

Remarks on some completeness conditions involved in several common fixed point theorems

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ABSTRACT.

In this note we discuss two subspace completeness conditions involved in some recent common fixed point theorems, show that they are indeed weaker than the completeness assumption of the whole ambient space and find a unifying condition for both. Using this fact, several common fixed point theorems are then reformulated under slightly more general conditions.

REFERENCES

- [1] Abbas, M. and Jungck, G., *Common fixed point results for noncommuting mappings without continuity in cone metric spaces*, J. Math. Anal. Appl. **341** (2008), 416-420
- [2] Babu, G. V. R., Sandhya, M. L. and Kameswari, M. V. R., *A note on a fixed point theorem of Berinde on weak contractions*, Carpathian J. Math. **24** (2008), No. 1, 8-12
- [3] Berinde, V., *Approximation fixed points of weak contractions using the Picard iteration*, Nonlinear Analysis Forum, **9** (2004), No. 1, 43-53
- [4] Berinde, V., *Iterative Approximation of Fixed Points*, 2nd Ed., Springer Verlag, Berlin Heidelberg New York, 2007
- [5] Berinde, V., *A common fixed point theorem for compatible quasi contractive self mappings in metric spaces*, Appl. Math. Comput., **213** (2009), 348-354
- [6] Berinde, V., *Approximating common fixed points of noncommuting discontinuous weakly contractive mappings in metric spaces*, Carpathian J. Math., **25** (2009), No. 1, 13-22
- [7] Berinde, V., *Some remarks on a fixed point theorem for Ćirić-type almost contractions*, Carpathian J. Math. **25** (2009), No. 2, 157-162
- [8] Berinde, V., *Approximating common fixed points of noncommuting almost contractions in metric spaces*, Fixed Point Theory **11** (2010) (in print)
- [9] Berinde, V., *Common fixed points of noncommuting discontinuous contractive type mappings in cone metric spaces*, Taiwanese J. Math. **14** (2010) (in print)
- [10] Berinde, V., *Common fixed points of noncommuting almost contractions in cone metric spaces*, Math. Commun. (accepted)
- [11] Bessaga, C., *On the converse of the Banach fixed point principle*, Colloq. Math., **7** (1959), 41-43
- [12] Chatterjea, S. K., *Fixed-point theorems*, C. R. Acad. Bulgare Sci. **25** (1972), 727-730
- [13] Choban, M. M., *Mappings of metric spaces*, Sov. Math., Dokl. **10** (1969), 258-260; translation from Dokl. Akad. Nauk SSSR **184** (1969), 1298-1300
- [14] Choban, M. M., *On decomposition of metric spaces into a union of discrete subspaces*, An. Univ. Timis., Ser. Mat.-Inform. **38** (2000), No. 1, 47-56
- [15] Haghi, R. H. and Rezapour, Sh., *Fixed points of multifunctions on regular cone metric spaces*, Expo. Math., **28** (2010), 7177
- [16] Harjani, J. and Sadarangani, K., *Generalized contractions in partially ordered metric spaces and applications to ordinary differential equations*, Nonlinear Analysis (2009), doi:10.1016/j.na.2009.08.003
- [17] Huang, L.-G. and Zhang, X., *Cone metric spaces and fixed point theorems of contractive mappings*, J. Math. Anal. Appl., **332** (2007), No. 2, 1467-1475
- [18] Jungck, G., Radenović, S., Radojević, S. and Rakocević, V., *Common fixed point theorems for weakly compatible pairs on cone metric spaces*, Fixed Point Theory Appl. 2009, art. No. 643840, 1-13
- [19] Kadelburg, Z., Radenović, S. and Rosić, B., *Strict contractive conditions and common fixed point theorems in cone metric spaces*, Fixed Point Theory and Appl., Vol. 2009, Article ID 173838, 14 Pages, doi:10.1155/2009/173838
- [20] Kannan, R., *Some results on fixed points*, Bull. Calcutta Math. Soc. **10** (1968), 71-76
- [21] Kuratowski, K., *Topology*, vol. 1, Academic Press: New York, 1966
- [22] Păcurar, M., *Approximating common fixed points of Presić-Kannan type operators by a multi-step iterative method*, An. Științ. Univ. Ovidius Constanța, Ser. Mat., **17** (2009), No. 1, 153-168
- [23] Păcurar, M., *A multi-step iterative method for approximating common fixed points of Presić-Rus type operators on metric spaces* (submitted)
- [24] Păcurar, M., *A multi-step iterative method for approximating fixed points of Presić-Kannan operators*, Acta Math. Univ. Comenianae **79** (2010) (in print)
- [25] Radenović, S., *Common fixed points under contractive conditions in cone metric spaces*, Comput. Math. Appl. **58** (2009), No. 6, 1273-1278
- [26] Rezapour, Sh. and Hambarani, R., *Some notes on the paper "Cone metric spaces and fixed point theorems of contractive mappings"*, J. Math. Anal. Appl., **345** (2008) No. 2, 719-724
- [27] Rus, I. A., *Principles and Applications of the Fixed Point Theory* (in Romanian), Editura Dacia, Cluj-Napoca, 1979
- [28] Rus, I. A., *Bessaga mappings*, in Approximation and optimization, Proc. Colloq., Cluj-Napoca/Rom. 1984, 165-172 (1985)
- [29] Rus, I. A., *Generalized Contractions and Applications*, Cluj University Press, Cluj-Napoca, 2001
- [30] Rus, I. A., Petrușel, A. and Petrușel, Gabriela, *Fixed Point Theory*, Cluj University Press, Cluj-Napoca, 2008
- [31] Vetro, P., *Common fixed points in cone metric spaces*, Rend. Circ. Mat. Palermo, **56** (2007) No. 3, 464-468

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