

On a family of first order difference inequalities used in the iterative approximation of fixed points

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ABSTRACT.

Several first order difference inequalities which are intensively used in proving convergence theorems for various fixed point iteration procedures are unified and detailedly demonstrated. In terms of the difference equations terminology, the obtained results basically show that the zero solution of these difference inequalities is globally asymptotically stable. Some new more general conditions under which the zero solution of these difference inequalities is globally asymptotically stable, which extends and includes corresponding related results obtained in recent literature, are also given.

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