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Supra soft b-connectedness I: Supra soft b-irresoluteness and separateness

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ABSTRACT. This work is divided into two parts. In this part, we introduce and study the notion of supra soft connectedness based on the notion of supra b-open soft sets and give basic definitions and theorems about it. Further, we introduce the notion of supra b-irresolute soft functions as a generalization to the supra b-continuous soft function and study their properties in detail. Finally, we show that, the surjective supra b-irresolute soft image of supra soft b-connected space is also supra soft b-connected.

1. INTRODUCTION

Theories such as theory of vague sets and theory of rough sets are considered as mathematical tools for dealing with uncertainties. But these theories have their own difficulties. The concept of soft sets was first introduced by Molodtsov [20] in 1999 as a general mathematical tool for dealing with uncertain objects, in order to solve complicated problems in economics, engineering and the like. In [20, 21], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [19], the properties and applications of soft set theory have been studied increasingly [2, 17, 21, 22]. Recently, in 2011, Shabir and Naz [25] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Hussain et al. [6] investigated the properties of open (closed) soft, soft nbd and soft closure which are fundamental for further research on soft topology.

It got some stability only after the introduction of soft topology [25] in 2011. In [8], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [15] introduced the notion of soft semi separation axioms. The notion of soft ideal was initiated for the first time by Kandil et al. [11]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al. [9, 10, 12, 13, 14, 16]. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [4]. The notion of b-open soft sets was initiated by El-sheikh and Abd El-latif [3] and extended in [23]. An applications on b-open soft sets were introduced in [1, 5]. The notion of supra b-open soft sets was initiated by Abd El-latif et al. [1].

In this paper, the concept of supra b-irresolute soft functions is introduced, as a generalization to the supra b-continuous soft functions and several properties of it is investigated. Further, we introduce and study the notion of supra soft b-connectedness and gave

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the basic definitions and theorems about it. Finally, we show that, the surjective supra b-irresolute soft image of supra soft b-connected space is also supra soft b-connected.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which may found in earlier studies [1, 18, 25].

Definition 2.1. [20] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \varphi$ i.e
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2. [25] Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.3. [25] Let $x \in X$. Then x_E denote the soft set over X for which $x_E(e) = \{x\}\ \forall e \in E$ and called the singleton soft point.

Definition 2.4. [6, 25] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

- **(1):** $\tilde{X}, \tilde{\varphi} \in \tau$, where $\tilde{\varphi}(e) = \varphi$ and $\tilde{X}(e) = X,\ \forall e \in E$,
- **(2):** the union of any number of soft sets in τ belongs to τ,
- **(3):** the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$. A soft set (F, E) over X is said to be closed soft set, if its relative complement $(F, E)^c$ is an open soft set.

Definition 2.5. [26] The soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $\alpha \in E$ such that $F(\alpha) = \{x\}$ and $F(\alpha^c) = \varphi$ for each $\alpha^c \in E - \{\alpha\}$, and the soft point (F, E) is denoted by x_α. The soft point x_α is said to be belonging to the soft set (G, A), denoted by $x_\alpha \tilde{\in} (G, A)$, if for the element $\alpha \in A$, $F(\alpha) \subseteq G(\alpha)$.

Definition 2.6. [25] Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a non-null subset of X. Then the sub soft set of (F, E) over Y denoted by (F_Y, E), is defined as follows:

$$F_Y(e) = Y \cap F(e)\ \forall e \in E.$$

In other words $(F_Y, E) = \tilde{Y} \tilde{\cap} (F, E)$.

Definition 2.7. [25] Let (X, τ, E) be a soft topological space and Y be a non-null subset of X. Then

$$\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}$$

is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Definition 2.8. [3] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then, (F, E) is called a b-open soft set if $(F, E) \tilde{\subseteq} cl(int(F, E)) \tilde{\cup} int(cl(F, E))$. The set of all b-open soft sets is denoted by $BOS(X, \tau, E)$, or $BOS(X)$ and the set of all b-closed soft sets is denoted by $BCS(X, \tau, E)$, or $BCS(X)$.

Definition 2.9. [4] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq SS(X)_E$ is called supra soft topology on X with a fixed set E if

(1): $\tilde{X}, \tilde{\varphi} \in \mu$,

(2): the union of any number of soft sets in μ belongs to μ.

The triplet (X, μ, E) is called supra soft topological space (or supra soft spaces) over X.

