

## Heuristic introduction to weakly Picard operator theory

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### ABSTRACT.

In this paper we study the impact of weakly Picard operator theory, [see I. A. Rus, *Picard operators and applications*, Sc. Math. Japonicae, **58** (2003), No. 1, 191–219] on the following problem: what can we do in order to find conditions under which a given operator is a weakly Picard operator?

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