geq 0} \subseteq R$ strictly increasing with $\lim_{n\to\infty} r_n = r \Big\}$.

Let $\overline{R} = R \cup R_\omega$. We defined the *set of ω-maximal jumps* of γ as

$$R_{\omega-max} = maximal(\overline{R}) = \{r' \in \overline{R} \mid \forall r \in \overline{R} - \{r'\} : r' \not\leq r\}.$$

The following are obvious properties of the set of ω-maximal jumps of a jumping net (the proofs are easy and can be found in [9]):

Proposition 2.1. *(1) $R_{\omega-max}$ is finite ; (2) $\forall\, r \in \overline{R}, \exists\, r' \in R_{\omega-max}$ such that $r \leq r'$; (3) $\forall\, r \in R_{\omega-max}, \exists\, \{r_n\}_{n\geq 0} \subseteq R$ such that $\lim_{n\to\infty} r_n = r$.*

A marked jumping net is called *reduced-computable jumping net* ([9]), abbreviated *mRCJPTN*, if it is R-reduced and the set $R_{\omega-max}$ is computable.

In [9] we also introduced reachability trees and graphs for jumping nets, by a straightforward extension of these structures from classical P / T-nets (i.e. by adding arcs, labelled by "j", for all the jumps of the net).

Now let us recall from [9] the definition of coverability trees and graphs generalized for reduced-computable jumping Petri nets.

Let $\gamma = (\Sigma, R, m_0)$ be a *mRCJPTN* with $R \neq \emptyset$. Then the set of ω-maximal jumps is non-empty and finite, i.e.

$$R_{\omega-max} = \{\ (m'_i, m''_i)\ |1 \leq i \leq n\},\ \text{with } n \geq 1.$$

Following the same line as in [7], we associated to γ the following P / T-nets: $\gamma_0 = (\Sigma, m_0)$ and $\gamma_i = (\Sigma, m''_i)$, for each $1 \leq i \leq n$, and we defined the notions of *Karp-Miller coverability trees / graphs* of the jumping net γ as being the tuples of the coverability trees / graphs of the P / T-nets γ_0,γ_1,..., γ_n:

$$\mathcal{KMT}(\gamma) = < \mathcal{KMT}(\gamma_0), \mathcal{KMT}(\gamma_1), \ldots, \mathcal{KMT}(\gamma_n) > \ ,$$

and respectively

$$\mathcal{KMG}(\gamma) = < \mathcal{KMG}(\gamma_0),\ \mathcal{KMG}(\gamma_1), \ldots,\ \mathcal{KMG}(\gamma_n)\ > .$$

Notice that it is possible that some of the P / T-nets γ_0,γ_1, ...,γ_n to have initial markings with ω-components.

3. THE FINITENESS OF THE REACHABILITY SET

In this section we will show how we can use the Karp-Miller coverability graph $\mathcal{KMG}(\gamma)$ to solve the finiteness of the reachability set problem for reduced-computable jumping Petri nets.

We have the following result:

Proposition 3.2. *Let γ be a mRCJPTN. Then $RS(\gamma)$ is infinite iff $\exists\, 0 \le i \le n$ such that $RS(\gamma_i)$ is infinite or $\Omega(m''_i) \neq \emptyset$ (i.e. the initial marking of γ_i has ω-components), where $\gamma_0, \gamma_1, \ldots, \gamma_n$ are the P/T-nets associated to the net γ.*

Proof. The proof follows easily proceeding from the two following remarks:

(*) $RS(\gamma')$ is infinite iff γ' is unbounded, which holds for P/T-nets (only P/T-nets with finite initial markings) as well as for jumping Petri nets;

(**) If $\gamma' = (\Sigma', m'_0)$ is a P/T-net with an infinite initial marking, (i.e. $\Omega(m'_0) \neq \emptyset$), then all places from the set $\Omega(m'_0)$ are unbounded, so the net γ' is unbounded. Still, the set $RS(\gamma')$ can be finite.

Indeed, by using (*), we have that:
$RS(\gamma)$ is infinite $\Leftrightarrow$ γ is unbounded $\Leftrightarrow$ $\exists s \in S$ such that s is unbounded in γ.

