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Estimating a special difference of harmonic numbers

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ABSTRACT. The aim of this paper is to investigate a special difference of harmonic numbers. We obtain some limits and inequalities involving harmonic numbers and in the last part of the paper some open problems for investigation are posed.

1. INTRODUCTION

Let H_n be the nth harmonic number, i.e.

$$H_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n},$$

and let $D_n = H_n - \ln n$. As it is well-known, the sequence $(D_n)_{n\in\mathbb{N}}$ is convergent and $\gamma = \lim_{n\to\infty} D_n$ is Euler's constant ([9, Chapter 9, pp. 69–79], [20, Chapter 1, pp. 7–60], [21, p. 22; problem 72, p. 23], [22, problem 1.20, p. 6]).

As we have already mentioned in [18], [19], [20, pp. 7, 8], various lower and upper estimates have been obtained for $D_n - \gamma$, such as:

$$\begin{aligned}
&\frac{1}{2(n+1)} < D_n - \gamma < \frac{1}{2(n-1)}, && n \in \mathbb{N}\setminus\{1\} && ([23]);\\
&\frac{1}{2(n+1)} < D_n - \gamma < \frac{1}{2n}, && n \in \mathbb{N} && ([15], [28]);\\
&\frac{1}{2n+1} < D_n - \gamma < \frac{1}{2n}, && n \in \mathbb{N} && ([26]);\\
&\frac{1}{2n+\frac{2}{5}} < D_n - \gamma < \frac{1}{2n+\frac{1}{3}}, && n \in \mathbb{N} && ([24], [25]);\\
&\frac{1}{2n+\frac{2\gamma-1}{1-\gamma}} \le D_n - \gamma < \frac{1}{2n+\frac{1}{3}}, && n \in \mathbb{N} && ([25], [1], [3]).
\end{aligned} \tag{1.1}$$

It is worth specifying that $\frac{2\gamma-1}{1-\gamma}$ and $\frac{1}{3}$, that appear in (1.1), are the best constants with this property, i.e. $\frac{2\gamma-1}{1-\gamma}$ cannot be replaced by a smaller one and $\frac{1}{3}$ cannot be replaced by a larger one, so that the inequalities to hold for all $n \in \mathbb{N}$ ([25], [1], [3]). Clearly,

$$\lim_{n\to\infty} n(D_n - \gamma) = \frac{1}{2}, \tag{1.2}$$

which means that the sequence $(D_n)_{n\in\mathbb{N}}$ converges to γ very slowly, more precisely, with order 1.

This is the reason why many mathematicians tried to discover sequences with higher rate of convergence to γ. A possible method of doing so is to modify the argument of the logarithmic term in D_n.

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For example, D. W. DeTemple considered

$$R_n = H_n - \ln\left(n + \frac{1}{2}\right),$$

and he proved in [4] that

$$\frac{1}{24(n+1)^2} < R_n - \gamma < \frac{1}{24n^2}, n \in \mathbb{N},$$

which implies that

$$\lim_{n\to\infty} n^2(R_n - \gamma) = \frac{1}{24},$$

so the sequence $(R_n)_{n\in\mathbb{N}}$ converges to γ with order 2.

Another way of increasing the speed of convergence to γ is by subtracting a rational term in D_n. In [27] it is shown that

$$\lim_{n\to\infty} n^2\left(\gamma - D_n + \frac{1}{2n}\right) = \frac{1}{12}, \tag{1.3}$$

hereby the sequence $\left(D_n - \frac{1}{2n}\right)_{n\in\mathbb{N}}$ converges to γ with order 2. B.-N. Guo and F. Qi proved in [7, Theorem 1] that

$$\frac{1}{12n^2 + \frac{6}{5}} < \gamma - \left(D_n - \frac{1}{2n}\right) \le \frac{1}{12n^2 + \frac{2(7-12\gamma)}{2\gamma-1}}, \quad n \in \mathbb{N}, \tag{1.4}$$

the numbers $\frac{6}{5}$ and $\frac{2(7-12\gamma)}{2\gamma-1}$ being the best constants with this property.

All the above-mentioned results, together with a problem (see [17], [11, problem 3.1.3, p. 187], [12, problem 1.3, part a), p. 8]) in which it is asked to be shown that

$$\gamma < H_p + H_q - H_{pq} \le 1, \quad p, q \in \mathbb{N},$$

inspired us in writing the present paper.

