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Some stability results for Picard and Mann iteration processes using contractive condition of integral type

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ABSTRACT.
In this paper, we shall establish some stability results for Picard and Mann iteration processes in metric space and normed linear space by employing a contractive condition of integral type.

1. INTRODUCTION

In this paper we shall establish some stability results for Picard and Mann iteration processes in metric space and normed linear space by employing a contractive condition of integral type. Our results are generalizations and extensions of some of the results of Berinde [2], Osilike [17], Osilike and Udomene [18], Rhoades [20], Rhoades [21], Rhoades [22], Harder and Hicks [8] as well as some of the results of the author [9, 14, 15, 16].

Let $(E,\ d)$ be a complete metric space and $T : E \to E$ a selfmap of E. Suppose that $F_T = \{p \in E \mid Tp = p\}$ is the set of fixed points of T.

There are several iteration processes in the literature for which the fixed points of operators have been approximated over the years by various authors. In a complete metric space, the Picard iteration process $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n,\ n = 0, 1, \cdots, \tag{1.1}$$

has been employed to approximate the fixed points of mappings satisfying the inequality relation

$$d(Tx, Ty) \le \alpha d(x, y),\ \forall\, x,\ y \in E \text{ and } \alpha \in [0, 1). \tag{1.2}$$

Condition (1.2) is called the Banach's contraction condition. Any operator satisfying (1.2) is called strict contraction. Also, condition (1.2) is significant in the celebrated Banach's fixed point theorem [1].

In the Banach space setting, we shall state some of the iteration processes generalizing (1.1) as follows:

For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n,\ n = 0, 1, \cdots, \tag{1.3}$$

where $\{\alpha_n\}_{n=0}^{\infty} \subset [0, 1]$, is called the Mann iteration process (see Mann [13]).

For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\left.\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n Tz_n \\ z_n &= (1 - \beta_n)x_n + \beta_n Tx_n \end{aligned}\right\} n = 0, 1, ..., \tag{1.4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ *and* $\{\beta_n\}_{n=0}^{\infty}$ are sequences in $[0, 1]$, is called the Ishikawa iteration process (see Ishikawa [10]).

Kannan [12] established an extension of the Banach's fixed point theorem by using the following contractive definition: For a selfmap T, there exists $\beta \in \left(0, \frac{1}{2}\right)$ such that

$$d(Tx, Ty) \le \beta\left[d(x, Tx) + d(y, Ty)\right],\ \forall\, x,\ y \in E. \tag{1.5}$$

Chatterjea [5] used the following contractive condition: For a selfmap T, there exists $\gamma \in \left(0, \frac{1}{2}\right)$ such that

$$d(Tx, Ty) \le \gamma\left[d(x, Ty) + d(y, Tx)\right],\ \forall\, x,\ y \in E. \tag{1.6}$$

Zamfirescu [25] established a nice generalization of the Banach's fixed point theorem by combining (1.2), (1.5) and (1.6). That is, for a mapping $T : E \to E$, there exist real numbers $\alpha,\ \beta, \gamma$ satisfying $0 \le \alpha < 1,\ 0 \le \beta < \frac{1}{2},\ 0 \le \gamma < \frac{1}{2}$ respectively such that for each $x,\ y \in E$, at least one of the following is true:

$$\left.\begin{aligned} &(z_1)\ d(Tx, Ty) \le \alpha d(x, y) \\ &(z_2)\ d(Tx, Ty) \le \beta\left[d(x, Tx) + d(y, Ty)\right] \\ &(z_3)\ d(Tx, Ty) \le \gamma\left[d(x, Ty) + d(y, Tx)\right]. \end{aligned}\right\} \tag{1.7}$$

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The mapping $T : E \to E$ satisfying (1.7) is called the *Zamfirescu contraction*. Any mapping satisfying condition (z_2) of (1.7) is called a Kannan mapping, while the mapping satisfying condition (z_3) is called Chatterjea operator. The contractive condition (1.7) implies

$$||Tx - Ty|| \leq 2\delta||x - Tx|| + \delta||x - y||, \ \forall \ x, \ y \in E, \tag{1.8}$$

where $\delta = max\left\{\alpha, \dfrac{\beta}{1-\beta}, \dfrac{\gamma}{1-\gamma}\right\}, \ 0 \leq \delta < 1.$

The following definition of stability of iteration process due to Harder and Hicks [8] shall be required in the sequel.

