

Generalizations of some divisibility relations in \mathbb{N}

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ABSTRACT.

The present work gives a number of generalizations for some divisibility relations in the set of natural numbers. Let $n \geq 2$ and x_1, x_2, \dots, x_n be natural numbers. It is well known that $(x_1, x_2)[x_1, x_2] = x_1x_2$, where (x_1, x_2) denotes the greatest common divisor (*gcd*) and $[x_1, x_2]$ the lowest common multiple (*lcm*) of the numbers x_1 and x_2 . In the present paper we derive the formulas for (x_1, x_2, \dots, x_n) , the *gcd* of the numbers x_1, x_2, \dots, x_n , in terms of the lowest common multiple (*lcm*) of the subsets x_{i_1}, \dots, x_{i_k} , with $1 \leq i_1 < \dots < i_k \leq n$. We also find the dual formula for $[x_1, x_2, \dots, x_n]$, the *lcm* of the numbers x_1, x_2, \dots, x_n .

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