

Triple fixed point theorems for mixed monotone Prešić-Kannan and Prešić-Chatterjea mappings in partially ordered metric spaces

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ABSTRACT.

The aim of this paper is to extend the Kannan fixed point theorem from single-valued self mappings $T : X \rightarrow X$ to mappings $F : X^3 \rightarrow X$ satisfying a Prešić-Kannan type contractive condition:

$$d(F(x, y, z), F(y, z, u)) \leq \frac{k}{8} [d(x, F(x, y, z)) + d(y, F(y, x, y)) + d(z, F(z, y, x)) + d(y, F(y, z, u)) + d(z, F(z, y, z)) + d(u, F(u, z, y))],$$

or a Prešić-Chatterjea type contractive condition:

$$d(F(x, y, z), F(y, z, u)) \leq \frac{k}{8} [d(x, F(y, z, u)) + d(y, F(z, y, z)) + d(z, F(u, z, y)) + d(y, F(x, y, z)) + d(z, F(y, x, y)) + d(u, F(z, y, x))].$$

The obtained tripled fixed point theorems extend and unify several related results in literature.

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REFERENCES

- [1] Berinde, V., *Iterative Approximation of Fixed Points*, Springer, Berlin Heidelberg New York, 2007
- [2] Berinde, V., *Generalized coupled fixed point theorems for mixed monotone mappings in partially ordered metric spaces*, *Nonlinear Anal.*, **74** (2011), No. 18, 7347–7355
- [3] Berinde, V., *Coupled coincidence point theorems for mixed monotone nonlinear operators*, *Comput. Math. Appl.*, **64** (2012), No. 6, 1770–1777
- [4] Berinde, V., *Coupled fixed point theorems for ϕ -contractive mixed monotone mappings in partially ordered metric spaces*, *Nonlinear Anal.*, **75** (2012), No. 6, 3218–3228
- [5] Berinde, V. and Borcut, M., *Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces*, *Nonlinear Anal.*, **74** (2011), No. 15, 4889–4897
- [6] Berinde, V. and Păcurar, M., *An iterative method for approximating fixed points of Prešić nonexpansive mappings*, *Rev. d'Anal. Numer. Theor. Approx.*, **38** (2009), No. 2, 144–153
- [7] Berinde, V. and Păcurar, M., *Two elementary applications of some Prešić type fixed point theorems*, *Creat. Math. Inform.*, **20** (2011), No. 1, 32–42
- [8] Berinde, V. and Păcurar, M., *Coupled fixed point theorems for generalized symmetric Meir-Keeler contractions in ordered metric spaces*, *Fixed Point Theory Appl.*, **2012**, 2012:115, 11 pp.
- [9] Bhaskar, T. Gnana and Lakshmikantham, V., *Fixed point theorems in partially ordered metric spaces and applications*, *Nonlinear Anal.*, **65** (2006), No. 7, 1379–1393
- [10] Bojor, F., *Fixed points of Kannan mappings in metric spaces endowed with a graph*, *An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat.*, **20** (2012), No. 1, 31–40
- [11] Borcut, M., *Tripled fixed point theorems for monotone mappings in partially ordered metric spaces*, *Carpathian J. Math.*, **28** (2012), No. 2, 215–222
- [12] Borcut, M., *Tripled coincidence theorems for monotone mappings in partially ordered metric spaces*, *Creat. Math. Inform.*, **21** (2012), No. 2, 135–142
- [13] Borcut, M., *Tripled coincidence theorems for contractive type mappings in partially ordered metric spaces*, *Appl. Math. Comput.*, **218** (2012), No. 14, 7339–7346
- [14] Borcut, M. and Berinde, V., *Tripled coincidence theorems for contractive type mappings in partially ordered metric spaces*, *Appl. Math. Comput.*, **218** (2012), No. 10, 5929–5936
- [15] Borcut, M., Păcurar, M. and Berinde, V., *Tripled fixed point theorems for mixed monotone Kannan type contractive mappings*, *J. Appl. Math.*, **2014**, Art. ID 120203, 8 pp.
- [16] Chen, Y.-Z., *A Prešić type contractive condition and its applications*, *Nonlinear Anal.*, **71** (2009), No. 12, e2012–e2017
- [17] Chatterjea, S. K., *Fixed-point theorems*, *C. R. Acad. Bulgare Sci.*, **25** (1972), 727–730
- [18] Choudhury, B. S. and Kundu, A., *A Kannan-like contraction in partially ordered spaces*, *Demonstratio Math.*, **46** (2013), No. 2, 327–334
- [19] Cirić, L. B. and Prešić, S. B., *On Prešić type generalization of the Banach contraction mapping principle*, *Acta Math. Univ. Comenianae*, **76** (2007), No. 2, 143–147
- [20] Kannan, R., *Some results on fixed points*, *Bull. Calcutta Math. Soc.* **10** (1968) 71-76
- [21] Karapinar, E. and Berinde, V., *Quadruple fixed point theorems for nonlinear contractions in partially ordered metric spaces*, *Banach J. Math. Anal.*, **6** (2012), No. 1, 74–89

- [22] Păcurar, M., *Approximating common fixed points of Prešić-Kannan type operators by a multi-step iterative method*, An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat., **17** (2009), No. 1, 153–168
- [23] Păcurar, M., *Iterative Methods for Fixed Point Approximation*, Risoprint, Cluj-Napoca, 2010
- [24] Păcurar, M., *A multi-step iterative method for approximating fixed points of Prešić-Kannan operators*, Acta Math. Univ. Comen. New Ser., **79** (2010), No. 1, 77–88
- [25] Păcurar, M., *A multi-step iterative method for approximating common fixed points of Prešić-Rus type operators on metric spaces*, Stud. Univ. Babeş-Bolyai Math., **55** (2010), No. 1, 149–162
- [26] Păcurar, M., *Fixed points of almost Prešić operators by a k-step iterative method*, An. Ştiinţ. Univ. Al. I. Cuza Iaşi, Ser. Noua, Mat., **57** (2011), Supliment 199–210
- [27] Păvăloiu, I., *Rezolvarea ecuaţiilor prin interpolare*, Editura Dacia, Cluj-Napoca, 1981
- [28] Păvăloiu, I. and Pop, N., *Interpolare şi aplicaţii*, Risoprint, Cluj-Napoca, 2005
- [29] Petric, M., *Some results concerning cyclical contractive mappings*, Gen. Math. **18** (2010), No. 4, 213–226
- [30] Petric, M., *Best proximity point theorems for weak cyclic Kannan contractions*. Filomat **25** (2011), No. 1, 145–154
- [31] Petric, M. and Zlatanov, B., *Best proximity points and fixed points for p-summing maps*. Fixed Point Theory Appl. **2012**, 2012: 86, 12 pp.
- [32] Prešić, S. B., *Sur une classe d' inéquations aux différences finites et sur la convergence de certaines suites*, Publ. Inst. Math. (Beograd)(N.S.), **5** (19) (1965), 75–78
- [33] Rus, I. A., *An iterative method for the solution of the equation $x = f(x, \dots, x)$* , Rev. Anal. Numer. Theor. Approx., **10** (1981), No.1, 95–100
- [34] Zamfirescu, T., *Fix point theorems in metric spaces*, Arch. Math. (Basel), **23** (1972), 292–298

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