Definition 1.1. Let (E, d) be a complete metric space and $T : E \to E$ a selfmap of E. Suppose that $F_T = \{p \in E \mid Tp = p\}$ is the set of fixed points of T. Let $\{x_n\}_{n=0}^{\infty} \subset E$ be the sequence generated by an iteration procedure involving T which is defined by

$$x_{n+1} = f(T, x_n), \ n = 0, 1, 2, \cdots, \tag{1.9}$$

where $x_0 \in X$ is the initial approximation and f is some function. Suppose $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T.

Let $\{y_n\}_{n=0}^{\infty} \subset E$ and set $\varepsilon_n = d(y_{n+1}, f(T, y_n)), \ n = 0, 1, 2, \cdots$. Then, the iteration procedure (1.9) is said to be $T-$stable or stable with respect to T if and only if $\lim\limits_{n\to\infty} \varepsilon_n = 0$ implies $\lim\limits_{n\to\infty} y_n = p$.

If in (1.9), $f(T, x_n) = Tx_n, \ n = 0, 1, 2, \cdots$, then we have the Picard iteration process, while we obtain the Mann iteration process if

$$f(T, x_n) = (1 - \alpha_n)x_n + \alpha_n Tx_n, \ n = 0, 1, 2, \cdots, \ \alpha_n \in [0, 1].$$

Several stability results established by various authors in metric space and normed linear space for different contractive definitions are available in the literature. Some of the various authors whose contributions are of colossal value in the study of stability of the fixed point iteration procedures are Ostrowski [19], Harder and Hicks [8], Rhoades [20, 22], Osilike [17], Osilike and Udomene [18], Jachymski [11], Berinde [2, 3] and Singh et al [24]. Harder and Hicks [8], Rhoades [20, 22], Osilike [17] and Singh et al [24] used the method of the summability theory of infinite matrices to prove various stability results for certain contractive definitions. The method has also been adopted to establish various stability results for certain contractive definitions in Olatinwo et al [14, 15]. Osilike and Udomene [18] introduced a shorter method of proof of stability results and this has also been employed by Berinde [2], Imoru and Olatinwo [9], Olatinwo et al [16] and some others. In Harder and Hicks [8], the contractive definition stated in (1.2) was used to prove a stability result for the Kirk's iteration process. The first stability result on $T-$ stable mappings was proved by Ostrowski [19] for the Picard iteration using condition (1.2). In addition to (1.2), the contractive condition in (1.7) was also employed by Harder and Hicks [8] to establish some stability results for both Picard and Mann iteration processes. Rhoades [20, 22] extended the stability results of [8] to more general classes of contractive mappings. Rhoades [20] extended the results of [8] to the following independent contractive condition: there exists $c \in [0, 1)$ such that

$$d(Tx, Ty) \leq c \max\{d(x, y), d(x, Ty), d(y, Tx)\}, \ \forall x, y \in E. \tag{1.10}$$

Rhoades [22] used the following contractive definition: there exists $c \in [0, 1)$ such that

$$d(Tx, Ty) \leq c\text{max}\left\{d(x, y), \frac{d(x, Tx) + d(y, Ty)}{2}, d(x, Ty), d(y, Tx)\right\}, \tag{1.11}$$

$\forall x, y \in E$.

Moreover, Osilike [17] generalized and extended some of the results of Rhoades [22] by using a more general contractive definition than those of Rhoades [20, 22]. Indeed, he employed the following contractive definition: there exist $a \in [0, 1]$, $L \geq 0$, such that

$$d(Tx, Ty) \leq Ld(x, Tx) + ad(x, y), \ \forall \ x, \ y \in E. \tag{1.12}$$

Osilike and Udomene [18] introduced a shorter method to prove stability results for the various iteration processes using the condition (1.12). Berinde [2] established the same stability results for the same iteration processes using the same set of contractive definitions as in Harder and Hicks [8] but the same method of shorter proof as in Osilike and Udomene [18].

