

Tripled coincidence and common fixed point theorems for hybrid pair of mappings

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ABSTRACT.

The tripled fixed point is a generalization of the well known concept of "coupled fixed point". In this paper, we establish tripled coincidence and tripled common fixed point theorems for a hybrid pair consisting of a multi-valued and a single valued mapping on a metric space. We give examples to illustrate our results.

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