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Weakly sp-θ-closed functions and semipre-Hausdorff spaces

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ABSTRACT. For weakly sp-θ-closed surjections between arbitrary topological spaces conditions are sought which assure a semipre-Hausdorff range.

1. INTRODUCTION

The purpose of the present paper is find under what conditions on a weakly sp-θ-closed surjection $f : (X, \tau) \to (Y, \sigma)$ between arbitrary topological spaces will Y be semipre-Hausdorff and find other properties of weak sp-θ-closedness.

Throughout this paper, (X, τ) and (Y, σ) (or simply, X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If S is any subset of a space X, then $Cl(S)$ and $Int(S)$ denote the closure and the interior of S respectively. A point $x \in X$ is called a θ-cluster [15] point of S if $S \cap Cl(U) \neq \emptyset$ for each open set U containing x. The set of all θ-cluster points of S is called the θ-closure of S and is denoted by $Cl_\theta(S)$. Hence, a subset S is called θ-closed [15] if $Cl_\theta(S) = S$. Note that X is Hausdorff if and only if $\{x\}$ is θ-closed for each $x \in X$ [7]. A subset $S \subset X$ is called β-open [2] or semi-preopen [3], if $S \subset Cl(Int(Cl(S)))$. The complement of a β-open set is called a β-closed [2] or semi-preclosed [3] set. The family of all semi-preopen sets of a space X is denoted by $SPO(X, \tau)$ or $SPO(X)$. We set $SPO(X, x) = \{U : x \in U \in SPO(X)\}$. The intersection of all β-closed sets containing S is called the semi-preclosure of S [3] or β-closure and is denoted by $spCl(S)$ or $\beta Cl(S)$. The union of all β-open sets contained in S is called the semi-preinterior or β-interior of S and is denoted by $spInt(S)$ or β-Int(S). A subset S of X is said to be semipre-regular (briefly sp-regular) if it is both β-open and β-closed in X.

A point $x \in X$ is called a semipre-θ-cluster point of S [12] if $spCl(U) \cap S \neq \phi$ for every β-open set U containing x. The set of all semipre-θ-cluster points of S is called the semipre-θ-closure of S and is denoted by $spCl_\theta(S)$. A subset S is called semipre-θ-closed (briefly sp-θ-closed) if $spCl_\theta(S) = S$. The complement of a semipre-θ-closed set is called a semipre-θ-open set (briefly sp-θ-open). The semipre-θ-interior of a subset S of X is the union of all sp-θ-open subsets of X contained in S, and is denoted by $spInt_\theta(S)$.

A space X is called extremally disconnected (E.D) [16] if the closure of each open set in X is open.

Recall that, a function $f : (X, \tau) \to (Y, \sigma)$ is said to be:
(i) strongly continuous [9, 1] if for every subset A of X, $f(Cl(A)) \subset f(A)$.
(ii) β-closed [2](resp. β-open [2]) if $f(F)$ is β-closed (resp. β-open) in Y for each closed (resp. open) set F of X.

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(iii) contra-sp-θ-open if $f(F)$ is sp-θ-closed in Y for each β-open set F of X.
(iv) sp-θ-closed (resp. sp-θ-open) if $f(F)$ is sp-θ-closed (resp. sp-θ-open) in Y for each closed (resp. open) set F of X.

The following theorem is given by T. Noiri [12].

Theorem 1.1. *For any subset A of X:*
(1) $spCl_\theta(spCl_\theta(A)) = spCl_\theta(A)$;
(2) $spCl_\theta(A)$ *is sp-θ-closed;*
(3) *Intersection of arbitrary collection of sp-θ-closed set in X is sp-θ-closed;*
(4) $spCl_\theta(A)$ *is the intersection of all sp-θ-closed sets each containing A;*
(5) *If $A \in SPO(X,\tau)$, then $spCl(A) = spCl_\theta(A)$.*

For other advances on topological spaces obtained by our research group we recommend [12, 6, 5, 4].

2. Weakly sp-θ-closed functions and semipre-Hausdorff

Recall that, a function $f : (X,\tau) \to (Y,\sigma)$ is said to be weakly-closed [14] if

$$Cl(f(Int(F))) \subset f(F)$$

for each closed set F of X.

