CREAT. MATH. INFORM.
Volume **26** (2017), No. 1,
Pages 45 - 52

Online version at https://creative-mathematics.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 286X; Online Edition: ISSN 1843 - 441X
DOI: https://doi.org/10.37193/CMI.2017.01.06

Some new types of decomposition of continuity

CARLOS CARPINTERO, JOHN MORENO and ENNIS ROSAS

ABSTRACT. Using the notion of μ-semi open set, we introduce the concept of locally μ-semi open set as a generalization of locally μ-closed set and give a new theory of decomposition of semi-continuity and some weak forms of continuity are studied.

1. INTRODUCTION AND PRELIMINARIES

In the last years, different forms of open sets are being studied. Recently, a significant contribution to the theory of generalized open sets have been presented by A. Császár [1], [2], [3]. Specifically, in 2002, A. Császár [1] introduced the notions of generalized topology and generalized continuity. It is observed that a large numbers of articles are devoted to the study of generalized open sets and certain type of sets associated to a topological spaces, containing the class of open sets and possessing properties more or less to those open sets. We recall some notions defined in [1]. Let X be a nonempty set and let exp$X$, denote the power set of X. We call a class $\mu \subseteq$ exp$X$ a generalized topology [1] (briefly, GT) if $\emptyset \in \mu$ and the union of elements of μ belong to μ. A set X with a GT μ on it is called a generalized topological space (briefly, GTS) and is denoted by (X, μ). The elements of μ are called μ-open sets and the complement of a μ-open set is called μ-closed set. For $A \subset X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and $i_\mu(A)$, the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A (see [1], [2]). It is easy to observe that i_μ and c_μ are idempotent and monotone (ρ : exp$X \mapsto$ exp$X$ is said idempotent if $A \subset X$ implies $\rho(\rho(A)) = \rho(A)$ and monotone, if $A \subset B$ implies $\rho(A) \subseteq \rho(B)$). It is also well known (see [1], [2]), that if μ is a GT on X, $x \in X$ and $A \subset X$, then $x \in c_\mu(A)$ if and only if $x \in M$ and $M \in \mu$, implies $M \cap A \neq \emptyset$ and $c_\mu(X \backslash A) = X \backslash i_\mu(A)$.

In the same form as in topological space, is defined the notion of semi open set, we obtain that if (X, μ) is a generalized topological space, $A \subseteq X$ is said to be μ-semi open if there exists a μ-open set W such that $W \subseteq A \subseteq c_\mu(W)$ or equivalently, A is μ-semi open if and only if $A \subseteq c_\mu(i_\mu(A))$, the collection of all μ-semi open sets in X is denoted by μ-$SO(X)$. The complement of a μ-semi open set is called μ-semi closed, the collection of all μ-semi closed sets in X is denoted by μ-$SC(X)$. For $A \subset X$, we denote by $sc_\mu(A)$ the intersection of all μ-semi closed sets containing A, i.e., the smallest μ-semi closed set containing A; and $si_\mu(A)$, the union of all μ-semi open sets contained in A, i.e., the largest μ-semi open set contained in A (see [6]). sc_μ is idempotent and monotone. Also $sc_\mu(A) = A \cup i_\mu(c_\mu(A))$, $sc_\mu(X \setminus A) = X \setminus si_\mu(A)$ and $si_\mu(X \setminus A) = X \setminus sc_\mu(A)$ [6].

In [7], the authors introduced the notion of locally μ-closed set, μ-t-set, μ-B-set, μ^*-open set and μ·-open set in order to unify the theory of decomposition of semi-continuity. In this article, using the notion of μ-semi open set, we introduce the concept of locally μ-semi

Received: 14.07.2016. In revised form: 14.12.2016. Accepted: 28.12.2016
2010 *Mathematics Subject Classification.* 54C05, 54C08, 54C10.
Key words and phrases. *Locally μ-s-closed, μ-st-set, μ-sB-set, (μ, σ)-s-continuous functions.*
Corresponding author: Carlos Carpintero; carpintero.carlos@gmail.com

open set as a generalization of locally μ-closed and give a new theory of decomposition of semi-continuity and some weak form are studied. Throughout this paper $cl(A)$ (respectively $int(A)$) denotes the closure (respectively interior) of A in a topological space X.

2. LOCALLY μ-SEMI CLOSED SETS

Definition 2.1. [7] Let μ be a GT on a topological space (X, τ). A subset A of X is called locally μ-closed if $A = U \cap F$ where $U \in \tau$ and F is μ-closed.