Definition 2.10. [4] Let (X, μ, E) be a supra soft topological space over and $(F, E) \in SS(X)_E$. Then, the supra soft interior of (G, E), denoted by $int^s(G, E)$ is the soft union of all supra open soft subsets of (G, E) i.e
$int^s(G, E) = \tilde{\cup}\{(H, E) : (H, E)\ is\ supra\ open\ soft\ set\ and\ (H, E)\tilde{\subseteq}(G, E)\}$. Also, the supra soft closure of (F, E), denoted by $cl^s(F, E)$ is the soft intersection of all supra closed super soft sets of (F, E). Clearly, $cl^s(F, E)$ is the smallest supra closed soft set over X which contains (F, E) i.e
$cl^s(F, E) = \tilde{\cap}\{(H, E) : (H, E)\ is\ supra\ closed\ soft\ set\ and\ (F, E)\tilde{\subseteq}(H, E)\}$.

Definition 2.11. [1] Let (X, μ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$. Then, (F, E) is called a supra $b-$open soft set if $(F, E)\tilde{\subseteq}cl^s(int^s(F, E))\tilde{\cup}int^s(cl^s(F, E))$. The complement of a supra $b-$open soft set is a supra $b-$closed soft set. The set of all supra $b-$open soft sets is denoted by $SBOS(X, \mu, E)$, or $SBOS_E(X)$ and the set of all supra $b-$closed soft sets is denoted by $SBCS(X, \mu, E)$, or $SBCS_E(X)$.

Definition 2.12. [1] Let (X, μ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$. Then, the supra b-soft interior of (F, E) is denoted by $bSint^s(F, E)$, where $bSint^s(F, E) = \tilde{\bigcup}\{(G, E) : (G, E)\tilde{\subseteq}(F, E),\ (G, E) \in SBOS_E(X)\}$.
Also, the supra b-soft closure of (F, E) is denoted by $bScl^s(F, E)$, where $bScl^s(F, E) = \tilde{\bigcap}\{(H, E) : (H, E) \in SBCS_E(X),\ (F, E)\tilde{\subseteq}(H, E)\}$. The supra soft b-boundary of a soft set (F, E), denoted by $b - Sbds(F, E)$, and is defined as $b - Sbd^s(F, E) = bScl^s(F, E) \tilde{\cap} bScl^s(F_E)^c = bScl^s(F, E) - bSint^s(F, E)$.

3. SUPRA b-IRRESOLUTE SOFT FUNCTIONS

In this section, we introduce a new type of soft functions called a supra b-irresolute soft function as a generalization to the supra b-continuoussoft functions and obtain some of their properties and characterizations.

Definition 3.13. Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ is called supra$b$-irresolute soft if $f_{pu}^{-1}(F, B) \in SBOS_A(X)$ for each $(F, B) \in SBOS_B(X)$.

Theorem 3.1. *Every suprab-irresolute soft function is supra b-continuoussoft.*

Proof. Straightforward. □

Remark 3.1. The converse of Theorem 3.1 is not true in general, as shown in the following example.

Example 3.1. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \rightarrow Y$ and $p : A \rightarrow B$ as follows:
$u(a) = y,\ u(b) = z\ ,\ u(c) = x,$
$p(e_1) = k_1,\ p(e_2) = k_2$. Let (X, τ_1, A) be a soft topological space over X where,
$\tau_1 = \{\tilde{X}, \tilde{\varphi}, (H_1, A), (H_2, A)\}$, where $(H_1, A), (H_2, A)$ are soft sets over X defined as follows: $H_1(e_1) = \{a\}$, $H_1(e_2) = \{a\}$, $H_2(e_1) = \{a, b\}$, $H_2(e_2) = \{a, b\}$.
The supra soft topology μ_1 is defined as follows, μ_1=$\{\tilde{X}, \tilde{\varphi}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$,

