A survey on the stability of mean value points and functional equations involving some special functions

SORINEL DUMITRESCU [1], MIHAI MONEA[1] and CRISTINEL MORTICI[2,3]

ABSTRACT.

The aim of this work is to put together some of the recent and classical results in the theory of stability. In the first part, we recall the results regarding the intermediary points arising from various Mean Value Theorems, then we study the stability of some functional equations involving the gamma and beta functions.

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[1] DEPARTMENT OF MATHEMATICS
PH. D. STUDENT, UNIVERSITY POLITEHNICA OF BUCHAREST
SPLAIUL INDEPENDENŢEI 313, BUCHAREST, ROMANIA
E-mail address: sorineldumitrescu@yahoo.com

[1] DEPARTMENT OF MATHEMATICS
PH. D. STUDENT, UNIVERSITY POLITEHNICA OF BUCHAREST
SPLAIUL INDEPENDENŢEI 313, BUCHAREST, ROMANIA
E-mail address: mihaimonea@yahoo.com

[2] DEPARTMENT OF MATHEMATICS
VALAHIA UNIVERSITY OF TÂRGOVIŞTE
BD. UNIRII 18, 130082 TÂRGOVIŞTE, ROMANIA

[3] DEPARTMENT OF MATHEMATICS
ACADEMY OF THE ROMANIAN SCIENTISTS
SPLAIUL INDEPENDENŢEI 54, 050094 BUCHAREST, ROMANIA
E-mail address: cristinel.mortici@hotmail.com