Remark 2.1. If μ is a GT on a topological space (X, τ), then every open set as well as a μ-closed set is locally μ-closed.

Definition 2.2. [5]Let μ be a GT on a topological space (X, τ). A subset A of X is called:

1. μ-t-set if $int(A) = int(c_\mu(A))$.
2. μ-B-set if $A = U \cap V$, $U \in \tau$, V is a μ-t-set.
3. μ^*-open set if $A \subseteq cl(i_\mu(A))$.
4. $\mu^{\cdot}$-open set if $A \subseteq int(c_\mu(A))$.

Definition 2.3. Let μ be a GT on a topological space (X, τ). A subset A of X is called generalized μ-closed or simple a gμ-closed if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \tau$.

Definition 2.4. Let μ be a GT on a topological space (X, τ). A subset A of X is called locally μ-semi closed or simple locally μ-s-closed if $A = U \cap F$ where $U \in \tau$ and F is μ-semi closed.

Remark 2.2. If μ is a GT on a topological space (X, τ), then every open set as well as a μ-semi closed set is locally μ-semi closed, also every locally μ-closed set is locally μ-semi closed.

Definition 2.5. Let μ be a GT on a topological space (X, τ). A subset A of X is called:

1. μ-st-set if $int(A) = int(sc_\mu(A))$.
2. μ-sB-set if $A = U \cap V$, $U \in \tau$, V is a μ-st-set.
3. μ^*-semi open set (briefly μ^*-s-open set) if $A \subseteq cl(si_\mu(A))$.
4. $\mu^{\cdot}$-semi open set (briefly $\mu^{\cdot}$-s-open set) if $A \subseteq int(sc_\mu(A))$.

The complement of a μ^*-semi open set is called μ^*-semi closed set (briefly μ^*-s-closed set).

In the following example, we calculate some sets according with Definition 2.5.

Example 2.1. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X\}$ and $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ be a GT on (X, τ). We obtain that:

1. μ-closed sets =$\{X, \{c\}, \{b, c\}, \{a, c\}\}$.
2. locally μ-closed sets =$\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$.
3. μ-$SO(X)sets = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$.
4. μ-$SC(X)sets = \{\emptyset, X, , \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$.
5. locally μ-semi closed sets =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$.
6. μ-st-sets =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$.
7. μ-sB-sets =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$.
8. μ^*-s-open sets =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$.
9. $\mu^{\cdot}$-s-open sets =$\{\emptyset, X, \{a, b\}\}$.

The following theorems characterizes the locally μ-semi closed sets.

Theorem 2.1. *Let μ be a GT on a topological space (X, τ). $A \subseteq X$ is locally μ-semi closed if and only if $X \setminus A$ is the union of a closed set and a μ-semi open set.*

Proof. Suppose that A is locally μ-semi closed, then $A = U \cap F$ where $U \in \tau$ and F is μ-semi closed. It follows that $X \setminus A = X \cap (U \cap F)^c = (U^c \cup F^c) = (X \setminus U) \cup (X \setminus F)$. Conversely, suppose that $X \setminus A = W \cup G$, W closed set and G μ-semi open. Then $A = X \setminus (X \setminus A) = X \setminus (W \cup G) = (X \setminus W) \cap (X \setminus G)$. □

Theorem 2.2. *Let μ be a GT on a topological space (X, τ). $A \subseteq X$ is locally μ-semi closed if and only if there exists an open set U such that $A = U \cap sc_\mu(A)$.*

Proof. Let A be a locally μ-semi closed subset of X, then $A = U \cap F$, where $U \in \tau$ and F is μ-semi closed. It follows that $A = A \cap U \subseteq U \cap sc_\mu(A) \subseteq U \cap sc_\mu(F) = U \cap F = A$. In consequence, $A = U \cap sc_\mu(A)$. Conversely, since $sc_\mu(A)$ is a μ-semi closed, it follows that A is locally μ-semi closed. □

In the following example, we can see that there exists a locally μ-semi closed set that is not open as well as μ-semi closed.

