

The computation of some integrals in compact form

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ABSTRACT.

Based on the Euler-Zagier multiple zeta function ζ and the extended zeta function $\tilde{\zeta}$, we compute integrals of the form

$$I_{k,l,r,m}^{\pm} = \int_0^1 \frac{(-\log(1 \pm x))^k}{(1 \pm x)^m} \cdot x^r (-\log x)^l dx,$$
$$J_{l,r,m,k}^{\pm} = \int_0^1 \frac{x^{kr+k-1}}{(1 \pm x^k)^m} \cdot (-\log x)^l dx$$

and $I_{l,r,m_1,m_2} = \int_0^1 \frac{x^r (-\log x)^l}{(1-x)^{m_1} (1+x)^{m_2}} dx$ where k, l, r, m, m_1, m_2 are integers.

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