Definition 2.1. A function $f : (X,\tau) \to (Y,\sigma)$ is said to be weakly sp-θ-closed if

$$spCl_\theta(f(spInt(F))) \subset f(F)$$

for each closed set F of X.

Remark 2.1. In fact weak sp-θ-closedness and weak-closedness are independent notions.

Example 2.1. (1) A weakly sp-θ-closed function which is not weakly-closed.
Let $X = \{a,b,c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a,b\}, X\}$ and $f : (X,\tau) \to (X,\tau)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is a weakly sp-θ-closed, but it is not a weakly-closed.
(2) A weakly-closed function which is not weakly sp-θ-closed.
Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}\}$, $\sigma = \{\emptyset, X, \{b\}, \{a,b\}, \{b,c\}\}$ and $f : (X,\tau) \to (X,\sigma)$ be the identity function. Then f is weakly-closed but it is not weakly sp-θ-closed since $spCl_\theta(f(spInt(\{b,c\}))) \not\subset f(\{b,c\})$.

Theorem 2.2. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following conditions are equivalent:*
(1) *f is weakly sp-θ-closed;*
(2) $spCl_\theta(f(U)) \subset f(Cl(U))$ *for each β-open set U of X.*

Proof. (1) $\Rightarrow$ (2): Let U be any β-open subset of X. Since $Cl(U)$ is a closed and $U \subset spIn(Cl(U)$ by (1) we have $spCl_\theta(f(U) \subset spCl_\theta(f(spInt(ClU))) \subset f(Cl(U))$.
(2) $\Rightarrow$ (1): Let F be any closed subset of X. Then,
$spCl_\theta(f(spInt(F))) \subset f(Cl(spInt(F))) \subset f(Cl(F)) = f(F)$. □

Theorem 2.3. *For a function $f : (X,\tau) \to (Y,\sigma)$ bijective, the following conditions are equivalent:*
(1) *f is weakly sp-θ-closed;*
(2) $spCl_\theta(f(U)) \subset f(Cl(U))$ *for each β-open subset U of X;*
(3) *For each subset F in Y and each open set U in X with $f^{-1}(F) \subset U$, there exists a sp-θ-open set A in Y with $F \subset A$ and $f^{-1}(F) \subset spCl(U)$;*

(4) *For each point y in Y and each open set U in X with $f^{-1}(y) \subset U$, there exists a sp-θ-open set A in Y containing y and $f^{-1}(A) \subset spCl(U)$.*

Proof. $(1) \Rightarrow (2)$: Theorem 2.2.
$(2) \Rightarrow (3)$: Let F be a subset of Y and let U be open in X with $f^{-1}(F) \subset U$. Then $f^{-1}(F) \cap Cl(X - spCl(U)) = \phi$ and consequently, $F \cap f(Cl(X - spCl(U))) = \phi$. By (2), $F \cap spCl_\theta(f(X - spCl(U))) = \phi$. Let $A = Y - spCl_\theta(f(X - spCl(U)))$. Then A is sp-θ-open with $F \subset A$ and $f^{-1}(A) \subset X - f^{-1}(spCl_\theta(f(X - spCl(U)))) \subset X - f^{-1}f(X - spCl(U)) \subset spCl(U)$.
$(3) \Rightarrow (4)$: Clear.
$(4) \Rightarrow (1)$: Let F be closed in X and let $y \in Y - f(F)$. Since $f^{-1}(y) \subset X - F$, there exists a sp-θ-open set A in Y with $y \in A$ and $f^{-1}(A) \subset spCl(X - F) = X - spInt(F)$ by (4). Therefore $A \cap f(spInt(F)) = \phi$, so that $y \in Y - spCl_\theta(f(spInt(F)))$. Thus $spCl_\theta(f(spInt(F))) \subset f(F)$. □

A space X is said to be semipre-Hausdorff [11] (resp. sp-θ-T_2) if for every pair of distinct points x and y, there exist two β-open (resp. sp-θ-open) sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \emptyset$.

Lemma 2.1. *X is semipre-Hausdorff if and only if $\{x\}$ is sp-θ-closed.*
(Thus the semipre-Hausdorff property can be viewed as a pointwise property).