Example 2.2. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b, c\}, \{c, d\}, \{c\}\}$ and $\mu = \{\emptyset, X, \{a, b, c\}, \{c, d\}\}$ be a GT on (X, τ). We obtain that:

1. μ-closed sets $=\{\emptyset, X, \{d\}, \{a, b\}\}$.
2. locally μ-closed sets $=\{\emptyset, X, \{a, b\}, \{c\}, \{a, b, c\}, \{c, d\}, \{d\}\}$.
3. μ-$SO(X) sets = \{\emptyset, X, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.
4. μ-$SC(X) sets = \{\emptyset, X, \{a, b\}, \{d\}, \{b\}, \{a\}\}$.
5. locally μ-semi closed sets $=\{\emptyset, X, \{a, b\}, \{d\}, \{b\}, \{a\}, \{a, b, c\}, \{c, d\}, \{c\}\}$.
6. μ-t-sets $=\{\emptyset, X, \{a\}, \{b\}, \{d\}, \{a, b\}\}$.
7. μ-st-sets $=\{\emptyset, X, \{a\}, \{b\}, \{d\}, \{a, b\}\}$.
8. μ-B-sets $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, b, c\}, \{c, d\}\}$.
9. μ-sB-sets $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b, c\}, \{c, d\}, \{a, b\}\}$.
10. μ^*-open sets $=\{\emptyset, X, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.
11. μ^*-s-open sets $=\{\emptyset, X, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.
12. μ'-open sets $=\{\emptyset, X, \{c\}, \{a, c\}, \{a, d\}, \{b, c\}\{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
13. μ'-s-open sets $=\{\emptyset, X, \{c\}, \{a, c\}, \{a, d\}, \{b, c\}\{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

The following examples will be useful to better understand the development of this article.

Example 2.3. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mu = \{\emptyset, \{a\}, \{a, c\}\}$ be a GT on (X, τ). We obtain that:

1. μ-closed sets $=\{X, \{b, c\}, \{b\}\}$.
2. locally μ-closed sets $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$.
3. μ-$SO(X) sets = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$.
4. μ-$SC(X) sets = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}, \{a, c\}\}$.
5. locally μ-semi closed sets $=\{\emptyset, X, \{b, c\}, \{c\}, \{b\}, \{a\}, \{a, b\}, \{a, c\}\}$.
6. μ-t-sets $=\{X, \{b\}, \{b, c\}\}$.
7. μ-st-sets $=\{\emptyset, X, \{b\}, \{c\}, \{a\}, \{b, c\}, \{a, c\}\}$.
8. μ-B-sets $=\{\emptyset, X, \{b\}, \{a\}, \{b, c\}, \{a, b\}\}$.
9. μ-sB-sets $=\{\emptyset, X, \{b\}, \{c\}, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
10. μ^*-open sets $=\{\emptyset, \{a\}, \{b\}, \{a, c\}\}$.
11. μ^*-s-open sets $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
12. μ'-open sets $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$.
13. μ'-s-open sets $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

Example 2.4. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b, c\}\}$ and $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}$ $\{a, b, c\}\}$ be a GT on (X, τ). We obtain that:

1. μ-closed sets $=\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{c\}, \{d\}\}$.
2. μ-$SO(X)sets = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}$, $\{a, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c\}, \{a, b, d\}\}$.
3. μ-$SC(X)sets = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{a, c\}$, $\{b, c\}, \{b\}, \{a\}, \{d\}, \{c\}\}$.
4. $\mu^{,}$-open sets $=\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\{a, b, d\}, \{b, c, d\}\}$.
5. $\mu^{,}$-s-open sets $=\{\emptyset, X, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$.

Note that $sc_\mu(A) \subseteq c_\mu(A)$ and $i_\mu(A) \subseteq si_\mu(A)$ for all $A \subseteq X$. If μ is a GT on a topological space (X, τ), we obtain the relationship between the sets given in Definitions 2.2 and 2.4.

Theorem 2.3. *Let μ be a GT on a topological space (X, τ) and $A \subset X$. Then*

1. *If A is a μ-t-set, then A is a μ-st-set.*
2. *If A is a μ-B-set, then A is a μ-sB-set.*
3. *If A is a μ^*-open, then A is a μ^*-s-open.*
4. *If A is a $\mu^{,}$-s-open, then A is a $\mu^{,}$-open.*

Observe that in Example 2.2, τ is not contained in μ. If we take $A = \{c\}$, A is locally μ-semi closed, $sc_\mu(A) = X$, $sc_\mu(A) \setminus A = \{a, b, d\}$ is not μ-semi closed and $A \cup (X \setminus sc_\mu(A)) = A = \{c\}$ is not a μ-semi open set, also A is not contained in $si_\mu(A \cup (X \setminus sc_\mu(A)))$, because $A \cup (X \setminus sc_\mu(A)) = \{c\}$ and $sc_\mu(\{c\}) = \emptyset$.
In the case that $\tau \subset \mu$, we have the following theorem.

