

## A numerical study on the robustness and efficiency of the PL homotopy algorithm for solving unconstrained optimization problems

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### ABSTRACT.

Our aim in this paper is to illustrate the relevance of the fixed point piecewise-linear homotopy algorithm for solving unconstrained optimization problems. The numerical tests are performed by using an implementation of the piecewise-linear homotopy algorithm in the modern programming language C#, as described previously in [Bozantan, A., *An implementation of the piecewise-linear homotopy algorithm for the computation of fixed points*, *Creat. Math. Inform.*, **19** (2010), No. 2, 140–148] and [Bozantan, A. and Berinde, V., *Applications of the PL homotopy algorithm for the computation of fixed points to unconstrained optimization problems*, *Creat. Math. Inform.*, **22** (2013), No. 1, 41–46]. As shown by the numerical experiments done on a set of classic test functions in optimization theory, the PL homotopy algorithm appears to be more reliable than the classical Newton's method and some other important methods for finding local or global minima.

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