Proof. Necessity. $\{x\} \subset spCl_\theta(\{x\})$ is always hold. We prove that $spCl_\theta(\{x\}) \subset \{x\}$. Let $y \notin \{x\}$. Then there exist $U \in SPO(X, x)$ and $V \in SPO(Y, y)$ such that $U \cap V = \emptyset$. Hence $spCl(V) \cap U = \emptyset$. Therefore $spCl(V) \cap \{x\} = \emptyset$. i.e., $y \notin spCl_\theta(\{x\})$ and $spCl_\theta(\{x\}) \subset \{x\}$ and $\{x\}$ is sp-θ-closed.
Sufficiency. Is clear since every sp-θ-open sets is β-open. □

Theorem 2.4. *For a topological space (X, τ), the following properties are equivalent:*
(1) *For every pair of distinct points $x, y \in X$, there exist $U \in SP\theta O(X, x)$ and $V \in SP\theta O(X, y)$ such that $spCl_\theta(U) \cap spCl_\theta(V) = \emptyset$;*
(2) *(X, τ) is sp-θ-T_2;*
(3) *(X, τ) is semipre-Hausdorff;*
(4) *For every pair of distinct points $x, y \in X$, there exist $U, V \in SPO(X)$ such that $x \in U$, $y \in V$ and $spCl(U) \cap spCl(V) = \emptyset$;*
(5) *For every pair of distinct points $x, y \in X$, there exist $U, V \in SPR(X)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

Proof. $(1) \Rightarrow (2)$. This is obvious.
$(2) \Rightarrow (3)$. Since $SP\theta O(X) \subset SPO(X)$, the proof is obvious.
$(3) \Rightarrow (4)$. This follows from Lemma 5.2 of [12].
$(4) \Rightarrow (5)$. By ([12], Theorem 3.1), $spCl(U) \in SPR(X)$ for every $U \in SPO(X)$ and the proof immediately follows.
$(5) \Rightarrow (1)$. By ([12], Theorem 3.5), every sp-regular set is sp-θ-open and sp-θ-closed. Hence the proof is obvious. □

Definition 2.2. For a function $f : (X, \tau) \to (Y, \sigma)$ we let $G(f) = \{(x, f(x)) : x \in X\} \subset X \times Y$ represent the graph of f and we say that $G(f)$ is strongly β-closed (resp. β-closed) if whenever $(x, y) \in (X \times Y) \backslash G(f)$, there exist $U \in SPO(X, x)$ and $V \in SPO(Y, y)$ such that $[U \times sPCl(V)] \cap G(f) = \emptyset$ (resp. $(U \times V)) \cap G(f) = \emptyset$).

Lemma 2.2. *A function $f : (X, \tau) \to (Y, \sigma)$, has a strongly β-closed (resp. β-closed) graph if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in SPO(X, x)$ and $V \in SPO(Y, y)$ such that $f(U) \cap spCl(V) = \emptyset$ (resp. $f(U) \cap V = \emptyset$).*

Recall that, two non-empty subsets A and B in X are strongly separated [14], if there exist open sets U and V in X with $A \subset U$ and $B \subset V$ such that $Cl(U) \cap Cl(V) = \emptyset$. If A and B are singleton sets we may speak of points being strongly separated. We will use the fact (see [8]) that in a normal space, disjoint closed sets are strongly separated.

Theorem 2.5. *If the surjection $f : (X, \tau) \to (Y, \sigma)$ has a strongly β-closed graph $G(f)$ then Y is semipre-Hausdorff.*

Proof. We will show that Y is semipre-Hausdorff at each point in the image of f. i.e., each $\{f(x)\}$ is sp-θ-closed (Lemma 2.1). Let $x \in X$ and let $z \in Y - \{f(x)\}$. Then $(x, z) \notin G(f)$ and there exist $U \in SPO(X, x)$ and $V \in SPO(Y, z)$ such that $[U \times spCl(V)] \cap G(f) = \emptyset$. Thus $spCl(V) \cap \{f(x)\} = \emptyset$ and $z \notin spCl_\theta(V)$. Hence $\{f(x)\}$ is sp-θ-closed. Since f is surjective, Y is semipre-Hausdorff. □

Definition 2.3. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be (*)-semipreopen if $spCl(f(U)) = spCl_\theta(f(U))$ for all open sets $U \subset X$.