Theorem 2.4. *Let μ be a GT on a topological space (X, τ) and $\tau \subset \mu$. If A is locally μ-semi closed, then:*

1. *$sc_\mu(A) \setminus A$ is μ-semi closed.*
2. *$A \cup (X \setminus sc_\mu(A))$ is μ-semi open set.*
3. *A is contained in $si_\mu(A \cup (X \setminus sc_\mu(A)))$.*

Proof. 1.- Suppose that A is a locally μ-semi closed subset of X, then there exists an open set U such that $A = U \cap sc_\mu(A)$. It follows that: $sc_\mu(A) \setminus A = sc_\mu(A) \setminus (U \cap sc_\mu(A)) = sc_\mu(A) \cap (X \setminus (U \cap sc_\mu(A))) = sc_\mu(A) \cap ((X \setminus U) \cup (X \setminus sc_\mu(A))) = sc_\mu(A) \cup (X \setminus U) \cap sc_\mu(A) \cap (X \setminus sc_\mu(A)) = sc_\mu(A) \cap (X \setminus U)$. Now, as $sc_\mu(A)$ is μ-semi closed, $X \setminus U$ is closed and $\tau \subset \mu$, we obtain that $X \setminus U$ is μ-semi closed and then $sc_\mu(A) \cap (X \setminus U)$ is μ-semi closed.
2.- Using (1), $sc_\mu(A) \setminus A$ is μ-semi closed, then its complement $X \setminus (sc_\mu(A) \setminus A)$ is μ-semi open, but $X \setminus (sc_\mu(A) \setminus A) = X \setminus (sc_\mu(A) \cap (X \setminus A) = A \cup (X \setminus sc_\mu(A))$.
3.- Using (2), $A \subset (A \cup (X \setminus sc_\mu(A))) = si_\mu(A \cup (X \setminus sc_\mu(A)))$. □

Definition 2.6. Let μ be a GT on a topological space (X, τ). A subset A of X is called generalized μ-semi closed or simple a gμ-s-closed if $sc_\mu(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \tau$.

Remark 2.3. Every generalized μ-closed set is a generalized μ-semi closed set, but the converse is not necessarily true as we can see in the following example

Example 2.5. In Example 2.2,

1. locally μ-closed sets = $\{\emptyset, X, \{a, b, c\}, \{a, b\}, \{c, d\}, \{c\}, \{d\}$, $\{c\}, \{d\}\}$.
2. μ-semi closed sets $=\{\emptyset, X, \{a, b\}, \{b\}, \{a\}, \{d\}\}$.
3. locally μ-semi closed sets $=\{\emptyset, X, \{a, b\}, \{d\}, \{b\}, \{a\}, \{a, b, c\}, \{c, d\}$, $\{c\}\}$.

4. gμ-semi closed sets $=\{\emptyset, X, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$.

In Example 2.3,

5. gμ-closed sets = $\{X, \{b\}, \{c\}, \{a,c\}, \{b,c\}\}$.
6. gμ-semi closed sets = $\{\emptyset, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}\}$.
7. locally μ-semi closed sets = $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.
8. μ-semi closed sets = $\{\emptyset, X, \{b,c\}, \{a,c\}, \{b\}, \{c\}\}$.

The following theorem characterize the μ-semi closed sets in terms of gμ-semi closed sets and locally μ-semi closed sets.

Theorem 2.5. *Let μ be a GT on a topological space (X,τ). $A \subset X$ is μ-semi closed if and only if A is gμ-semi closed and locally μ-semi closed.*

Proof. Suppose that A is μ-semi closed in X and $A \subset U$, with $U \in \tau$. Since $A = sc_\mu(A)$, we obtain that A is gμ-semi closed and locally μ-semi closed.
Conversely, suppose that A is gμ-semi closed and locally μ-semi closed, then $A = U \cap F$, where $U \in \tau$ and F is μ-semi closed, therefore, $A \subset U$ and $A \subset F$, in consequence, $sc_\mu(A) \subset U$ and $sc_\mu(A) \subset F$ and hence $sc_\mu(A) \subset U \cap F = A$. So A is μ-semi closed. □

Example 2.6. In Examples 2.2, {a,b,c} is locally μ-semi closed but is not gμ-semi closed, in consequence is not μ-semi closed. In the same form {a,d} is gμ-semi closed but is not locally μ-semi closed, in consequence is not μ-semi closed.