where $(F_1, A), (F_2, A), (F_3, A), (F_4, A)$ are soft sets over X defined as follows:
$F_1(e_1) = \{a\}$, $F_1(e_2) = \{a\}$, $F_2(e_1) = \{a, b\}$, $F_2(e_2) = \{a, b\}$, $F_3(e_1) = \{a\}$, $F_3(e_2) = \{b\}$, $F_4(e_1) = \{a\}$, $F_4(e_2) = \{a, b\}$.
Let (Y, τ_2, B) be a soft topological space over Y where,
$\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, where (G, B) is a soft set over Y defined by:
$G(k_1) = \{x, y\}$, $G(k_2) = \{x, y\}$.
The supra soft topology μ_2 is defined as follows, $\mu_2 = \{\tilde{X}, \tilde{\varphi}, (G_1, B), (G_2, B)\}$, where $(G_1, B), (G_2, B)$ are soft sets over Y defined as follows:
$G_1(e_1) = \{x, y\}$, $G_1(e_2) = \{x, y\}$, $G_2(e_1) = \{y, z\}$, $G_2(e_2) = \{y, z\}$. Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Hence, f_{pu} is a supra b-continuous soft function. On the other hand, the soft set (M, B), where (M, B) is a soft set over Y defined by:
$M(k_1) = \{x, z\}$, $M(k_2) = \{x, z\}$.
Then, (H, B) is supra b-open soft set in Y. Also, $f^{-1}_{pu}((H, B)) = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}$ is not supra b-open soft set over X. Therefore, f_{pu} is not supra b-irresolute soft function.

Theorem 3.2. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $u : X \to Y$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, the following are equivalent:*

- **(1):** *f_{pu} is suprab-irresolute soft function.*
- **(2):** $f^{-1}_{pu}(H, B) \in SBCS_A(X)\ \forall\ (H, B) \in \mu_2^c$.
- **(3):** $f_{pu}(bScl^s_{\mu_1}(G, A)) \tilde{\subseteq} bScl^s_{\mu_2}(f_{pu}(G, A))\ \forall\ (G, A) \in SS(X)_A$.
- **(4):** $bScl^s_{\mu_1}(f^{-1}_{pu}(H, B)) \tilde{\subseteq} f^{-1}_{pu}(bScl^s_{\mu_2}(H, B))\ \forall\ (H, B) \in SS(Y)_B$.
- **(5):** $f^{-1}_{pu}(BSint^s_{\mu_2}(H, B)) \tilde{\subseteq} BSint^s_{\mu_1}(f^{-1}_{pu}(H, B))\ \forall\ (H, B) \in SS(Y)_B$.

Proof.

$(1) \Rightarrow (2)$: Let$(H, B) \in \mu_2^c$. Then, $(H, B)^c \in \mu_2$ and $f^{-1}_{pu}(H, B)^c \in SBOS_A(X)$ from Definition 3.13. Since $f^{-1}_{pu}(H, B)^c = (f^{-1}_{pu}(H, B))^c$ from [[26], Theorem 3.14]. Thus, $f^{-1}_{pu}(H, B) \in SBCS_A(X)$.

$(2) \Rightarrow (3)$: Let $(G, A) \in SS(X)_A$. Since $(G, A) \tilde{\subseteq} f^{-1}_{pu}(f_{pu}(G, A)) \tilde{\subseteq} f^{-1}_{pu}\ (bScl^s_{\mu_2}(f_{pu}(G, A))) \in BCS(Y)$ from (2) and [[26], Theorem 3.14]. Then, $(G, A) \tilde{\subseteq} bScl^s_{\mu_1}(G, A) \tilde{\subseteq} f^{-1}_{pu}(bScl^s_{\mu_2} (f_{pu}(G, A)))$. Hence, $f_{pu}(bScl(G, A) \tilde{\subseteq} f_{pu}(f^{-1}_{pu}(bScl^s_{\mu_2}(f_{pu}(G, A)))) \tilde{\subseteq} bScl^s_{\mu_2}(f_{pu}(G, A)))$ from [[26], Theorem 3.14]. Thus, $f_{pu}(bScl^s_{\mu_1}(G, A)) \tilde{\subseteq} bScl^s_{\mu_2}(f_{pu}(G, A))$.

$(3) \Rightarrow (4)$: Let $(H, B) \in SS(Y)_B$ and $(G, A) = f^{-1}_{pu}(H, B)$. Then, $f_{pu}(bScl^s_{\mu_1}(f^{-1}_{pu}(H, B))) \tilde{\subseteq} bScl^s_{\mu_2}(f_{pu}(f^{-1}_{pu}(H, B)))$ From (3). Hence, $bScl^s_{\mu_1}(f^{-1}_{pu}(H, B)) \tilde{\subseteq} f^{-1}_{pu}(f_{pu}(bScl^s_{\mu_1}(f^{-1}_{pu} (H, B)))) \tilde{\subseteq} f^{-1}_{pu}(bScl^s_{\mu_2}(f_{pu}(f^{-1}_{pu}(H, B)))) \tilde{\subseteq} f^{-1}_{pu}(bScl^s_{\mu_2}(H, B))$ from [[26], Theorem 3.14]. Thus, $bScl^s_{\mu_1}(f^{-1}_{pu}(H, B)) \tilde{\subseteq} f^{-1}_{pu}(bScl^s_{\mu_2}(H, B))$.