More recently, Imoru and Olatinwo [9] established some stability results which are generalizations of some of the results of [2, 8, 17, 18, 20, 22]. In Imoru and Olatinwo [9], the following contractive definition was employed: there exist $a \in [0, 1)$ and a monotone increasing function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\varphi(0) = 0$, such that

$$d(Tx, Ty) \leq \varphi(d(x, Tx)) + ad(x, y), \ \forall \ x, \ y \in E. \tag{1.13}$$

In a recent paper of Branciari [4], a generalization of Banach [1] was established. In that paper, Branciari [4] employed the following contractive integral inequality condition: there exists $c \in [0, 1)$ such that $\forall \ x, \ y \in E$, we have

$$\int_0^{d(f(x), f(y))} \varphi(t)dt \leq c\int_0^{d(x,y)} \varphi(t)dt, \tag{1.14}$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0$, $\int_0^\varepsilon \varphi(t)dt > 0$.

Rhoades [23] used the conditions

$$\int_0^{d(f(x),f(y))} \varphi(t)dt \leq k \int_0^{m(x,y)} \varphi(t)dt, \ \forall\, x,\ y \in E, \tag{1.15}$$

where

$$m(x,y) = max\left\{d(x,y), d(x,f(x)), d(y,f(y)), \frac{d(x,f(y)) + d(y,f(x))}{2}\right\},$$

and

$$\int_0^{d(f(x),f(y))} \varphi(t)dt \leq k \int_0^{M(x,y)} \varphi(t)dt, \ \forall\, x,\ y \in E, \tag{1.16}$$

with

$$M(x,y) = max\left\{d(x,y), d(x,f(x)), d(y,f(y)), d(x,f(y)), d(y,f(x))\right\},$$

where $k \in [0,1)$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ in both cases is as defined in (1.14). Condition (1.16) is the integral form of Ciric's condition in Ciric [7].

In the next section, we shall give some new contractive conditions of integral type to obtaining our results.

2. Preliminaries

Following Branciari [4] and Rhoades [23], we now state the following contractive conditions of integral type which shall be employed in establishing our results.

For a selfmapping $T : E \to E$, there exist a real number $k \in [0,1)$ and monotone increasing functions $\nu,\ \psi : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\psi(0) = 0$ and $\forall\, x,\ y \in E$, we have

$$\int_0^{d(Tx,Ty)} \varphi(t)d\nu(t) \leq \psi\left(\int_0^{d(x,Tx)} \varphi(t)d\nu(t)\right) + k\int_0^{d(x,y)} \varphi(t)d\nu(t), \tag{2.1}$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each

$$\varepsilon > 0,\ \int_0^\varepsilon \varphi(t)d\nu(t) > 0.$$

Remark 2.1. If in condition (2.1), we have $\varphi(t) = 1$ *and* $\nu(t) = t$, then we get condition (1.13) employed in Imoru and Olatinwo [9]. Also, if in condition (2.1), we have

$$\varphi(t) = 1,\ \nu(t) = t \ \text{ and } \ \psi(u) = Lu,\ L \geq 0,\ \forall u \in \mathbb{R}^+,$$

then we obtain condition (1.12) in this paper used in Osilike [17] and Osilike and Udomene [18].

In this paper, we shall consider the Picard and Mann iteration processes to establish some stability results for self-mappings in metric space and normed linear space by employing the contractive condition of integral type defined in (2.1). Our results are generalizations and extensions of those of [2, 8, 9, 14, 15, 16, 17, 18, 20, 22]. For more on the study of fixed point iteration processes and various contractive conditions, our interested readers can consult Berinde [3], Ciric [6, 7], Rhoades [20] and others in the reference section of this paper.

We shall require the following lemmas in the sequel.

Lemma 2.1. (Berinde [2, 3]) *If δ is a real number such that $0 \leq \delta < 1$, and $\{\varepsilon'_n\}_{n=0}^\infty$ is a sequence of positive numbers such that $\lim_{n\to\infty} \varepsilon'_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^\infty$ satisfying*

$$u_{n+1} \leq \delta u_n + \varepsilon'_n,\ n = 0, 1, \cdots,$$

we have $\lim_{n\to\infty} u_n = 0$.