Theorem 2.6. *Let μ be a GT on a topological space (X,τ) and A, B subsets of X.*

1. *A is a μ-st-set if and only if A is a μ^*-s-closed.*
2. *If A is μ-semi closed, then A is μ-st-set.*
3. *If A and B are μ-st-sets, then $A \cap B$ is μ-st-set.*
4. *If A is μ-st-set, then A is μ-sB-set.*
5. *Every locally μ-semi closed set is μ-sB-set.*

Proof. 1. Suppose that A is a μ-st-set, then $int(A) = int(sc_\mu(A))$ and hence $int(sc_\mu(A)) \subset A$, in consequence, A is a μ^*-s-closed. Conversely, if A is a μ^*-s-closed, then $int(sc_\mu(A)) \subset A$ and hence $int(sc_\mu(A)) \subset int(A) \subset A \subset int(sc_\mu(A))$. Therefore, $int(sc_\mu(A)) \subset int(A) \subset int(A) \subset int(sc_\mu(A))$. In consequence, $int(sc_\mu(A)) = A$, so A is a μ^*-s-closed.
2. If A is μ-semi closed, then $A = sc_\mu(A)$, and hence $int(A) = int(sc_\mu(A))$. Therefore, A is μ-st-set.
3. Suppose that A and B are μ-st-sets. Since $A \cap B \subseteq sc_\mu(A \cap B)$, we obtain that $int(A \cap B) \subseteq int(sc_\mu(A \cap B)) \subseteq int(sc_\mu(A) \cap sc_\mu(B)) = int(sc_\mu(A)) \cap int(sc_\mu(B)) = int(A) \cap int(B) = int(A \cap B)$. In consequence, $int(A \cap B) = int(sc_\mu(A \cap B))$.
4. Since $X \in \tau$ and $A = A \cap X$, then A is a μ-sB-set.
5. Suppose that A is locally μ-semi closed set of X, then $A = U \cap F$, where $U \in \tau$ and F is a μ-semi closed. Using (2), F is μ-st-set, then by (4), follows that $A = U \cap F$, where $U \in \tau$ and F is a μ-st-set and therefore, A is μ-sB-set. □

The following examples show that the converse of the above theorem is not necessarily true.

Example 2.7. In Example 2.2, $\{c\}$ is μ-sB-set, but $\{c\}$ is not μ-st-set. Also in Example 2.3, $\{a\}$ is μ-st-set, but $\{a\}$ is not μ-semi closed.

Example 2.8. *Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{c\}\}$ and $\mu = \{\emptyset, \{a\}, \{a,b\}\}$. Then $\{a\}$ is a μ-sB-set but is not locally μ-semi closed.*

Theorem 2.7. *Let μ be a GT on a topological space (X,τ). $A \subset X$ is open if and only if A is μ'-s-open and μ-sB-set.*

Proof. Let A be an open set, then $A = int(A) \subseteq int(sc_\mu(A))$ and hence, A is μ'-s-open set. Since $A = A \cap X$, where X is μ-st-set, then A is μ-sB-set.
Conversely, since A is a μ-sB-set, $A = U \cap V$, where $U \in \tau$ and V is a μ-st-set. By hypothesis, $A \subseteq int(sc_\mu(A)) = int(sc_\mu(U \cap V)) \subseteq int(sc_\mu(U) \cap sc_\mu(V)) = int(sc_\mu(U)) \cap int(sc_\mu(V)) = int(sc_\mu(U)) \cap int(V)$. But $A = U \cap V = (U \cap V) \cap U \subseteq (int(sc_\mu(U)) \cap int(V)) \cap U = (int(sc_\mu(U)) \cap U \cap int(V)) = U \cap int(V) \subseteq U \cap V = A$. Therefore, A is an open set. □

Example 2.9. In Example 2.2, $\{b,c,d\}$ is μ'-s-open but not μ-sB-set. In the same form, $\{a\}$ is μ-sB-set but is not μ'-s-open, in consequence, is not open.

3. (μ,σ)-S-CONTINUOUS FUNCTIONS

In [7] it was defined the notion of (μ,σ)-continuous functions. In this section, we generalize this notion and obtain some characterizations of it.

