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The intersection convolution of relations

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ABSTRACT.

We initiate a systematic study of the intersection convolution of relations on one groupoid to another. The intersection convolution allows of some natural generalizations of the Hahn-Banach type extension theorems.

1. INTRODUCTION

If F and G are relations on one groupoid X to another Y, then we define a relation $F * G$ on X to Y such that

$$(F * G)(x) = \bigcap \{ F(u) + G(v) : \quad x = u + v, \quad u \in D_F, \ v \in D_G \}$$

for all $x \in X$, where D_F and D_G are the domains of F and G, respectively. The relation $F * G$ will be called the intersection convolution of F and G.

This definition closely resembles to that of the infimal convolution studied mainly by Moreau [16] and Strömberg [23]. It is actually a particular case of a more general definition of the infimal convolution of two partial functions on a groupoid to an infimum-complete partially ordered groupoid.

However, for a natural generalization of the Hahn-Banach extension theorems [4], it is enough to consider only the intersection convolution of a sublinear relation of one vector space to another and linear partial selection of it [24]. Therefore, we shall only investigate here the above definition.

For instance, we show that if X is, in particular, a group, then for any $x \in X$ we have

$$(F * G)(x) = \bigcap \{ F(x - v) + G(v) : \quad v \in (-D_F + x) \cap D_G \}.$$

Therefore, $(F * G)(x) = \bigcap_{v \in D_G} \big(F(x - v) + G(v) \big)$ whenever $X = D_F$.

Moreover, by using the binary intersection property of Nachbin [17], we give some sufficient conditions in order that the intersection convolution $F * G$ of an odd, subadditive relation F and an odd, semi-subadditive relation G, on one commutative group X to another Y, be total in the sense that $X = D_{F*G}$.

The necessary prerequisites concerning relations and groupoids, which may be unfamiliar to the reader, will be briefly laid out in the next preparatory section. Unfortunately, our present terminology may differ from the earlier one.

2. A FEW BASIC FACTS ON RELATIONS AND GROUPOIDS

A subset F of a product set $X \times Y$ is called a relation on X to Y. If in particular $F \subset X^2$, then we may simply say that F is a relation on X. The same terminology can also be applied when Y need not be specified.

If F is a relation on X to Y, then for any $x \in X$ and $A \subset X$ the sets $F(x) = \{ y \in Y : \ (x, y) \in F \}$ and $F[A] = \bigcup_{a \in A} F(a)$ are called the images of x and A under F, respectively.

If F is a relation on X to Y, then the values $F(x)$, where $x \in X$, uniquely determine F, since we have $F = \bigcup_{x \in X} \{x\} \times F(x)$. Therefore, a relation F on X to Y can be naturally defined by specifying the values $F(x)$ for all $x \in X$.

For instance, if F is a relation on X to Y and G is a relation on Y to Z, then the composition $G \circ F$ and the inverse F^{-1} can be defined such that $(G \circ F)(x) = G[F(x)]$ for all $x \in X$ and $F^{-1}(y) = \{ x \in X : \ y \in F(x) \}$ for all $y \in Y$.

If F is a relation on X to Y, then the sets $R_F = F[X]$ and $D_F = F^{-1}[Y]$ are called the range and domain of F, respectively. If in particular $X = D_F$, then we say that F is a relation of X to Y, or that F is a total relation on X to Y.

If F is a relation on X to Y and $U \subset D_F$, then the relation $F | U = F \cap (U \times Y)$ is called the restriction of F to U. Moreover, F and G are relations on X to Y such that $D_F \subset D_G$ and $F = G | D_F$, then G is called an extension of F.

In particular, a relation f on X to Y is called a function if for each $x \in D_f$ there exists $y \in Y$ such that $f(x) = \{y\}$. In this case, by identifying singletons with their elements, we may simply write $f(x) = y$.

If F is a relation on X to Y, then a relation Φ of D_F to Y is called a selection relation of F if $\Phi \subset F$. Thus, the axiom of choice can be briefly reformulated by saying that every relation has a selection function.

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If X is a set and $+$ is a function of X^2 to X, then the function $+$ is called an operation in X and the ordered pair $X(+)=(X,+)$ is called a groupoid even if X is void.

