

Properties and inequalities for the h - and (h, m) -logarithmically convex functions

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ABSTRACT.

In the paper, the authors introduce definitions of h -logarithmically convex functions and (h, m) -logarithmically convex functions respectively, discuss some properties of these kinds of functions, and establish some integral inequalities for these kinds of functions.

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