$(4) \Rightarrow (2)$: Let (H, B) be a closed soft set over Y. Then, $bScl^s_{\mu_1}(f^{-1}_{pu}(H, B)) \tilde{\subseteq} f^{-1}_{pu}(bScl^s_{\mu_2} (H, B)) = f^{-1}_{pu}(H, B) = f^{-1}_{pu}(H, B)\ \forall\ (H, B) \in SS(Y)_B$ from (4), but clearly $f^{-1}_{pu}(H, B) \tilde{\subseteq} bScl^s_{\mu_1}(f^{-1}_{pu}(H, B))$. This means that, $f^{-1}_{pu}(H, B) = bScl^s_{\mu_1}(f^{-1}_{pu}(H, B)) \in SBCS_A(X)$.

$(1) \Rightarrow (5)$: Let $(H, B) \in SBOS_A(X)$. Then, $f^{-1}_{pu}(bSint^s_{\mu_2}(H, B)) \in SBOS_A(X)$ from (1). Hence, $f^{-1}_{pu}(bSint^s_{\mu_2}(H, B)) = bSint^s_{\mu_1}(f^{-1}_{pu} bSint^s_{\mu_2}(H, B)) \tilde{\subseteq} bSint^s_{\mu_1}\ (f^{-1}_{pu}(H, B))$. Thus, $f^{-1}_{pu}(bSint^s_{\mu_2}(H, B)) \tilde{\subseteq} bSint^s_{\mu_1}(f^{-1}_{pu}(H, B))$.

$(5) \Rightarrow (1)$: Let $(H, B) \in SBOS_A(X)$. Then, $bSint^s_{\mu_2}(H, B) = (H, B)$ and $f^{-1}_{pu}(bSint^s_{\mu_2}(H,B)) = f^{-1}_{pu}((H,B)) \tilde{\subseteq} bSint^s_{\mu_1}(f^{-1}_{pu}(H,B))$ from (5). But, we have $bSint^s_{\mu_1}(f^{-1}_{pu}(H, B)) \tilde{\subseteq} f^{-1}_{pu}(H,B)$. This means that, $bSint^s_{\mu_1}(f^{-1}_{pu}(H, B)) = f^{-1}_{pu}(H, B) \in SBOS_A(X)$. Thus, f_{pu} is suprab-irresolute soft function.

□

4. Soft b-separateness

In this section, we will research the notion of supra soft b-separated sets in supra soft topological spaces and study its basic properties in detail.

Definition 4.14. Two non-null soft sets G_E and H_E of a soft topological space (X, τ, E) are said to be supra soft b-separated sets if $G_E \tilde{\cap} bScl^s(H_E) = \tilde{\varphi}$ and $bScl^s(G_E) \tilde{\cap} (H, E) = \tilde{\varphi}$.

Definition 4.15. A soft set F_E of a supra soft topological space (X, μ, E) is said to be supra b-clopen soft set if it is both supra b-open soft set and supra b-closed soft set.

The following theorem follows from the above definition.

Theorem 4.3.

- **(1):** *Each two supra soft b-separated sets are always disjoint*
- **(2):** *Each two disjoint soft sets, in which both of them either supra b-open soft sets or supra b-closed soft sets, are supra soft b-separated.*

Theorem 4.4. *Let G_E and H_E be non-null soft sets of a soft topological space (X, τ, E). Then, the following statements hold:*

- **(1):** *If G_E and H_E are supra soft b-separated, $G_{1E} \tilde{\subseteq} G_E$ and $H_{1E} \tilde{\subseteq} H_E$, then G_{1E} and H_{1E} are supra soft b-separated sets.*
- **(2):** *If G_E and H_E are supra b-open soft sets, $U_E = G_E \tilde{\cap} (\tilde{X_E} - H_E)$ and $V_E = H_E \tilde{\cap} (\tilde{X_E} - G_E)$, then U_E and V_E are supra soft b-separated sets.*

Proof. **(1):** Since $G_{1E} \tilde{\subseteq} G_E$. Then, $bScl^s(G_{1E}) \tilde{\subseteq} bScl^s(G_E)$. Hence, $H_{1E} \tilde{\cap} bScl^s(G_{1E}) \tilde{\subseteq} H_E \tilde{\cap} bScl^s(G_E) = \tilde{\varphi}$. Similarly, $G_{1E} \tilde{\cap} bScl^s(H_{1E}) = \tilde{\varphi}$. Thus, G_{1E} and H_{1E} are supra soft b-separated sets.