In this case, we may simply write $x+y$ in place of $+\big((x,y)\big)$ for any $x, y \in X$. Moreover, we may also simply write X in place of $X(+)$ whenever the operation $+$ is clearly understood.

In the practical applications, instead of groupoids, it is usually sufficient to consider only semigroups (associative groupoids). However, several definitions and theorems on semigroups can be naturally extended to groupoids.

For instance, if X is a groupoid, then for any $A, B \subset X$, we may naturally write $A+B=\{a+b: \ a \in A, \ b \in B\}$. Moreover, we may also write $x+A=\{x\}+A$ and $A+x=A+\{x\}$ for any $x \in X$.

Note that if in particular X is a group, then we may also naturally write $-A=\{-a: \ a \in A\}$ and $A-B=A+(-B)$ for any $A, B \subset X$. Though, the family $\mathcal{P}(X)$ of all subsets of X is only a monoid (semigroup with zero).

A relation F on one groupoid X to another Y is called subadditive if $F(x+y) \subset F(x)+F(y)$ for all $x, y \in X$. Moreover, the relation F is called semi-subadditive if the above inclusion is required to hold only for all $x, y \in D_F$.

While, a relation F on one groupoid X to another Y is called superadditive if $F(x)+F(y) \subset F(x+y)$ for all $x, y \in X$. Moreover, a relation F on one group X to another Y is called odd if $-F(x) \subset F(-x)$ for all $x \in X$.

Note that if F is an odd relation on one group X to another Y, then we also have $-F(-x) \subset F\big(-(-x)\big)=F(x)$, and hence $F(-x) \subset -F(x)$ for all $x \in X$. Therefore, the corresponding equality is also true.

In this respect, it is also worth mentioning that if F is a nonvoid, superadditive relation on a group X to a monoid Y which is quasi-odd in the sense that $0 \in F(x)+F(-x)$ for all $x \in D_F$, then $0 \in F(0)$ and F is quasi-additive in the sense that $F(x+y)=F(x)+F(y)$ for all $x, y \in X$ with either $x \in D_F$ or $y \in D_F$.

In [11], we have also proved that if F is a quasi-odd, superadditive relation on one group X to another Y, then there exists a selection function f of F which is odd-like and representing in the sense that $-f(x) \in F(-x)$ and $F(x)=f(x)+F(0)=F(0)+f(x)$ for all $x \in D_F$.

References show that the various additivity and homogeneity properties of relations have formerly been investigated by several authors. Certainly, the translation, the convexity and the linearity properties are the most important ones.

It is noteworthy that Banach's closed graph and open mapping theorems can, most naturally, be generalized in terms of superadditive relations. While, the corresponding generalizations of the Banach–Steinhaus, Hahn–Banach and Hyers–Ulam theorems need subadditive relations.

3. The intersection convolution of relations

Definition 3.1. If X is a groupoid, then we define a relation γ on X to X^2 such that

$$\gamma(x)=\big\{(u,v) \in X^2: \quad x=u+v\big\}$$

for all $x \in X$.

Remark 3.1. Thus, γ is just the inverse relation of the operation $+$ in X. Therefore, the properties of $+$ can also be expressed in terms of γ.

Definition 3.2. If X is a groupoid, then for any $A, B \subset X$, we define

$$\Gamma(x, A, B)=\gamma(x) \cap (A \times B).$$

Remark 3.2. Thus, for any $u, v \in X$, we have $(u,v) \in \Gamma(x, A, B)$ if and only if $u \in A$ and $v \in B$ such that $x=u+v$.

Definition 3.3. If F and G are relations on one groupoid X to another Y, then we define a relation $F*G$ on X to Y such that

$$\big(F*G\big)(x)=\bigcap\big\{F(u)+G(v): \quad (u,v) \in \Gamma(x, D_F, D_G)\big\}$$

for all $x \in X$. The relation $F*G$ will be called the intersection convolution of the relations F and G.

Remark 3.3. A particular case of the intersection convolution was already introduced in [24]. For some closely related notions, see also Moreau [16], Strömberg [23] and Beg [3]. The latter author has extended some of the results of [24] to fuzzy-set-valued functions.

