

The intersection convolution of relations

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ABSTRACT.

We initiate a systematic study of the intersection convolution of relations on one groupoid to another. The intersection convolution allows of some natural generalizations of the Hahn-Banach type extension theorems.

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