

Fixed points for mappings defined on pseudometric spaces

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ABSTRACT.

In this article, the distinct classes of continuous pseudometrics and metrics (perfect, quasi-perfect, sequentially complete pseudometrics and metrics) are defined and studied in depth. The conditions under which the set of fixed points of a given mapping of a space with concrete pseudometric is non-empty are determined. Some examples are proposed. For spaces with pseudometrics there are proved the Bishop-Phelps, Takahashi, Caristi and other theorems.

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* Dedicated to Professor Emeritus Constantin Corduneanu on the occasion of his 85th birthday

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