In Section 2 it is proved that

$$\lim_{n\to\infty} n[pH_n - H_{n^p} - (p-1)\gamma] = \frac{p}{2},$$

for $p \in \mathbb{N} \setminus \{1\}$.

We obtain the best constants a and b with the property that the inequalities

$$\frac{1}{n+a} \le 2H_n - H_{n^2} - \gamma < \frac{1}{n+b}$$

hold for every $n \in \mathbb{N}$. These constants are $a = \frac{\gamma}{1-\gamma}$ and $b = \frac{2}{3}$.

Also, we obtain that

$$\lim_{n\to\infty} n^2\left[\gamma - \left(2H_n - H_{n^2} - \frac{1}{n}\right)\right] = \frac{2}{3}$$

and

$$\lim_{n\to\infty} n^2\left[(p-1)\gamma - \left(pH_n - H_{n^p} - \frac{p}{2n}\right)\right] = \frac{p}{12},$$

for $p \in \mathbb{N} \setminus \{1, 2\}$, and we prove some inequalities regarding $\gamma - \left(2H_n - H_{n^2} - \frac{1}{n}\right)$.

We mention that only after obtaining our results, we saw the paper [10].

In Section 3 some open problems for investigation are posed.

Interesting results involving Euler–Mascheroni type sequences can be found in [2]. Other results on harmonic numbers can be found in [8], [13], [16].

Further on we collect some formulae that we need in our analysis. Recall that the digamma function ψ is the logarithmic derivative of the gamma function, i.e.

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}, \quad x \in (0, +\infty).$$

It is known that ([6, Section 8.365, Entry 4, p. 904], [14, Section 5.4, Entry 5.4.14, p. 137])

$$\psi(n+1) = -\gamma + H_n, \quad n \in \mathbb{N}. \tag{1.5}$$

From the recurrence formula ([6, Section 8.365, Entry 1, p. 904], [14, Section 5.5, Entry 5.5.2, p. 138])

$$\psi(x+1) = \psi(x) + \frac{1}{x}, \quad x \in (0, +\infty),$$

and the asymptotic formula ([14, Section 5.11, Entry 5.11.2, p. 140], see also [6, Section 8.367, Entry 13, p. 906])

$$\psi(x) \sim \ln x - \frac{1}{2x} - \frac{1}{12x^2} + \frac{1}{120x^4} - \frac{1}{252x^6} + \cdots \quad (x \to \infty),$$

we get

$$\psi(x+1) \sim \ln x + \frac{1}{2x} - \frac{1}{12x^2} + \frac{1}{120x^4} - \frac{1}{252x^6} + \cdots \quad (x \to \infty). \tag{1.6}$$

2. Double inequalities for harmonic numbers

We mention that the idea for the solution of the following limit was provided to the author by O. Furdui (see also [5, problem 1.28, p. 5] for a problem on a double Euler–Mascheroni sequence).

Proposition 2.1. *Let $p \in \mathbb{N} \setminus \{1\}$. We have*

$$\lim_{n\to\infty} n[pH_n - H_{n^p} - (p-1)\gamma] = \frac{p}{2}.$$

Proof. We obtain, based on (1.2), that

$$\begin{aligned} &\lim_{n\to\infty} n[pH_n - H_{n^p} - (p-1)\gamma] \\ &= \lim_{n\to\infty} \left[pn(H_n - \ln n - \gamma) - \frac{n^p(H_{n^p} - \ln n^p - \gamma)}{n^{p-1}} \right] = p \cdot \frac{1}{2} - 0 = \frac{p}{2}. \end{aligned}$$

□

Hence, for $p = 2$ in Proposition 2.1 we get that

$$\lim_{n\to\infty} n(2H_n - H_{n^2} - \gamma) = 1.$$

Now we give our first main result.

Theorem 2.1. *We have*

$$\frac{1}{n + \frac{\gamma}{1-\gamma}} \le 2H_n - H_{n^2} - \gamma < \frac{1}{n + \frac{2}{3}},$$

for every $n \in \mathbb{N}$. Moreover, the constants $\frac{\gamma}{1-\gamma}$ and $\frac{2}{3}$ are the best possible with this property.

Proof. As one can easily see,

$$2H_n - H_{n^2} - \gamma = 2(D_n - \gamma) - (D_{n^2} - \gamma).$$

Having in view (1.1), we are able to write that

$$\frac{2}{2n + \frac{2\gamma-1}{1-\gamma}} - \frac{1}{2n^2 + \frac{1}{3}} < 2H_n - H_{n^2} - \gamma < \frac{2}{2n + \frac{1}{3}} - \frac{1}{2n^2 + \frac{2\gamma-1}{1-\gamma}},$$

for all $n \in \mathbb{N}$.