Lemma 2.2. *Let $(E,\ d)$ be a complete metric space and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative, and such that for each $\varepsilon > 0$, $\int_0^\varepsilon \varphi(t)d\nu(t) > 0$. Suppose that $\{u_n\}_{n=0}^\infty,\ \{v_n\}_{n=0}^\infty \subset E$ and $\{a_n\}_{n=0}^\infty \subset (0,1)$ are sequences such that*

$$\left|d(u_n,v_n) - \int_0^{d(u_n,v_n)} \varphi(t)d\nu(t)\right| \leq a_n,$$

with $\lim_{n\to\infty} a_n = 0$. Then,

$$d(u_n,v_n) - a_n \leq \int_0^{d(u_n,v_n)} \varphi(t)d\nu(t) \leq d(u_n,v_n) + a_n. \tag{2.2}$$

Proof. By letting

$$y = d(u_n, v_n) - \int_0^{d(u_n,v_n)} \varphi(t)d\nu(t)$$

and using the definition of modulus function in $|y|$ yields (2.2). □

Remark 2.2. Lemma 2.2 is also applicable in normed linear space setting since metric is induced by norm. That is, we have

$$d(x, y) = ||x - y||, \ \forall\, x,\ y \in E,$$

whenever we are working in a normed linear space.

3. Main Results

Theorem 3.1. *Let $(E,\ d)$ be a complete metric space and $T : E \to E$ a selfmapping of E satisfying condition (2.1). Suppose T has a fixed point p. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Picard iteration process defined by (1.1). Let $\nu,\ \psi : \mathbb{R}^+ \to \mathbb{R}^+$ be monotone increasing functions such that $\psi(0) = 0$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)d\nu(t) > 0$. Then, the Picard iteration process is T-stable.*

Proof. Let $\{y_n\}_{n=0}^{\infty} \subset E$ and $\varepsilon_n = d(y_{n+1}, Ty_n)$. Assume $\lim\limits_{n\to\infty} \varepsilon_n = 0$. Then, we shall establish that $\lim\limits_{n\to\infty} y_n = p$ by using condition (2.1), Lemma 2.2 and the triangle inequality as follows. Let $\{a_n\}_{n=0}^{\infty} \subset (0, 1)$. Then, by Lemma 2.2, we have

$$\int_0^{d(y_{n+1},\ p)} \varphi(t)d\nu(t) \leq d(y_{n+1},\ p) + a_n$$

$$\leq \psi\left(\int_0^{d(p,\ Tp)} \varphi(t)d\nu(t)\right) + k\int_0^{d(p,\ y_n)} \varphi(t)d\nu(t) + \int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n$$

$$= k\int_0^{d(y_n,\ p)} \varphi(t)d\nu(t) + \int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n. \tag{3.1}$$

We can now express (3.1) in the form $u_{n+1} \leq \delta u_n + \varepsilon'_n$,
where

$$0 \leq \delta = k < 1,\ u_n = \int_0^{d(y_n,\ p)} \varphi(t)d\nu(t)$$

and

$$\varepsilon'_n = \int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n,$$

with

$$\lim_{n\to\infty} \varepsilon'_n = \lim_{n\to\infty}\left(\int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n\right) = 0,$$

so that by Lemma 2.1 and the fact that $\int_0^{\varepsilon} \varphi(t)d\nu(t) > 0$, for each $\varepsilon > 0$, we have that $\lim\limits_{n\to\infty} \int_0^{d(y_n,\ p)} \varphi(t)d\nu(t) = 0$ from which it follows that $\lim\limits_{n\to\infty} d(y_n,\ p) = 0$, that is $\lim\limits_{n\to\infty} y_n = p$.

Conversely, let $\lim\limits_{n\to\infty} y_n = p$. Then, by the contractive condition (2.1), Lemma 2.2 and the triangle inequality again, we have

$$\int_0^{\varepsilon_n} \varphi(t)d\nu(t) = \int_0^{d(y_{n+1},\ Ty_n)} \varphi(t)d\nu(t)$$

$$\leq \int_0^{d(y_{n+1},\ p)} \varphi(t)d\nu(t) + k\int_0^{d(p,\ y_n)} \varphi(t)d\nu(t) + 3a_n \to 0$$

as $n \to \infty$.