Theorem 2.6. *If $f : (X, \tau) \to (Y, \sigma)$ is β-open then it is (*)-semipreopen.*

Proof. Let $U \subset X$ be open. Since f is β-open, $f(U) \in SPO(Y)$. By Theorem 1.1(5), $spCl(f(U)) = spCl_\theta(f(U))$ and so f is (*)-semipreopen. □

Theorem 2.7. *If $f : (X, \tau) \to (Y, \sigma)$ is (*)-semipreopen and $G(f)$ is β-closed then $G(f)$ is strongly β-closed.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$. Since $G(f)$ is β-closed there exist $U \in SPO(X, x)$ and $V \in SPO(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$. Then $spCl(f(U)) \cap V = \emptyset$. Therefore $spCl_\theta(f(U)) \cap V = \emptyset$. Thus, there is a β-open set $W \subset Y$ with $y \in W$ and $spCl(W) \cap f(U) = \emptyset$. Then, $(x, y) \in U \times W$ and $(U \times spCl(W)) \cap G(f) = \emptyset$ so that $G(f)$ is strongly β-closed. □

Corollary 2.1. *If $f : (X, \tau) \to (Y, \sigma)$ is a (*)-semipreopen surjection with β-closed graph $G(f)$, then Y is semipre-Hausdorff.*

Proof. It follows from Theorem 2.7 and 2.5. □

Theorem 2.8. *If $f : (X, \tau) \to (Y, \sigma)$ is weakly sp-θ-closed with all fibers $f^{-1}(y)$ θ-closed, then $G(f)$ is β-closed.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$. Since $f^{-1}(y)$ is θ-closed, There are disjoint open sets W and U with $x \in W$ and $f^{-1}(y) \subset U$. By Theorem 2.3 and weak sp-θ-closedness of f there is a sp-θ-open set $V \subset Y$ with $y \in V$ and $f^{-1}(y) \subset spCl(U) \subset Cl(U) \subset X - W$. Thus $(x, y) \in W \times V$ and $(W \times V) \cap G(f) = \emptyset$. □

Corollary 2.2. *If $f : (X, \tau) \to (Y, \sigma)$ is (*)-semipreopen weakly sp-θ-closed surjection with all fibers θ-closed, then Y is semipre-Hausdorff.*

Proof. It is suffices to apply Theorem 2.8 and Corollary 2.1. □

Theorem 2.9. *If $f : (X, \tau) \to (Y, \sigma)$ is weakly sp-θ-closed surjection and all pairs of disjoint fibers are strongly separated then f is semipre-Hausdorff.*

Proof. Let y and z be two distinct points in Y. Let G and H be open sets in X such that $f^{-1}(y) \in G$ and $f^{-1}(z) \in H$ with $Cl(G) \cap Cl(H) = \emptyset$. By weak sp-$\theta$-closedness there are sp-θ-open sets U and V in Y such that $y \in U$ and $z \in V$, $f^{-1}(U) \subset spCl(G) \subset Cl(G)$ and $f^{-1}(V) \subset spCl(H) \subset Cl(H)$. Therefore $U \cap V = \emptyset$ because $Cl(G) \cap Cl(H) = \emptyset$ and f surjective. Then by Theorem 2.4, Y is semipre-Hausdorff. □

Corollary 2.3. *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly sp-θ-closed surjection with all fibers closed and X is normal, then Y is semipre-Hausdorff.*

The next result follows from Corollary 2.3.

Corollary 2.4. *If $f : (X, \tau) \to (Y, \sigma)$ is continuous weakly sp-θ-closed surjection with X a compact Hausdorff space and Y a T_1 space, then Y is a compact semipre-Hausdorff space.*

Proof. Since f is a continuous surjection and Y is a T_1 space, Y is a compact and all fibers are closed. Since X is normal, Y is also semipre-Hausdorff. □

3. Other properties of weak sp-θ-closedness

Recall that, a function $f : (X, \tau) \to (Y, \sigma)$ is called sp-θ-closed if $f(F)$ is sp-θ-closed in Y for every closed set F of X.

Clearly, every sp-θ-closed function is weakly sp-θ-closed, but the converse is not true.

Example 3.2. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $f : (X, \tau) \to (Y, \tau)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is a weakly sp-θ-closed, but it is not a sp-θ-closed function since for $F = \{b, c\}$, $f(F)$ is not sp-θ-closed in Y.

