A convergence criteria for multiple harmonic series

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ABSTRACT.

In this note we study the convergence of multiple harmonic series of the following form

$$\sum_{k_1,\cdots,k_n \geq 1} \frac{k_1^{l_1} k_2^{l_2} \cdots k_n^{l_n}}{\left(k_1^{p_1} + \cdots + k_n^{p_n}\right)^p} \quad \text{and} \quad \sum_{k_1,\cdots,k_n \geq 1} \frac{\left(k_1^{l_1} + k_2^{l_2} + \cdots + k_n^{l_n}\right)^m k_1^{b_1} \cdots k_n^{b_n}}{\left(k_1^{p_1} + \cdots + k_n^{p_n}\right)^p},$$

where m, l_i, b_i are nonnegative real numbers and p and p_i are positive real numbers. We determine exactly when the two harmonic series converge.

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