

k -Combinations of an unlabelled graph

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ABSTRACT.

In this paper we extend the notion of the binomial coefficient $\binom{n}{k}$ into a new notion $\binom{[G]}{k}$, where $[G]$ is an unlabelled graph with n vertices and $0 \leq k \leq n$. We call $\binom{[G]}{k}$ as the graph binomial coefficient and a version of the graph binomial expansion is also studied. As an application of this notion, we enumerate the number of ways to color vertices of a path and beads of a necklace.

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