Theorem 3.10. *Let $f : (X, \tau) \to (Y, \sigma)$ be weakly sp-θ-closed. If for each closed subset F of X and each fiber $f^{-1}(y) \subset X - F$ there exists an open set U of X such that $f^{-1}(y) \subset U \subset Cl(U) \subset X - F$, then f is sp-θ-closed.*

Proof. Let F be any closed subset of X and $y \in Y - f(F)$. Then $f^{-1}(y) \cap F = \emptyset$ and hence $f^{-1}(y) \subset X - F$. By hypothesis, there exists an open set U of X such that $f^{-1}(y) \subset U \subset Cl(U) \subset X - F$. Since f is weakly sp-θ-closed, there exists a sp-θ-open V in Y with $y \in V$ and $f^{-1}(V) \subset spCl(U) \subset Cl(U)$. Therefore , we obtain $f^{-1}(V) \cap F = \emptyset$ and hence $V \cap f(F) = \emptyset$. This shows that $y \notin spCl_\theta(f(F))$. Therefore, $f(F)$ is sp-θ-closed in Y and f is a sp-θ-closed function. □

Theorem 3.11. *Let $f : (X, \tau) \to (Y, \sigma)$ be weakly sp-θ-closed. If for each closed subset F of X and each fiber $f^{-1}(y) \subset X - F$ there exists an open set U of X for which $F \subset U$ and $f^{-1}(y) \cap U = \emptyset$, then f is sp-θ-closed.*

Proof. Let F be any closed subset of X and $y \in Y - f(F)$, thus $f^{-1}(y) \subset X - F$ and hence there exists an open subset U of X for which $F \subset U$ and $f^{-1}(y) \cap U = \emptyset$. Then $y \in Y - f(U) \subset Y - f(F)$. Since f is weakly sp-θ-closed, $spCl_\theta(f(spInt(F))) \subset f(F)$. Hence, we obtain $y \in spInt_\theta(Y - f(spInt(F)))$. Let $H_y = spInt_\theta(Y - f(spInt(F)))$. Then H_y is a sp-θ-open subset of Y containing y. Hence $Y - f(F) = \cup\{H_y : y \in Y - f(F)\}$ is sp-θ-open and hence $f(F)$ is sp-θ-closed. □

Corollary 3.5. *If $f : (X, \tau) \to (Y, \sigma)$ is weakly sp-θ-closed with all closed fibers, then f is sp-θ-closed.*

Proof. For any closed subset F and any fiber $f^{-1}(y) \subset X - F$, let $U = X - f^{-1}(y)$. Then U is open set with $F \subset U$ and $f^{-1}(y) \cap U = \emptyset$. □

Theorem 3.12. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra sp-θ-open , then f is weakly sp-θ-closed.*

Proof. Let F be a closed subset of X. Then, $spCl_\theta(f(spInt(F))) = f(spInt(F)) \subset f(F)$. □

Theorem 3.13. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is strongly continuous, then the following are equivalent:*
(1) *f is weakly sp-θ-closed;*
(2) *f is contra sp-θ-open.*

Proof. (1) $\Rightarrow$ (2): Let U be an β-open subset of X. By hypothesis and Theorem 2.2, we have $spCl_\theta(f(U)) \subset f(Cl(U)) \subset f(U)$. Hence $f(U)$ is sp-θ-closed.
(2) $\Rightarrow$ (1): It follows from Theorem 3.12. □

Theorem 3.14. *Every weakly sp-θ-closed strongly continuous bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is sp-θ-open (and sp-θ-closed).*

Proof. Let U be an open subset of X. Since f is weakly sp-θ-closed $spCl_\theta(f(spInt(X - U))) \subset f(X - U)$. Hence and since f is bijective, we obtain $f(U) \subset spInt_\theta(f(Cl(U))) = spInt_\theta(f((U))) \subset f(U)$. Therefore $f(U)$ is sp-θ-open.
By other hand, by Theorem 3.13 f is contra sp-θ-open so, $f(U)$ is also sp-θ-closed.
In particular by ([12], Theorem 3.5), $f(U)$ is sp-regular for each open $U \subset X$. □

Theorem 3.15. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly sp-θ-closed bijection, then for every subset F in Y and every open set U in X with $f^{-1}(F) \subset U$, there exists a sp-θ-closed set B in Y such that $F \subset B$ and $f^{-1}(B) \subset Cl(U)$.*