The intersection convolution can be used to to easily prove some natural generalizations of the Hahn-Banach type extension theorems of Rodrígues-Salinas and Bou [21], Ioffe [14], Gajda, Smajdor and Smajdor [9] and Smajdor and Szczawińska [22]. Actually, it has already been implicitly used in the above mentioned papers.

Remark 3.4. Moreover, the intersection convolution and the Hahn-Banach type extension theorems can also be used to prove certain forms of the Hyers-Ulam type selection theorems of Gajda and Ger [8] and the present author [25].

The importance of ideas of W. Smajdor, Z. Gajda and R. Ger has been recognized by Rassias [20] and Czerwik [5]. Moreover, Popa and Nikodem [19], [18] and Badora, Ger and Páles [1] have provided substantial generalizations.

Remark 3.5. In connection with Definition 3.3, it is also worth mentioning that, in contrast to the intersection convolution, the union one need not be introduced.

Namely, in [10], for the global sum

$$F \oplus G = \{(x+z,\ y+w): \quad (x,\ y) \in F, \quad (z,\ w) \in G\}$$

of the relations F and G we have proved that

$$(F \oplus G)(x) = \bigcup \{ F(u) + G(v): \quad (u,\ v) \in \Gamma(x,\ D_F,\ D_G)\}$$

for all $x \in X$. Note that here we may simply write $\gamma(x)$ in place $\Gamma(x,\ D_F,\ D_G)$.

In contrast to the global sum $F \oplus G$, the intersection convolution $F * G$ is not an increasing function of its factors. However, by using the corresponding definitions, we can easily prove the following

Theorem 3.1. *If F, G, H and K are relations on one groupoid X to another Y such that*

(1) $D_H \subset D_F$ and $F \,|\, D_H \subset H$;

(2) $D_K \subset D_G$ and $G \,|\, D_K \subset K$;

then $F * G \subset H * K$.

Proof. By the corresponding definitions, it is clear that

$$\begin{aligned}(F * G)(x) = \bigcap \{ F(u) + G(v): \quad (u,\ v) \in \Gamma(x,\ D_F,\ D_G)\} \\ \subset \bigcap \{ F(u) + G(v): \quad (u,\ v) \in \Gamma(x,\ D_H,\ D_K)\} \\ \subset \bigcap \{ H(u) + K(v): \quad (u,\ v) \in \Gamma(x,\ D_H,\ D_K) \} = (H * K)(x)\end{aligned}$$

for all $x \in X$. Therefore, the required inclusion is also true. □

Now, as an immediate consequence of this theorem, we can also state

Corollary 3.1. *If F, G, H and K are relations on one groupoid X to another Y such that F and G are extensions of H and K, respectively, then $F * G \subset H * K$.*

Moreover, as some immediate consequences of Theorem 3.1, we can also state

Corollary 3.2. *If F, G and H are relations on one groupoid X to another Y such that $D_H \subset D_F$ and $F \,|\, D_H \subset H$, then $F * G \subset H * G$.*

Corollary 3.3. *If F, G and H are relations on one groupoid X to another Y such that $D_H \subset D_G$ and $G \,|\, D_H \subset H$, then $F * G \subset F * H$.*

4. Some important particular cases of Definition 3.3

A simple reformulation of an important particular case of Definition 3.3 immediately yields the following

Theorem 4.2. *If F and G are relations on a group X to a groupoid Y, then for any $x \in X$ we have*

$$(F * G)(x) = \bigcap \{ F(x-v) + G(v): \quad v \in (-D_F + x) \cap D_G\}.$$

Proof. If $y \in (F * G)(x)$, then

$$y \in \bigcap \{ F(u) + G(v): \quad (u,\ v) \in \Gamma(x,\ D_F,\ D_G)\}.$$

Therefore, for any $(u,\ v) \in \Gamma(x,\ D_F,\ D_G)$, we have $y \in F(u) + G(v)$. Now, if $v \in (-D_F + x) \cap D_G$, then by noticing that $(x-v,\ v) \in \Gamma(x,\ D_F,\ D_G)$ we can see that $y \in F(x-v) + G(v)$. Therefore,

$$y \in \bigcap \{ F(x-v) + G(v): \quad v \in (-D_F + x) \cap D_G\}$$

also holds.