And by using (*) and (**), we have that:
$\exists 0 \le i \le n$ such that $RS(\gamma_i)$ is infinite or $\Omega(m''_i) \neq \emptyset$ $\Leftrightarrow$ $\exists 0 \le i \le n$ such that γ_i is unbounded $\Leftrightarrow$ $\exists 0 \le i \le n, \exists s \in S$ such that s is unbounded in γ_i $\Leftrightarrow$ $\exists s \in S, \exists 0 \le i \le n$ such that s is unbounded in γ_i.

Thus, to finish the proof, it is sufficient to show that s is unbounded in γ $\Leftrightarrow$ $\exists 0 \le i \le n$ such that s is unbounded in γ_i. But this statement is true (we proved it in [11]). □

Theorem 3.1. *Let γ be a mRCJPTN. The reachability set of γ, $RS(\gamma)$, is infinite iff there is at least one infinite node (i.e. a pseudo-marking which contains at least one symbol ω) in at least one graph from the Karp-Miller coverability graph $\mathcal{KMG}(\gamma)$.*

Proof. This statement follows easily from the definition of the Karp-Miller coverability graph $\mathcal{KMG}(\gamma)$, from the previous proposition, and from the similar results for P/T-nets (with finite or infinite initial markings). □

Theorem 3.1 holds for every finite coverability graph of γ, not only for the Karp-Miller graph, and so to decide the properties listed in the theorem it is sufficient to compute any finite coverability graph, particularly the minimal one.

From Theorem 3.1 and the similar one from P/T-nets ([4]) we conclude that the following decision problem is solvable by using the Karp-Miller coverability graph for marked reduced-computable jumping Petri nets (or any other finite coverability graph):

Corollary 3.1. *The finiteness of the reachability set problem is decidable for mRCJPTN.*

The use of the minimal coverability graph for solving these decision problems is important from the computational point of view because it is, generally speaking, smaller than the Karp-Miller graph.

4. Conclusions and future work

In this paper we have extended the decidability result concerning the finiteness of the reachability set of a net from classical Petri nets to jumping Petri nets.

An open problem which remains, is to study if there are more efficient algorithms for this decision problem.

References

[1] Desel, J., Reisig, W. and Rozenberg, G., (Eds.), *Lectures on Concurrency and Petri Nets. Advances in Petri Nets 2004*, Lecture Notes in Computer Science **3098**, Springer-Verlag, 2004

[2] Finkel, A., *The minimal coverability graph for Petri nets*, in *Advanced in Petri Nets 1993*, Lecture Notes in Computer Science **674**, pp. 210-243, Springer-Verlag, 1993

[3] Jucan T. and Ţiplea, F.L., *Petri Nets. Theory and Application* (in romanian), Romanian Academy Publishing House, Bucharest, 2nd edition, 1999

[4] Karp, R.M. and Miller, R.E., *Parallel program schemata*, Journal of Computer and System Sciences, **3** (1969), No. 3, 147-195

[5] Reisig, W., *Petri Nets. An Introduction*, Springer-Verlag, Berlin, 1985

[6] Reisig, W., *Place/Transition systems*, In *Petri Nets: Central Models and Their Properties. Advanced in Petri Nets 1986, Part I*, volume 254 of *Lecture Notes in Computer Science*, pp. 117-141, Springer-Verlag, 1987

[7] Ţiplea, F.L. and Jucan, T., *Jumping Petri nets*, Foundations of Computing and Decision Sciences, **19** (1994), No. 4, 319-332

[8] Ţiplea, F.L. and Mäkinen, E., *Jumping Petri nets. Specific properties*, Fundamenta Informaticae, **32** (1997), 373-392

[9] Vidraşcu, C. and Jucan, T., *On coverability structures for jumping Petri nets*, Scientific Annals of the University "Al. I. Cuza" of Iaşi, Computer Science Section, **IX** (2000), 1-26

[10] Vidraşcu, C., *Structural properties of Petri nets. PhD thesis* (in romanian), Technical report of the University "Al. I. Cuza" of Iaşi, Computer Science Department, **tr04-04** (2004)

[11] Vidraşcu, C., *Boundedness problems for jumping Petri nets*, ROMAI Journal, **3** (2007), No. 1, 213-232

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