

Cubic differential systems with 2-invariant straight lines

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ABSTRACT.

For cubic differential system with a singular point $O(0, 0)$ of a center or a focus type having three invariant straight lines of which one is 2-invariant it is proved that the origin is a center if and only if the first five Lyapunov quantities vanish.

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