Definition 3.7. Let μ be a GT on a topological space (X,τ). A function $f : (X,\tau) \to (Y,\sigma)$ is said to be (μ,σ)-s-continuous if $f^{-1}(V)$ is μ-semi open in X for each open set V of Y.

Theorem 3.8. *Every (μ,σ)-continuous function is (μ,σ)-s-continuous but not conversely.*

Example 3.10. *Let $X = \mathbb{R}$ be the set of real numbers, $\mu = \{\emptyset, \mathbb{R} \setminus \mathbb{Q}\}$ and $\sigma = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ where $\mathbb{Q}$ denotes the set of all rational numbers and $\mathbb{R} \setminus \mathbb{Q}$ denotes the set of all irrational numbers. Define $f : (\mathbb{R},\sigma) \to (\mathbb{R},\sigma)$ as the identity function. Then f is (μ,σ)-s-continuous but not (μ,σ)-continuous.*

Theorem 3.9. *Let μ be a GT on a topological space (X,τ) and $f : (X,\tau) \to (Y,\sigma)$ a function. Then the following are equivalent :*

1. *f is (μ,σ)-s-continuous.*
2. *For each $x \in X$ and each open set V of Y with $f(x) \in V$, there exists a μ-semi open set U containing x such that $f(U) \subseteq V$.*
3. *For each $x \in X$ and each open set V of Y with $f(x) \in V$, $f^{-1}(V)$ is a μ-semi open neighborhood of x.*
4. *The inverse image of each closed set in Y is μ-semi closed.*
5. *$sc_\mu(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for every $B \subseteq Y$.*
6. *$f(sc_\mu(A)) \subseteq cl(f(A))$ for every $A \subseteq X$.*
7. *$f^{-1}(int(B)) \subseteq si_\mu(f^{-1}(B))$ for every $B \subseteq Y$.*

Proof. $1. \Rightarrow 2.$ Let $x \in X$ and V an open set in Y such that $f(x) \in V$. Since f is (μ,σ)-s-continuous, $f^{-1}(V)$ is μ-semi open. By putting $U = f^{-1}(V)$, $x \in U$ and $f(U) \subseteq V$.
$2. \Rightarrow 3.$ Let $x \in X$ and V an open in Y such that $f(x) \in V$. By 2, there exists a μ-semi open set U containing x such that $f(U) \subseteq V$. So each $x \in U \subseteq f^{-1}(V)$ and hence $f^{-1}(V)$ is a μ-semi open neighborhood of x.
$3. \Rightarrow 1.$ Let $x \in X$ and V an open in Y such that $f(x) \in V$. By 3, $f^{-1}(V)$ is a μ-semi open neighborhood of x. Thus for each $x \in f^{-1}(V)$, there exists a μ-semi open set U_x containing x such that $x \in U_x \subseteq f^{-1}(V)$. Hence $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$ and so $f^{-1}(V) \in \mu\text{-}SO(X)$.
$1. \Leftrightarrow 4.$ It is obvious.
$1. \Rightarrow 5.$ Let B be a subset of Y. Since $cl(B)$ is closed and f is (μ,σ)-s-continuous, $f^{-1}(cl(B)$ is μ-semi closed. Therefore, $sc_\mu(f^{-1}(B)) \subseteq sc_\mu(f^{-1}(cl(B))) = f^{-1}(cl(B))$.
$5. \Rightarrow 6.$ Let A be a subset of X. By 5, we have $sc_\mu(f^{-1}(f(A))) \subseteq f^{-1}(cl(f(A)))$. But $sc_\mu(A) \subseteq sc_\mu(f^{-1}(f(A)))$. Therefore $f(sc_\mu(A)) \subseteq cl(f(A))$.

6. $\Rightarrow$ 7. Let B be a subset of Y. By 6, $f(sc_\mu(X \setminus f^{-1}(B)) \subseteq cl(f(X \setminus f^{-1}(B))$ and $f(X \setminus si_\mu(f^{-1}(B)) \subseteq cl(Y \setminus B) = Y \setminus int(B)$. Therefore $X \setminus si_\mu(f^{-1}(B)) \subseteq f^{-1}(Y \setminus int(B))$ and $f^{-1}(int(B)) \subseteq si_\mu(f^{-1}(B))$.
7. $\Rightarrow$ 1. Let B be an open in Y and $f^{-1}(int(B)) \subseteq si_\mu(f^{-1}(B))$. Then $f^{-1}(B) \subseteq si_\mu(f^{-1}(B))$. But $si_\mu(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = si_\mu(f^{-1}(B))$. Therefore $f^{-1}(B)$ is μ-semi open. □

As an immediate consequence of Theorem 3.9, we have the following result.