Taking into account that

$$\begin{aligned}
&\frac{2}{2n+\frac{2\gamma-1}{1-\gamma}} - \frac{1}{2n^2+\frac{1}{3}} - \frac{1}{n+\frac{\gamma}{1-\gamma}} \\
&\quad = \frac{3(1-\gamma)(n-2)(2\gamma n+1) - 6\gamma^2 - 4\gamma + 7}{3(1-\gamma)^2\left(2n+\frac{2\gamma-1}{1-\gamma}\right)\left(2n^2+\frac{1}{3}\right)\left(n+\frac{\gamma}{1-\gamma}\right)} > 0, \quad n \geq 2,
\end{aligned}$$

as a straightforward calculation shows, it follows that

$$\frac{1}{n+\frac{\gamma}{1-\gamma}} < \frac{2}{2n+\frac{2\gamma-1}{1-\gamma}} - \frac{1}{2n^2+\frac{1}{3}}, \quad n \geq 2.$$

So,

$$\frac{1}{n+\frac{\gamma}{1-\gamma}} \leq 2H_n - H_{n^2} - \gamma,$$

for every $n \in \mathbb{N}$, and the constant $\frac{\gamma}{1-\gamma}$ is the best possible with this property (the equality holds only when $n = 1$).

Also, a straightforward calculation shows that

$$\frac{2}{2n+\frac{1}{3}} - \frac{1}{2n^2+\frac{2\gamma-1}{1-\gamma}} < \frac{1}{n+\frac{2}{3}}, \quad n \in \mathbb{N},$$

because

$$\begin{aligned}
&\frac{1}{n+\frac{2}{3}} - \frac{2}{2n+\frac{1}{3}} + \frac{1}{2n^2+\frac{2\gamma-1}{1-\gamma}} \\
&\quad = \frac{15(1-\gamma)n - 20\gamma + 11}{9(1-\gamma)\left(n+\frac{2}{3}\right)\left(2n+\frac{1}{3}\right)\left(2n^2+\frac{2\gamma-1}{1-\gamma}\right)} > 0, \quad n \in \mathbb{N},
\end{aligned}$$

which means that

$$2H_n - H_{n^2} - \gamma < \frac{1}{n+\frac{2}{3}},$$

for every $n \in \mathbb{N}$. It remains to prove that the constant $\frac{2}{3}$ is the best possible with this property, fact which we get from the following two assertions.

We have just proved that

$$\frac{1}{2H_n - H_{n^2} - \gamma} - n > \frac{2}{3}, \quad n \in \mathbb{N}.$$

On the other hand, using (1.5) and (1.6), we have

$$\begin{aligned}
\frac{1}{2H_n - H_{n^2} - \gamma} - n &= \frac{1}{2\psi(n+1) - \psi(n^2+1)} - n \\
&= \frac{1}{\frac{1}{n} - \frac{2}{3n^2} + O\left(\frac{1}{n^4}\right)} - n \\
&= \frac{\frac{2}{3} + O\left(\frac{1}{n^2}\right)}{1 + O\left(\frac{1}{n}\right)} \to \frac{2}{3} \quad (n \to \infty).
\end{aligned}$$

$\square$

Proposition 2.2. *Let $p \in \mathbb{N} \setminus \{1\}$. We have*

$$\lim_{n\to\infty} n^2 \left[(p-1)\gamma - \left(pH_n - H_{n^p} - \frac{p}{2n}\right)\right] = \begin{cases} 2\cdot\frac{1}{12} - 0\cdot\frac{1}{12} + \frac{1}{2} = \frac{2}{3}, & p = 2, \\ p\cdot\frac{1}{12} - 0\cdot\frac{1}{12} + 0 = \frac{p}{12}, & p \geq 3. \end{cases}$$

Proof. We obtain, based on (1.3), that

$$\begin{aligned} &\lim_{n\to\infty} n^2 \left[(p-1)\gamma - \left(pH_n - H_{n^p} - \frac{p}{2n}\right)\right] \\ &= \lim_{n\to\infty} \left[pn^2\left(\gamma - H_n + \ln n + \frac{1}{2n}\right)\right. \\ &\qquad \left. - \frac{1}{n^{2p-2}} \cdot n^{2p}\left(\gamma - H_{n^p} + \ln n^p + \frac{1}{2n^p}\right) + \frac{1}{2n^{p-2}}\right] \\ &= \begin{cases} 2\cdot\frac{1}{12} - 0\cdot\frac{1}{12} + \frac{1}{2} = \frac{2}{3}, & p = 2, \\ p\cdot\frac{1}{12} - 0\cdot\frac{1}{12} + 0 = \frac{p}{12}, & p \geq 3. \end{cases} \end{aligned}$$

□

The other main result of our paper is the following theorem.

