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# On a functional equation arising in mathematical biology and theory of learning

VASILE BERINDE[1,2] and ABDUL RAHIM KHAN[2]

ABSTRACT.

V. Istrăţescu [Istrăţescu, V. I., *On a functional equation*, J. Math. Anal. Appl., **56** (1976), No. 1, 133–136] used the Banach contraction mapping principle to establish an existence and approximation result for the solution of the functional equation

$$\varphi(x) = x\varphi((1-\alpha)x+\alpha) + (1-x)\varphi((1-\beta)x),\ x \in [0,1],\ (0 < \alpha \leq \beta < 1),$$

which is important for some mathematical models arising in biology and theory of learning.

This equation has been studied by Lyubich and Shapiro [A. P. Lyubich, Yu. I. and Shapiro, A. P., *On a functional equation* (Russian), Teor. Funkts., Funkts. Anal. Prilozh. **17** (1973), 81–84] and subsequently, by Dmitriev and Shapiro [Dmitriev, A. A. and Shapiro, A. P., *On a certain functional equation of the theory of learning* (Russian), Usp. Mat. Nauk **37** (1982), No. 4 (226), 155–156].

The main aim of this note is to solve this functional equation with more general arguments for $\varphi$ on the right hand side, by using appropriate fixed point tools.

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[1] DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
NORTH UNIVERSITY CENTER AT BAIA MARE
TECHNICAL UNIVERSITY OF CLUJ-NAPOCA
VICTORIEI 76, 430122 BAIA MARE ROMANIA
*E-mail address*: vberinde@ubm.ro

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[2] DEPARTMENT OF MATHEMATICS AND STATISTICS
KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DHAHRAN, SAUDI ARABIA
*E-mail address*: arahim@kfupm.edu.sa
*E-mail address*: vberinde@ubm.ro