Corollary 3.1. *Let μ be a GT on a topological space (X, τ) and $f : (X, \tau) \to (Y, \sigma)$ a (μ, σ)-s-continuous function, then the following are equivalent:*

1. $si_\mu(sc_\mu(f^{-1}(B))) \cap sc_\mu(si_\mu(f^{-1}(B))) \subseteq f^{-1}(cl(B))$ *for each B in Y.*
2. $f[si_\mu(sc_\mu(A)) \cap sc_\mu(si_\mu(A))] \subseteq cl(f(A)))$ *for each A in X.*

Definition 3.8. Let μ be a GT on a topological space (X, τ) and $A \subseteq X$. Then the μ-s-kernel of A denoted by μ-s-$Ker(A)$ is defined to be the set, μ-s-$Ker(A) = \cap\{U : U \in \mu - SO(X), A \subseteq U\}$.

Theorem 3.10. *Let μ be a GT on a topological space (X, τ) and $x \in X$. Then $y \in \mu$-s-$Ker(\{x\})$ if and only if $x \in sc_\mu(\{y\})$.*

Proof. Assume that $y \notin \mu$-s-$Ker(\{x\})$. Then there exists a μ-semi open set U containing x such that $y \notin U$. Therefore, we have $x \notin sc_\mu(\{y\})$. The converse is similarly shown. □

Theorem 3.11. *Let μ be a GT on a topological space (X, τ) and A a subset of X. Then μ-s-$Ker(A) = \{x : x \in X, sc_\mu(\{x\}) \cap A \neq \emptyset\}$.*

Proof. Let $x \in \mu$-s-$Ker(A)$ and $sc_\mu(\{x\}) \cap A = \emptyset$. Then, $x \notin X \setminus sc_\mu(\{x\})$ which is a μ-semi open set containing A. But this is impossible, since $x \in \mu$-s-$Ker(A)$. Consequently, $sc_\mu(\{x\}) \cap A \neq \emptyset$.
Conversely, let $x \in X$ such that $sc_\mu(\{x\}) \cap A \neq \emptyset$. Suppose that $x \notin \mu$-s-$Ker(A)$. Then there exists a μ-semi open set U containing A and $x \notin U$. Let $y \in sc_\mu(\{x\}) \cap A$. Then $y \in sc_\mu(\{x\})$ and $y \in A$. Thus $x \in \mu$-s-$Ker(\{y\})$ and $y \in U \in \mu$ implies $x \in U \in SO(X)$. By this contradiction, $x \in \mu$-s-$Ker(A)$. □

Theorem 3.12. *The following are equivalent for any points x and y in a GTS (X, μ) :*

(1) μ-s-$Ker(\{x\}) \neq \mu$-s-$Ker(\{y\})$.
(2) $sc_\mu(\{x\}) \neq sc_\mu(\{y\})$.

Theorem 3.13. *Let μ be a GT on a topological space (X, τ) and $A \subseteq X$. Then*

1. *$x \in \mu$-s-$Ker(A)$ if and only if $A \cap F \neq \emptyset$ for any μ-semi closed subset F of X with $x \in F$.*
2. *$A \subseteq \mu$-s-$Ker(A)$ and $A = \mu$-s-$Ker(A)$ if A is μ-semi open in X.*
3. *If $A \subseteq B$ then μ-s-$Ker(A) \subseteq \mu$-s-$Ker(B)$.*

Proof. 1. Let $x \in \mu$-s-$Ker(A)$. Then by Theorem 3.11, $A \cap sc_\mu(\{x\}) \neq \emptyset$. Conversely, assume that $A \cap F \neq \emptyset$. By taking $F = sc_\mu(\{x\})$, we have $A \cap sc_\mu(\{x\}) \neq \emptyset$ which implies $x \in \mu$-s-$Ker(A)$.
2. Let A be μ-semi open in X. Then always $A \subseteq \mu$-s-$Ker(A)$. On the other hand, assume that $x \in \mu$-s-$Ker(A)$. Then $x \in \bigcap\{U : U \in \mu - SO(X), A \subseteq U\}$. Since A is μ-semi open implies that $x \in A$. Thus μ-s-$Ker(A) \subseteq A$. Hence $A = \mu$-s-$Ker(A)$.
3. It is obvious. □

As an immediate consequence of Theorems 3.9 and 3.13, we have the following result.

