

Remark on the Laplacian-energy-like and Laplacian incidence energy invariants of graphs

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ABSTRACT.

Let G be an undirected connected graph with n vertices and m edges, $n \geq 3$, and let $\mu_1 \geq \mu_2 \geq \dots \geq \mu_{n-1} > \mu_n = 0$ and $\rho_1 \geq \rho_2 \geq \dots \geq \rho_{n-1} > \rho_n = 0$ be Laplacian and normalized Laplacian eigenvalues of G , respectively. The Laplacian-energy-like (LEL) invariant of graph G is defined as $LEL(G) = \sum_{i=1}^{n-1} \sqrt{\mu_i}$. The Laplacian incidence energy of graph is defined as $LIE(G) = \sum_{i=1}^{n-1} \sqrt{\rho_i}$. In this paper, we consider lower bounds of graph invariants $LEL(G)$ and $LIE(G)$ in terms of some graph parameters, that refine some known results.

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