Theorem 2.2. *We have*

$$\frac{2}{3n^2 + \frac{2}{\gamma} - 3} \leq \gamma - \left(2H_n - H_{n^2} - \frac{1}{n}\right) < \frac{2}{3n^2 + \frac{43}{100}},$$

for every $n \in \mathbb{N}$. Moreover, the constant $\frac{2}{\gamma} - 3$ is the best possible with this property.

Proof. Let $v_n = 2H_n - H_{n^2} - \frac{1}{n}$, for $n \in \mathbb{N}$. We have

$$\gamma - v_n = 2\left(\gamma - H_n + \ln n + \frac{1}{2n}\right) - \left(\gamma - H_{n^2} + \ln n^2 + \frac{1}{2n^2}\right) + \frac{1}{2n^2}.$$

We can therefore write, having in view (1.4), that

$$\frac{1}{6n^2 + \frac{3}{5}} - \frac{1}{12n^4 + \frac{2(7-12\gamma)}{2\gamma-1}} + \frac{1}{2n^2} < \gamma - v_n < \frac{1}{6n^2 + \frac{7-12\gamma}{2\gamma-1}} - \frac{1}{12n^4 + \frac{6}{5}} + \frac{1}{2n^2},$$

for all $n \in \mathbb{N}$.

It is not difficult to check that

$$\begin{aligned} &\frac{1}{6n^2 + \frac{3}{5}} - \frac{1}{12n^4 + \frac{2(7-12\gamma)}{2\gamma-1}} + \frac{1}{2n^2} - \frac{2}{3n^2 + \frac{2}{\gamma} - 3} \\ &= \frac{1}{10\gamma(2\gamma-1)n^2\left(6n^2 + \frac{3}{5}\right)\left(6n^4 + \frac{7-12\gamma}{2\gamma-1}\right)\left(3n^2 + \frac{2}{\gamma} - 3\right)} \\ &\quad \times\{(n-2)(n+2)[(-1656\gamma^2 + 1788\gamma - 480)n^4 + (-6570\gamma^2 + 7077\gamma - 1896)n^2 \\ &\qquad -24786\gamma^2 + 26466\gamma - 7018] - 99036\gamma^2 + 105729\gamma - 28030\} > 0, \quad n \geq 2, \end{aligned}$$

and this implies that

$$\frac{2}{3n^2 + \frac{2}{\gamma} - 3} < \frac{1}{6n^2 + \frac{3}{5}} - \frac{1}{12n^4 + \frac{2(7-12\gamma)}{2\gamma-1}} + \frac{1}{2n^2}, \quad n \geq 2.$$

So,

$$\frac{2}{3n^2 + \frac{2}{\gamma} - 3} \leq \gamma - v_n,$$

for every $n \in \mathbb{N}$, and the constant $\frac{2}{\gamma} - 3$ is the best possible with this property (the equality holds only when $n = 1$).

Also, it can be shown that

$$\frac{1}{6n^2 + \frac{7-12\gamma}{2\gamma-1}} - \frac{1}{12n^4 + \frac{6}{5}} + \frac{1}{2n^2} < \frac{2}{3n^2 + \frac{43}{100}}, \quad n \in \mathbb{N},$$

because

$$\begin{aligned} &\frac{2}{3n^2 + \frac{43}{100}} - \frac{1}{6n^2 + \frac{7-12\gamma}{2\gamma-1}} + \frac{1}{12n^4 + \frac{6}{5}} - \frac{1}{2n^2} \\ &= \frac{-1}{1000(2\gamma-1)n^2\left(3n^2 + \frac{43}{100}\right)\left(6n^2 + \frac{7-12\gamma}{2\gamma-1}\right)\left(6n^4 + \frac{3}{5}\right)} \\ &\quad \times \{(n-1)(n+1)[(38640\gamma - 22320)n^4 + (38580\gamma - 22500)n^2 \\ &\quad +46824\gamma - 27137] + 45276\gamma - 26234\} > 0, \quad n \in \mathbb{N}. \end{aligned}$$