Corollary 3.2. *Let μ be a GT on a topological space (X, τ) and $f : (X, \tau) \to (Y, \sigma)$ a function (μ, σ)-s-continuous. Then the following are equivalent :*

1. *For every subset A of X, $f(si_\mu(A)) \subset Ker(f(A))$.*
2. *For every subset B of Y, $si_\mu(f^{-1}(B)) \subseteq f^{-1}(Ker(B))$.*

Definition 3.9. Let μ be a GT on a topological space (X, τ). Then $f : (X, \tau) \to (Y, \sigma)$ is said to be gμ-s-continuous (respectively contra locally μ-s-continuous) if $f^{-1}(F)$ is a gμ-s-closed (respectively locally μ-s-closed) for each closed set F of (Y, σ).

Example 3.11. In Example 2.3, take $f : (X, \tau) \to (X, \tau)$, defined as: $f(a) = b$, $f(b) = c$ and $f(c) = a$, then f is contra locally μ-s-continuous but is not gμ-s-continuous. In the same form if in Example 2.8, we define $f : (X, \tau) \to (X, \tau)$, as: $f(a) = b$, $f(b) = a$ and $f(c) = c$, then f is μ-s-continuous but is not contra locally μ-s-continuous. Observe that in each case f is not (μ, τ)-s-continuous.

The following theorem is a direct consequence of Theorem 3.9 and Theorem 2.5

Theorem 3.14. *Let μ be a GT on a topological space (X, τ). Then $f : (X, \tau) \to (Y, \sigma)$ is (μ, σ)-s-continuous if and only if it is gμ-s-continuous and contra locally μ-s-continuous.*

The following example shows the existence of a function that is contra locally μ-s-continuous but not is gμ-s-continuous, in consequence is not (μ, τ)-s-continuous.

Example 3.12. In Example 2.2, define $f : (X, \tau) \to (X, \tau)$ as follows: $f(a) = a, f(b) = b, f(c) = d$ and $f(d) = c$. Then f is contra locally μ-s-continuous but not is gμ-s-continuous, in consequence is not (μ, τ)-s-continuous.

Theorem 3.15. *Let μ be a GT on a topological space (X, τ). Then a contra continuous function $f : (X, \tau) \to (Y, \sigma)$ is (μ, σ)-s-continuous if and only if it is gμ-s-continuous*

Proof. Suppose that f is a contra continuous and (μ, σ)-s-continuous. Let F be a closed set in Y, then $f^{-1}(F)$ is open and μ-semi closed in X. Since each μ-semi closed is gμ-s-closed, then f is gμ-s-continuous.
Conversely, let F be a closed set in Y, then $f^{-1}(F)$ is open and gμ-s-closed in X. Since each open set is locally μ-s-closed, then $f^{-1}(F)$ is locally μ-s-closed and gμ-s-closed, by Theorem 2.5, f is (μ, σ)-s-continuous. $\square$

References

[1] Császár, A., *Generalized topology, generalized continuity*, Acta Math. Hungar., **96** (2002), 351–357

[2] Császár, A., *Generalized open sets in generalized topologies*, Acta Math. Hungar., **96** (2005), 53–66

[3] Császár, A., *δ and θ-modifications of generalized topologies*, Acta Math. Hungar, **120** (2008), No. 3, 275–279

[4] Maki, H., Chandrasekhara, Rao K. and Nagoor Gani, A., *On generalizing semi-open sets and preopen sets*, Pure Appl. Math. Math. Sci, **49** (1999), 17–29

[5] Noiri, T. and Roy, B., *Unification of generalized open sets on topological spaces*, Acta Math. Hungar., **130** (2011), 349–357

[6] Rosas, E., Rajesh, N. and Carpintero, C., *Some New Types of Open and Closed Sets in minimal Structure part I*, Int. Math. Forum, **4** (2009), No. 44, 2169–2184

[7] Roy, B. and Sen, R., *On a type of decomposition of continuity*, Afr. Mat., **26** (2015), No. (1-2), 153–158

Department of Mathematics
Universidad de Oriente
Cumaná, Venezuela
Email address: carpintero.carlos@gmail.com
Email address: ennisrafael@gmail.com

Department of Basic Sciences
Universidad de la Costa
Barranquilla, Colombia
Email address: jbmorenobarrios@gmail.com