(2): Let G_E and H_E be supra b-open soft sets. Then, ($\tilde{X_E}$ - G_E) and ($\tilde{X_E}$ - H_E) are supra b-closed soft sets. Assume that, $U_E = G_E \tilde{\cap} (\tilde{X_E} - H_E)$ and $V_E = H_E \tilde{\cap} (\tilde{X_E} - G_E)$. Then, $U_E \tilde{\subseteq} (\tilde{X_E} - H_E)$ and $V_E \tilde{\subseteq} (\tilde{X_E} - G_E)$. Hence, $bScl^s(U_E) \tilde{\subseteq} (\tilde{X_E} - H_E) \tilde{\subseteq} (\tilde{X_E} - V_E)$ and $bScl^s(V_E) \tilde{\subseteq} \tilde{X_E} - G_E \tilde{\subseteq} (\tilde{X_E} - U_E)$. Consequently, $bScl^s(U_E) \tilde{\cap} V_E = \tilde{\varphi}$ and $bScl^s(V_E) \tilde{\cap} U_E = \tilde{\varphi}$. Therefore, U_E and V_E are supra soft b-separated sets. □

Theorem 4.5. *Any two soft sets G_E and H_E of a soft topological space (X, τ, E) are supra soft b-separated sets if and only if there exist supra b-open soft sets U_E and V_E such that $G_E \tilde{\subseteq} U_E$, $H_E \tilde{\subseteq} V_E$ and $G_E \tilde{\cap} V_E = \tilde{\varphi}$, $H_E \tilde{\cap} U_E = \tilde{\varphi}$.*

Proof. **Necessity:** Let G_E and H_E be supra soft b-separated sets. Then, $G_E \tilde{\cap} bScl^s(H_E) = \tilde{\varphi}$ and $bScl^s(G_E) \tilde{\cap} H_E = \tilde{\varphi}$. Let $V_E = \tilde{X_E} - bScl^s(G_E)$ and $U_E = \tilde{X_E} - bScl^s(H_E)$. Thus, U_E and V_E are supra b-open soft sets such that $G_E \tilde{\subseteq} U_E$, $H_E \tilde{\subseteq} V_E$, $G_E \tilde{\cap} V_E$ =$\tilde{\varphi}$ and $H_E \tilde{\cap} U_E = \tilde{\varphi}$.

Sufficient: Let U_E, V_E be supra b-open soft sets such that $G_E \tilde{\subseteq} U_E$, $H_E \tilde{\subseteq} V_E$ and $G_E \tilde{\cap} V_E = \tilde{\varphi}$, $H_E \tilde{\cap} U_E = \tilde{\varphi}$. Since ($\tilde{X_E}$ - V_E) and ($\tilde{X_E}$ - U_E) are supra b-closed soft sets. Then, $bScl^s(G_E) \tilde{\subseteq} (\tilde{X_E}$ - $V_E) \tilde{\subseteq} (\tilde{X_E}$ - $H_E)$ and $bScl^s(H_E) \tilde{\subseteq} (\tilde{X_E}$ - $U_E) \tilde{\subseteq} (\tilde{X_E}$ - $G_E)$. Thus, $bScl^s(G_E) \tilde{\cap} H_E = \tilde{\varphi}$ and $bScl^s(H_E) \tilde{\cap} G_E = \tilde{\varphi}$. This means that, G_E and H_E are supra soft b-separated sets. □

Theorem 4.6. *Let (X_1, τ_1, A) and (X_2, τ_1, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively and $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective suprab-irresolute soft function and $(F, B), (G, B)$ are supra soft b-separated sets in X_2, then $f_{pu}^{-1}(F, B), f_{pu}^{-1}(G, B)$ are supra soft b-separated sets in X_1.*