It follows that

$$\gamma - v_n < \frac{2}{3n^2 + \frac{43}{100}},$$

for every $n \in \mathbb{N}$. □

3. Comments and open problems

In Theorem 2.1 we have obtained estimates for $2H_n - H_{n^2} - \gamma$, $n \in \mathbb{N}$, with best possible constants. As it was proved in the beginning of Section 2, we have the following limit $\lim_{n\to\infty} n[pH_n - H_{n^p} - (p-1)\gamma] = \frac{p}{2}$, for $p \in \mathbb{N} \setminus \{1\}$. Therefore, it is natural to investigate the problem of finding the best possible constants a and b such that

$$\frac{p}{2n+a} \leq pH_n - H_{n^p} - (p-1)\gamma < \frac{p}{2n+b}, \tag{3.7}$$

for every $n \in \mathbb{N}$. In order to do so we consider the sequence $(b_n)_{n\in\mathbb{N}}$ defined by the equality

$$pH_n - H_{n^p} - (p-1)\gamma = \frac{p}{2n+b_n}.$$

Thus

$$b_n = \frac{p}{pH_n - H_{n^p} - (p-1)\gamma} - 2n.$$

We have

$$b_1 = \frac{p}{(p-1)(1-\gamma)} - 2$$

and

$$\begin{aligned} b_n &= \frac{p}{p\psi(n+1) - \psi(n^p+1)} - 2n \\ &= \frac{p}{\frac{p}{2n} - \frac{p}{12n^2} + \frac{p}{120n^4} - \frac{1}{2n^p} + \frac{1}{12n^{2p}} + O\left(\frac{1}{n^6}\right)} - 2n \\ &= \frac{\frac{p}{6n} - \frac{p}{60n^3} + \frac{1}{n^{p-1}} - \frac{1}{6n^{2p-1}} + O\left(\frac{1}{n^5}\right)}{\frac{p}{2n} - \frac{p}{12n^2} + \frac{p}{120n^4} - \frac{1}{2n^p} + \frac{1}{12n^{2p}} + O\left(\frac{1}{n^6}\right)} \\ &\xrightarrow{n\to\infty} \begin{cases} \frac{4}{3}, & p = 2, \\ \frac{1}{3}, & p \geq 3. \end{cases} \end{aligned}$$

For $p \geq 3$, it remains to study the preceding problem (3.7), with $a = b_1 = \frac{p}{(p-1)(1-\gamma)} - 2$ and $b = \lim\limits_{n\to\infty} b_n = \frac{1}{3}$. We mention that, for $p = 2$, we recover the result from Theorem 2.1, with $\frac{a}{2} = \frac{b_1}{2} = \frac{\gamma}{1-\gamma}$ and $\frac{b}{2} = \frac{1}{2}\lim\limits_{n\to\infty} b_n = \frac{2}{3}$.

Another problem that can be investigated is related to Theorem 2.2. We consider the sequence $(\tilde{b}_n)_{n\in\mathbb{N}}$ defined by the equality

$$\gamma - \left(2H_n - H_{n^2} - \frac{1}{n}\right) = \frac{2}{3n^2 + \tilde{b}_n}.$$

Thereby

$$\tilde{b}_n = \frac{2}{\gamma - \left(2H_n - H_{n^2} - \frac{1}{n}\right)} - 3n^2.$$

Clearly, $\tilde{b}_1 = \frac{2}{\gamma} - 3$ and

$$\begin{aligned} \tilde{b}_n &= \frac{2}{\psi(n^2+1) - 2\psi(n+1) + \frac{1}{n}} - 3n^2 \\ &= \frac{2}{\frac{2}{3n^2} - \frac{1}{10n^4} + O\left(\frac{1}{n^6}\right)} - 3n^2 \\ &= \frac{\frac{3}{10} + O\left(\frac{1}{n^2}\right)}{\frac{2}{3} + O\left(\frac{1}{n^2}\right)} \xrightarrow{n\to\infty} \frac{9}{20}. \end{aligned}$$

It would be of interest to study if in Theorem 2.2, the constant $\frac{43}{100}$ can be replaced by $\lim\limits_{n\to\infty} \tilde{b}_n = \frac{9}{20}$ as the best constant.

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