Commuting Regularity degree of finite semigroups

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ABSTRACT.

A pair (x, y) of elements x and y of a semigroup S is said to be a commuting regular pair, if there exists an element $z \in S$ such that $xy = (yx)z(yx)$. In a finite semigroup S, the probability that the pair (x, y) of elements of S is commuting regular will be denoted by $dcr(S)$ and will be called the Commuting Regularity degree of S. Obviously if S is a group, then $dcr(S) = 1$. However for a semigroup S, getting an upper bound for $dcr(S)$ will be of interest to study and to identify the different types of non-commutative semigroups. In this paper, we calculate this probability for certain classes of finite semigroups. In this study we managed to present an interesting class of semigroups where the probability is $\frac{1}{2}$. This helps us to estimate a condition on non-commutative semigroups such that the commuting regularity of (x, y) yields the commuting regularity of (y, x).

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