Proof. Let $(F,B),(G,B)$ be supra soft b-separated sets in X_2. Then, $F_B\tilde{\cap}bScl^s(G_B)=\tilde{\varphi}_B$ and $bScl^s(F_B)\tilde{\cap}(G,B)=\tilde{\varphi}_B$. Since $bScl^s_{\tau_1}(f^{-1}_{pu}(H,B))\tilde{\subseteq}f^{-1}_{pu}(bScl^s_{\tau_2}(H,B))\ \forall\ (H,B)\in SS(Y)_B$ from Theorem 3.2 (4). It follows that, $f^{-1}_{pu}(G,B)\tilde{\cap}bScl^s_{\tau_1}(f^{-1}_{pu}(H,B))\tilde{\subseteq}f^{-1}_{pu}(G,B)$ $\tilde{\cap}f^{-1}_{pu}(bScl^s_{\tau_2}(H,B))=f^{-1}_{pu}[(G,B)\tilde{\cap}bScl^s_{\tau_2}(H,B)]=f^{-1}_{pu}(\tilde{\varphi}_B)=\tilde{\varphi}_A$. By a similar way, we have $f^{-1}_{pu}(H,B)\tilde{\cap}bScl^s_{\tau_1}(f^{-1}_{pu}(G,B))=\tilde{\varphi}_A$. Therefore, $f^{-1}_{pu}(F,B),f^{-1}_{pu}(G,B)$ are supra soft b-separated sets in X_1. □

5. SUPRA SOFT b-CONNECTEDNESS

In this section, we will research the notion of supra soft b-connectedness in supra soft topological spaces be means of supra b-open soft sets, supra b-closed soft sets, supra soft b-separated sets and study its basic properties.

Definition 5.16. Let (X,τ,E) be a soft topological space. A supra soft b-separation of $\tilde{X}$ is a pair of non-null proper supra b-open soft sets in τ such that $(F,E)\tilde{\cap}(G,E)=\tilde{\varphi}$ and $\tilde{X}=(F,E)\tilde{\cup}(G,E)$.

Definition 5.17. A soft topological space (X,τ,E) is said to be b-soft connected if and only if there is no supra soft b-separations of $\tilde{X}$. If (X,τ,E) has such supra soft b-separations, then (X,τ,E) is said to be supra soft b-disconnected.

Remark 5.2. **(1):** $\tilde{\varphi}$ is always supra soft b-connected.

(2): If G_E, H_E are non-null supra soft b-separated sets. Then, the pair G_E, H_E is called the supra soft b-disconnection of $\tilde{X}$.

Theorem 5.7. *Let (X,τ,E) be a soft topological space , then the following statements are equivalent:*

(1): *$\tilde{X}$ is supra soft b-connected.*

(2): *$\tilde{X}$ can not be expressed as a soft union of two non-null disjoint supra b-open soft sets.*

(3): *$\tilde{X}$ can not be expressed as a soft union of two non-null disjoint supra b-closed soft sets.*

(4): *There is no proper supra b-clopen soft set in (X,τ,E).*

(5): *$\tilde{X}$ can not be expressed as a soft union of two non-null soft b-separated sets.*

Proof.

$(1)\Leftrightarrow(2)$**:** It is obvious from Definition 5.17.

$(2)\Rightarrow(3)$**:** Suppose that $\tilde{X}=(F,E)\tilde{\cup}(G,E)$ for some supra b-closed soft sets (F,E) and (G,E) such that $(F,E)\tilde{\cap}(G,E)=\tilde{\varphi}$. Then, $(F,E)=(G,E)^c$ which is b-open soft set, $\tilde{X}=(G,E)\tilde{\cup}(G,E)^c$ and $(G,E)\tilde{\cap}(G,E)^c=\tilde{\varphi}$, which is a contradiction with (2).

$(3)\Rightarrow(4)$**:** Suppose that there is a proper supra b-clopen soft subset (F,E) of $\tilde{X}$. Then, $(F,E)^c$ is supra b-clopen soft set, where $\tilde{X}=(F,E)\,\tilde{\cup}\,(F,E)^c$ and (F,E) $\tilde{\cap}\,(F,E)^c=\tilde{\varphi}$, which is a contradiction with (3).