Again, using the condition on φ yields $\lim\limits_{n\to\infty} \varepsilon_n = 0$. □

Remark 3.3. Theorem 3.1 is a generalization and extension of Theorem 3.1 of Imoru and Olatinwo [9], both Theorem 1 and Theorem 2 of Berinde [2], Theorem 1 of Osilike [17], Theorem 5 of Osilike and Udomene [18], Theorem 1 of Rhoades [20], Theorem 23 of Rhoades [21], Theorem 1 of Rhoades [22] as well as Theorem 2 of Harder and Hicks [8].

Theorem 3.2. *Let $(E,\ \|.\|)$ be a normed linear space and $T : E \to E$ a selfmapping of E satisfying condition (2.1). Suppose T has a fixed point p. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iteration process defined by (1.3), where $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence in $[0, 1]$ such that $0 < \alpha \leq \alpha_n$ $(n = 0, 1, \cdots)$. Let $\nu,\ \psi : \mathbb{R}^+ \to \mathbb{R}^+$ be monotone increasing functions such that $\psi(0) = 0$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)d\nu(t) > 0$. Then, the Mann iteration process is T-stable.*

Proof. Suppose that

$$\{y_n\}_{n=0}^{\infty} \subset E,\ \varepsilon_n = ||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n||,\ n = 0, 1, \cdots,$$

and let $\lim_{n\to\infty} \varepsilon_n = 0$. Then, we shall establish that $\lim_{n\to\infty} y_n = p$, using the contractive condition (2.1), Lemma 2.2 and the triangle inequality as follows: Let $\{a_n\}_{n=0}^{\infty} \subset (0,1)$. Then, by Lemma 2.2, we have

$$\begin{aligned}\int_0^{\|y_{n+1}-p\|} \varphi(t)d\nu(t) &\le [\|y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n\| - a_n] \\ &\quad + (1-\alpha_n)[\|y_n - p\| - a_n] + \alpha_n[\|Tp - Ty_n\| - a_n] + 3a_n \\ &\le [1-(1-k)\alpha_n] \int_0^{\|y_n-p\|} \varphi(t)d\nu(t) + \int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n \\ &\le [1-(1-k)\alpha] \int_0^{\|y_n-p\|} \varphi(t)d\nu(t) + \int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n. \end{aligned} \tag{3.2}$$

Expressing (3.2) in the form $u_{n+1} \le \delta u_n + \varepsilon'_n$,
where

$$0 \le \delta = 1 - (1-k)\alpha < 1,\ u_n = \int_0^{||y_n - p||} \varphi(t)d\nu(t)$$

and

$$\varepsilon'_n = \int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n,$$

with

$$\lim_{n\to\infty} \varepsilon'_n = \lim_{n\to\infty} \left(\int_0^{\varepsilon_n} \varphi(t)d\nu(t) + 3a_n \right) = 0,$$

so that by Lemma 2.1 and the fact that $\int_0^{\varepsilon} \varphi(t)dt > 0$, for each $\varepsilon > 0$, we have that $\lim_{n\to\infty} \int_0^{||y_n-p||} \varphi(t)dt = 0$ from which it follows that $\lim_{n\to\infty} ||y_n - p|| = 0$, that is $\lim_{n\to\infty} y_n = p$.

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by the contractive condition (2.1), Lemma 2.2 and the triangle inequality again, we have

$$\int_0^{\varepsilon_n} \varphi(t)d\nu(t) = \int_0^{||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n||} \varphi(t)d\nu(t)$$

$$\le \int_0^{||y_{n+1}-p||} \varphi(t)d\nu(t) + [1-(1-k)\alpha] \int_0^{||y_n-p||} \varphi(t)d\nu(t) + 3a_n \to 0$$

as $n \to \infty$. □

Remark 3.4. Theorem 3.2 is a generalization and extension of Theorem 3.2 of Imoru and Olatinwo [9], Theorem 3 of Berinde [2], Theorem 2 of Rhoades [22], Theorem 24 of Rhoades [21], Theorem 2 of Rhoades [20], Theorem 3 of Harder and Hicks [8] as well as some of the results of the author [14, 15, 16].