Conversely, if

$$y \in \bigcap \{ F(x-v) + G(v): \quad v \in (-D_F + x) \cap D_G\},$$

then for any $v \in (-D_F + x) \cap D_G$ we have $y \in F(x-v) + G(v)$. Now, if $(u,\ v) \in \Gamma(x,\ D_F,\ D_G)$, then by noticing that $u = x - v$ and $v \in (-D_F + x) \cap D_G$ we can see that $y \in F(u) + G(v)$. Therefore,

$$y \in \bigcap \{ F(u) + G(v): \quad (u,\ v) \in \Gamma(x,\ D_F,\ D_G)\},$$

and thus $y \in (F * G)(x)$ also holds. □

In addition to the above theorem, it is also worth establishing the following

Theorem 4.3. *If F and G are relations on a group X to a groupoid Y, then for any $x \in X$ we have*

$$(F * G)(x) = \bigcap \big\{ F(u) + G(-u+x) : \quad u \in D_F \cap (x - D_G) \big\}.$$

Proof. This theorem can actually be derived from Theorem 4.2, by noticing that if $v \in (-D_F + x) \cap D_G$ and $u = x - v$, then $v = -u + x$ and $u \in D_F \cap (x - D_G)$. Moreover, if $u \in D_F \cap (x - D_G)$ and $v = -u + x$, then $u = x - v$ and $v \in (-D_F + x) \cap D_G$. □

From the above two theorems, by using that $-X + x = X$ and $x - X = X$ for all $x \in X$, we can immediately get

Corollary 4.4. *If F and G are relations on a group X to a groupoid Y, then for any $x \in X$ we have*

(1) $\big(F * G\big)(x) = \bigcap\limits_{v \in D_G} \big(F(x-v) + G(v) \big)$ *whenever F is total;*

(2) $\big(F * G\big)(x) = \bigcap\limits_{u \in D_F} \big(F(u) + G(-u+x) \big)$ *whenever G is total.*

Hence, it is clear that in particular we also have

Corollary 4.5. *If F and G are relations of a group X to a groupoid Y, then for any $x \in X$ we have*

$$(F * G)(x) = \bigcap_{v \in X} \big(F(x-v) + G(v) \big) = \bigcap_{u \in X} \big(F(u) + G(-u+x) \big).$$

Remark 4.6. A multiplicative form of the first statement of this corollary quite closely resembles to the definition of ordinary convolution of integrable functions.

5. Some sufficient conditions for the totality of the intersection convolution

Definition 5.4. A family $\mathcal{A}$ of sets is said to have the binary intersection property if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{A}$.

Remark 5.7. This terminology differs from that of Nachbin [17]. (See also Ingleton [13], Rodríguez-Salinas and Bou [21], and Fuchssteiner and Horváth [7].)

However, it is in accordance with the usual definition of the finite intersection property of Kelley [15, p. 135]. Therefore, we shall use the above terminology.

Theorem 5.4. *If F and G are relations on one commutative group X to another Y such that:*

(1) *D_F and D_G are subgroups of X;*

(2) *$F(x) \cap G(x) \neq \emptyset$ for all $x \in D_F \cap D_G$;*

(3) *F is odd and subadditive and G is odd and semi-subadditive;*

then for any $x \in X$ the family

$$\big\{ F(x-v) + G(v) : \quad v \in (-D_F + x) \cap D_G \big\}$$

has the binary intersection property.

Proof. If $v, t \in (-D_F + x) \cap D_G$, then $v, t \in -D_F + x$ and $v, t \in D_G$. Hence, by using (1), we can infer that

$$t - v \in -D_F + x - (-D_F + x) = -D_F + x - x + D_F = -D_F + D_F = D + F + D_F = D_F$$

and $t - v \in D_G - D_G = D_G + D_G = D_G$. Therefore, $t - v \in D_F \cap D_G$, and thus by (2) we have $F(t-v) \cap G(t-v) \neq \emptyset$. Hence, by using (3) and the commutativity of X and Y, we can already infer that

$$\begin{aligned} 0 \in \big(F(t-v) - G(t-v) \big) &= F\big((x-v) - (x-t)\big) - G(t-v) \\ &\subset F(x-v) + F\big(-(x-t)\big) - \big(G(t) + G(-v)\big) \\ &= F(x-v) - F(x-t) - \big(G(t) - G(v)\big) \\ &= F(x-v) + G(v) - \big(F(x-t) + G(t)\big). \end{aligned}$$

Therefore, $\big(F(x-v) + G(v)\big) \cap \big(F(x-t) + G(t)\big) \neq \emptyset$ is also true. □

Definition 5.5. A family $\mathcal{A}$ of subsets of a set X is called a Nachbin system in X if for every subfamily $\mathcal{B}$ of $\mathcal{A}$, having the binary intersection property, we have $\bigcap \mathcal{B} \neq \emptyset$.

