

Bivariate uniform approximation via bivariate Lagrange interpolation polynomials

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ABSTRACT.

In the present note, we extend some univariate uniform approximation results by means of Lagrange interpolating polynomials [Ivan, M., *Elements of Interpolation Theory*, Mediamira Science Publisher, Cluj-Napoca (2004)] to the bivariate case. It is well known that generally, in the univariate case, the sequence of Lagrange interpolation polynomials does't converges to the approximated function. This fact was first observed by G. Faber (see [9]), which constructed an example when the sequence of Lagrange interpolation polynomials diverges. The result of G. Faber was more generalized by I. Muntean (see [12]).

M. Ivan established first sufficient conditions for the uniform convergence of the sequence of Lagrange interpolation polynomials associated to a univariate real valued function.

First, we represent the remainder term of bivariate Lagrange interpolation formula in terms of bivariate divided difference. Using this representation we establish sufficient conditions for the uniform convergence of the sequence of bivariate Lagrange interpolation polynomials to the approximated function.

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