Proof. Let F be a subset of Y and U be an open subset of X with $f^{-1}(F) \subset U$. Put $B = spCl_\theta(f(spInt(Cl(U))))$, then B is a sp-θ-closed set of Y such that $F \subset B$ since $F \subset f(U) \subset f(spInt(Cl(U))) \subset spCl_\theta(f(spInt(Cl(U)))) = B$. And since f is weakly sp-θ-closed, we have $f^{-1}(B) \subset Cl(U)$. □

Recall that, a set F in a topological space X is θ-compact [14] if for each cover Ω of F by open sets U in X, there is a finite family $U_1, ..., U_n$ in Ω such that $F \subset Int(\cup\{Cl(U_i) : i = 1, 2, ..., n\})$.

Every compact space is θ-compact, but the converse does not hold as we can see in the following example.

Example 3.3. In the real line with the usual topology, consider the $A = \{1/n : n \in N\}$. it is easy to see that A is θ-compact but does not is compact.

Theorem 3.16. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly sp-θ-closed bijection with all fibers θ-closed in X, then $f(F)$ is sp-θ-closed for each θ-compact set F in X.*

Proof. Let F be θ-compact and $y \in Y - f(F)$. Then $f^{-1}(y) \cap F = \emptyset$ and for each $x \in F$ there is an open U_x in X containing x such that $Cl(U_x) \cap f^{-1}(y) = \emptyset$. Clearly $\Omega = \{U_x : x \in F\}$ is an open cover of F and since F is θ-compact, there is a finite family $\{U_{x_1}, ..., U_{x_n}\}$ in Ω such that $F \subset Int(A)$, where $A = \cup\{Cl(U_{x_i}) : i = 1, ..., n\}$. Since f is weakly sp-θ-closed, there exists a sp-θ-open B in Y with $f^{-1}(y) \subset f^{-1}(B) \subset spCl(X - A) \subset Cl(X - A) = X - Int(A) \subset X - F$. Therefore $y \in B$ and $B \cap f(F) = \emptyset$. Thus $y \in Y - pCl_\theta(f(F))$. This shows that $f(F)$ is sp-θ-closed. □

Definition 3.4. A topological space X is said to be:
(i) quasi H-closed [13] if every open cover of X has a finite subfamily whose closures

cover X. A subset A of a space X is quasi H-closed relative to X if every cover of A by open sets of X has a finite subfamily whose closures cover A.
(ii) almost β-θ-compact space if every cover of X with sp-θ-open sets has a finite subfamily of members whose closures cover X. And a subset A of a space X is almost β-θ-compact relative to X if every cover of A with sp-θ-open subsets has a finite subfamily of members whose closures cover A.

Lemma 3.3. [10] *A function $f : (X, \tau) \to (Y, \sigma)$ is open if and only if for each $B \subset Y$, $f^{-1}(Cl(B)) \subset Cl(f^{-1}(B))$.*

Theorem 3.17. *Let (X, τ) be an extremally disconnected space and $f : (X, \tau) \to (Y, \sigma)$ be an open and weakly sp-θ-closed function with quasi H-closed fibers. Then $f^{-1}(G)$ is quasi H-closed for each almost β-θ-compact set $G \subset Y$.*

Proof. Let $\{V_\alpha : \alpha \in I\}$ be an open cover of $f^{-1}(G)$. Then for each $y \in G$, $f^{-1}(y) \subset \cup\{Cl(V_\alpha) : \alpha \in I(y)\} = H_y$ for some finite $I(y) \subset I$. Then H_y is closed and open since X is extremally disconnected. So, by weak sp-θ-closedness, there exists a sp-θ-open set U_y containing y such that $f^{-1}(U_y) \subset spCl(H_y) \subset Cl(H_y) = H_y$. Then, $\{U_y : y \in G\}$ is a cover of G by sp-θ-open sets and $G \subset \cup\{Cl(U_y) : y \in K\}$ for some finite subset K of G. Hence, by Lemma 3.3, $f^{-1}(G) \subset \cup\{Cl(f^{-1}(U_y)) : y \in K\}$. Thus $f^{-1}(G) \subset \cup\{Cl(V_\alpha) : \alpha \in I(y)$ and $y \in K\}$. Therefore $f^{-1}(G)$ is quasi H-closed. □

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