Remark 5.8. Quite similarly a family of subsets of a set may be called a Riesz system if every subfamily of it having the finite intersection property has a nonvoid intersection.

Moreover, a family of subsets of a uniform space may be called a Cantor system if every subfamily of it containing small sets and having the finite intersection property has a nonvoid intersection.

Namely, according to Kelley [15, pp. 136 and 193], this terminology allows us to briefly state that a topological (uniform) space is compact (complete) if and only if the family of its closed subsets forms a Riesz (Cantor) system.

Example 5.1. It can be easily seen that the family of all closed balls in $\mathbb{R}$ is a Nachbin system. While, the family all closed balls in $\mathbb{R}^2$ is already not a Nachbin system.

More generally, it can be shown that if Ω is a nonvoid set, then the family of closed balls in the supremum-normed space of all bounded functions of Ω to $\mathbb{R}$ is also a Nachbin system.

Now, as an immediate consequence of Theorems 5.4 and 4.2, we can also state the following

Theorem 5.5. *If F is a and G are as in Theorem 5.4 and there exists a Nachbin system $\mathcal{A}$ in Y such that:*

(4) $F(x-v)+G(v) \in \mathcal{A}$ *for all* $x \in X$ *and* $v \in (-D_F+x) \cap D_G$;

then the relation $F * G$ is total.

Proof. If $x \in X$, then by Theorem 5.4 the family

$$\big\{ F(x-v) + G(v) : \quad v \in (-D_F+x) \cap D_G \big\}$$

has the binary intersection property. Hence, by Theorem 4.2 and condition (4), it is clear that

$$(F*G)(x) = \bigcap \big\{ F(x-v) + G(v) : \qquad v \in (-D_F+x) \cap D_G \big\} \neq \emptyset.$$

Therefore, the required assertion is true. □

Remark 5.9. In addition to Example 5.1, we can note that the family of all closed balls in a commutative normed group X is a translation-invariant subset of the commutative monoid $\mathcal{P}(X)$.

Therefore, as an important particular case of Theorem 5.5, it is also worth stating

Corollary 5.6. *If F is a relation and g is a function on one commutative group X to another Y and $\mathcal{A}$ is a translation-invariant Nachbin system in Y such that:*

(1) $F(x) \in \mathcal{A}$ *for all* $x \in D_F$;

(2) D_F *and* D_g *are subgroups of* X;

(3) $g(x) \in F(x)$ *for all* $x \in D_F \cap D_g$;

(4) F *is odd and subadditive and* g *is semi-additive;*

then the relation $F * g$ is total.

Proof. If $x \in X$ and $v \in (-D_F+x) \cap D_g$, then $x - v \in D_F$ and $v \in D_g$. Thus, $F(x-v) \in \mathcal{A}$ and $g(v) \in Y$. Hence, by the translation-invariance of $\mathcal{A}$, it follows that $F(x-v) + g(v) \in \mathcal{A}$. Thus, Theorem 5.5 can be applied to get the required assertion. Namely, an additive function of one group to another is necessarily odd. Therefore, g is also odd. □

From the above corollary, it is clear that in particular we also have

Corollary 5.7. *If F is an odd, subadditive relation of one commutative group X to another Y and there exists a translation-invariant Nachbin system $\mathcal{A}$ in Y such that $F(x) \in \mathcal{A}$ for all $x \in X$, then $F * f$ is total relation for every additive function f of a subgroup D of X to Y such that $f(x) \in F(x)$ for all $x \in D$.*

Note. This paper is a shortened and improved version of some parts of our former manuscript [26] submitted to the present journal in 2008.

Continuations of the results of [26] have been published in [2], [6] and [27]. Moreover, the results of [24] have been extended to groups in [12].

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