$(4)\Rightarrow(3)$**:** Suppose that $\tilde{X}=(F,E)\tilde{\cup}(G,E)$ for some supra b-closed soft sets (F,E) and (G,E) such that $(F,E)\tilde{\cap}(G,E)=\tilde{\varphi}$. Then, $(F,E)=(G,E)^c$ and $(G,E)=(F,E)^c$. Thus, (F,E) and (G,E) are proper supra b-clopen soft sets, which is a contradiction with (4).

$(3)\Rightarrow(5)$**:** Suppose that $\tilde{X}=(H,E)\tilde{\cup}(G,E)$ for some soft b-separated sets (H,E) and (G,E). Then, $G_E\tilde{\cap}bScl^s(H_E)=\tilde{\varphi}$ and $bScl^s(G_E)\tilde{\cap}(H,E)=\tilde{\varphi}$. It follows that, $(H,E)\tilde{\cap}(G,E)=\tilde{\varphi}$. Hence, $(H,E)=(G,E)^c$ and $(G,E)=(H,E)^c$. Therefore, $bScl^s(H_E)\tilde{\subseteq}G^c_E=H_E$ and $bScl^s(G_E)\tilde{\subseteq}H^c_E=(G,E)$. But, $H_E\tilde{\subseteq}bScl^s(H_E)$ and $G_E\tilde{\subseteq}bScl^s(G_E)$. Thus, (H,E) and (G,E) are supra b-closed soft sets, which is a contradiction with (3).

$(5) \Rightarrow (1)$**:** Suppose that $\tilde{X} = (F,E)\tilde{\cup}(G,E)$ for some supra b-open soft sets (F,E) and (G,E) such that $(F,E)\tilde{\cap}(G,E) = \tilde{\varphi}$. Then $(F,E) = (G,E)^c$ and $(G,E) = (F,E)^c$. Thus, (F,E) and (G,E) are supra b-clopen soft sets. Hence, (F,E) and (G,E) are soft b-separated sets, which is a contradiction with (5).

□

Corollary 5.1. *Let (X,τ,E) be a soft topological space , then the following statements are equivalent:*

(1): *$\tilde{X}$ is supra soft b-connected.*
(2): *If $\tilde{X} = (F,E)\tilde{\cup}(G,E)$ for some supra b-open soft sets (F,E) and (G,E) such that $(F,E)\tilde{\cap}(G,E) = \tilde{\varphi}$. Then, either $(F,E) = \tilde{\varphi}$ or $(G,E) = \tilde{\varphi}$.*
(3): *If $\tilde{X} = (F,E)\tilde{\cup}(G,E)$ for some supra b-closed soft sets (F,E) and (G,E) such that $(F,E)\tilde{\cap}(G,E) = \tilde{\varphi}$. Then, either $(F,E) = \tilde{\varphi}$ or $(G,E) = \tilde{\varphi}$.*

Proof. Obvious from Theorem 5.7. □

Theorem 5.8. *Let (X_1,τ_1,A) and (X_2,τ_1,B) be soft topological spaces and $f_{pu} : (X_1,\tau_1,A) \rightarrow (X_2,\tau_2,B,)$ be a bijective suprab-irresolute soft function. If (G,A) is supra soft b-connected in X_1, then $f_{pu}(G,A)$ is supra soft b-connected in X_2.*

Proof. Suppose that $f_{pu}(G,A)$ is not supra soft b-connected in X_2. Then, $f_{pu}(G,A) = (M,B)\tilde{\cup}(N,B)$ for some supra soft b-separated sets $(M,B),(N,B)$ of $f_{pu}(G,A)$ in X_2 from Theorem 5.7. By Theorem 4.6, $f_{pu}^{-1}(M,B)$ and $f_{pu}^{-1}(N,B)$ are supra soft b-separated in X. Since f_{pu} is bijective soft function. So, $(G,A) = f_{pu}^{-1}(f_{pu}(G,A)) = f_{pu}^{-1}(M,B)\tilde{\cup}f_{pu}^{-1}(N,B)$. It follows that, (G,A) is not supra soft$b$-connected in X_1, which is a contradiction. Thus, $f_{pu}(G,A)$ is supra soft b-connected in X_2. □

Corollary 5.2. *Let (X_1,τ_1,A) and (X_2,τ_1,B) be soft topological spaces and $f_{pu} : (X_1,\tau_1,A) \rightarrow (X_2,\tau_2,B,)$ be a surjective supra b-irresolute soft function. If X_1 is supra soft b-connected space, then so X_2.*

Proof. Follows from Theorem 5.8. □

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