References

[1] Banach, S., *Sur les Operations dans les Ensembles Abstraits et leur Applications aux Equations Integrales*, Fund. Math. **3** (1922), 133-181

[2] Berinde, V., *On the stability of some fixed point procedures*, Bul. Ştiint. Univ. Baia Mare, Ser. B, Matematica-Informatica, **18** (2002), No. 1, 7-14

[3] Berinde, V.,*Iterative approximation of fixed points*, Second edition, Springer-Verlag Berlin Heidelberg, New York, 2007

[4] Branciari, A., *A fixed point theorem for mappings satisfying a general contractive condition of integral type*, Int. J. Math. Math. Sci., **29** (2002), No, 9, 531-536

[5] Chatterjea, S. K., *Fixed-Point theorems*, C. R. Acad. Bulgare Sci. **10** (1972), 727-730

[6] Ciric, Lj. B., *Generalized Contractions and Fixed Point Theorems*, Publ. Inst. Math. (Beograd) (N. S.) **12** (1971), No. 26, 19-26

[7] Ciric, Lj. B., *A Generalization of Banach's Contraction Principle*, Proc. Amer. Math. Soc. **45** (1974), 267-273

[8] Harder, A. M. and Hicks, T. L., *Stability Results for Fixed Point Iteration Procedures*, Math. Japonica **33** (1988), No. 5, 693-706

[9] Imoru, C. O. and Olatinwo, M. O., *On the Stability of Picard and Mann Iteration Processes*, Carp. J. Math. **19** (2003), No. 2, 155-160

[10] Ishikawa, S., *Fixed Point by a New Iteration Method*, Proc. Amer. Math. Soc. **44** (1974), No. 1, 147-150

[11] Jachymski, J. R., *An Extension of A. Ostrowski's Theorem on The Round-off Stability of Iterations*, Aequ. Math., **53** (1997), 242-253

[12] Kannan, R., *Some Results on Fixed Points*, Bull. Calcutta Math. Soc. **10** (1968), 71-76

[13] Mann, W. R., *Mean Value Methods in Iteration*, Proc. Amer. Math. Soc. **44** (1953), 506-510

[14] Olatinwo, M. O., Owojori, O. O. and Imoru, C. O., *On Some Stability Results for Fixed Point Iteration Procedure*, J. Math. Stat. **2** (2006), No. 1, 339-342

[15] Olatinwo, M. O., Owojori, O. O. and Imoru, C. O., *Some Stability Results on Krasnoselskij and Ishikawa Fixed Point Iteration Procedures*, J. Math. Stat. **2** (2006), No. 1, 360-362

[16] Olatinwo, M. O., Owojori, O. O. and Imoru, C. O., *Some Stability Results for Fixed Point Iteration Processes*, Aus. J. Math. Anal. Appl. **3** (2006), No. 2, Article 8, 1-7

[17] Osilike, M. O., *Some Stability Results for Fixed Point Iteration Procedures*, J. Nigerian Math. Soc. **14/15** (1995), 17-29

[18] Osilike, M. O. and Udomene, A., *Short Proofs of Stability Results for Fixed Point Iteration Procedures for a Class of Contractive-type Mappings,* Indian J. Pure Appl. Math. **30** (1999), No. 12, 1229-1234

[19] Ostrowski, A. M., *The Round-off Stability of Iterations,* Z. Angew. Math. Mech. **47** (1967), 77-81

[20] Rhoades, B. E., *Fixed Point Theorems and Stability Results for Fixed Point Iteration Procedures,* Indian J. Pure Appl. Math. **21** (1990), No. 1, 1-9

[21] Rhoades, B. E., *Some Fixed Point Iteration Procedures,* Int. J. Math. Sci. **14** (1991), No. 1, 1-16

[22] Rhoades, B. E., *Fixed Point Theorems and Stability Results for Fixed Point Iteration Procedures* II, Indian J. Pure Appl. Math. **24** (1993), No. 11, 691-703

[23] Rhoades, B. E., *Two Fixed Point Theorems for Mappings Satisfying A General Contractive Condition of Integral Type,* Int. J. Math. Math. Sci. **63** (2003), 4007-4013

[24] Singh, S. L., Bhatnagar, C. and Mishra, S. N., *Stability of Jungck-Type Iterative Procedures,* Int. J. Math. & Math. Sc. **19** (2005), 3035-3043

[25] Zamfirescu, T., *Fix Point Theorems in Metric Spaces,* Arch. Math. **23** (1972), 292-298

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