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Best proximity point theorems for weak cyclic Bianchini contractions

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ABSTRACT. Following the technique introduced in [Eldred, A. A. and Veeramani, P., *Existence and convergence of best proximity points*, J. Math. Anal. Appl., **323** (2006), 1001–1006], in this paper we will extend Bianchini's fixed point theorem to best proximity point type theorem. We introduce a new class of contractive conditions, called weak cyclic Bianchini contractions.

1. INTRODUCTION

Kirk, Srinivasan and Veeramani in [9], introduced the notion of contractions under cyclic conditions. Actually the contractive condition imply the sets involved in that condition to have a nonempty intersection. Moreover the fixed point problem $Tx = x$ as a unique solution situated in the intersection of the sets. But, as the fixed point problem does not posses a solution when the intersection of the sets is empty, it is natural to find an approximate solution such that the error is minimum. Indeed, best proximity point theorems investigate the existence of such optimal approximate solutions, called *best proximity points*, to the fixed point problem when there is no exact solution. Let us mention that for an operator $T : A \cup B \to A \cup B$, where A and B are nonempty subsets of a uniformly convex Banach space, a best proximity point is a point $x \in A \cup B$ such that

$$\|x - Tx\| = D(A, B) = \inf\{\|x - y\| \;:\; x \in A, y \in B\}.$$

The result of Kirk, Srinivasan and Veeramani given in [9] is a very important extension of one of the most important result in fixed point theory, i.e. the Banach Contraction Mapping Principle. Their theorem is:

Theorem 1.1. [9] *Let A and B be two nonemplty closed subsets of a complete metric space, and suppose $T : A \cup B \to A \cup B$ satisfiesthe following conditions:*

$$T(A) \subset B \quad and \quad T(B) \subset A \tag{1.1}$$

and

$$d(Tx, Ty) \leq a\, d(x, y), \forall x \in A,\ y \in B$$

where $a \in (0, 1)$. Then T has a unique fixed point in $A \cap B$.

An interestin feature about the above result is that continuity of T is no longer needed. Starting this point many papers appeared on the topic of cyclic operatos: [2], [7], [10], [16], [5]. Among them we mention here the fixed point theorem for Kannan operators in the presence of cyclical condition, for two sets:

Theorem 1.2. [12] *Let A and B be two nonemplty closed subsets of a complete metric space, and suppose $T : A \cup B \to A \cup B$ is a cyclic operator such that there exists $a \in \left(0, \frac{1}{2}\right)$ a constant and*

$$d(Tx, Ty) \leq a\left[d(x, Tx) + d(y, Ty)\right], \forall x \in A,\ y \in B.$$

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Then T has a unique fixed point in $A \cap B$.

If an operator satisfies the above contractive condition then it is called cyclic Kannan operator. The best proximity point result corresponding to the above theorem was published in [11] were they introduce the class of weak cyclic Kannan contractions. Regarding this class of contraction appeared the papers: [18], [19], [20], [13].

In 1972 R. M. T. Bianchini gives the following theorem as a generalization of Kannan's fixed point theorem:

Theorem 1.3. [3] *Let (X, d) be a complete metric space and $T : X \to X$ be a mapping for which there exist a real number $0 < h < 1$ such that for each $x, y \in X$ we have:*

$$d(Tx, Ty) \leq h \max\{d(x, Tx), d(y, Ty)\}. \tag{1.2}$$

Then T has a unique fixed point in X.

In [14] the above theorem was extended by imposing an additional cyclical condition. We state here the theorem in the case of two sets:

Theorem 1.4. *Let (X, d) be a complete metric space and A, B two nonempty closed subsets of X. Let $T : A \cup B \to A \cup B$ be a cyclical operator, i.e.satifiec (1.1), and there exist a real number $0 < h < 1$ such that for each $x \in A$ and $y \in B$ we have:*

$$d(Tx, Ty) \leq h \max\{d(x, Tx), d(y, Ty)\}. \tag{1.3}$$

Then T has a unique fixed point in $A \cap B$.

We will give an example of cyclical Bianchini operator which satisfies the above theorem.

Example 1.1. Let $X = \mathbb{C}$ with the usual norm and $A = [-1; 1]$ and $B = [-i; i]$ nonempty closed subsets of $\mathbb{C}$. Define the mapping $T : A \cup B \to A \cup B$ by $Tx = \dfrac{ix}{4}, \quad \forall x \in A \cup B$.

It is easy to see that T is a cyclic operator, i.e. satisfies the cyclical condition (1.1). Now we will check if T satisfies (1.3). Let $x \in A$ and $y \in B$. Then we have:

$$\begin{aligned} d(Tx, Ty) &= |Tx - Ty| \\ &= \left| \frac{ix}{4} - \frac{iy}{4} \right| \\ &= \frac{1}{4}|x - y| \\ &\leq \frac{1}{4}\left(|x - Tx| + |Tx - Ty| + |Ty - y|\right) \\ &= \frac{1}{4}\left(d(x, Tx) + d(Tx, Ty) + d(y, Ty)\right) \end{aligned}$$

which yields

$$d(Tx, Ty) \leq \frac{1}{3}\left(d(x, Tx) + d(y, Ty)\right).$$

So

$$d(Tx, Ty) \leq \frac{2}{3} \max\left\{d(x, Tx), d(y, Ty)\right\}.$$

Hence we have $h = \dfrac{2}{3} \in (0, 1)$. We can apply Theorem 1.4 and therefore T has a unique fixed point $x^* \in A \cap B = \{0\}$.

Rhoades proved in [15] that if T is Kannan operator then T is Bianchini operator but not conversly, by taking $Tx = \frac{x}{3},\ x \in (0,1)$.

Many papers have appeared regarding the cyclic version of Bianchini's fixed point theorem, see for example [4], [1], [8],[17].

2. Preliminaries

In order to obtain the existence and uniqueness of a best proximity point we will use two important convergence lemmas given in [6].

Lemma 2.1. ([6]) *Let A be a nonempty closed subset and B be a nonempty closed subset of a uniformly convex Banach space. Let $\{x_n\}$ and $\{z_n\}$ be sequences in A and $\{y_n\}$ be a sequence in B satisfying:*

(i) $\|z_n - y_n\| \to D(A,B)$;
(ii) for every $\epsilon > 0$ there exists N_0 such that for all $m > n \geq N_0$,

$$\|x_m - y_n\| \leq D(A,B) + \epsilon.$$

Then, for every $\epsilon > 0$ there exists N_1 such that for all $m > n \geq N_1$,

$$\|x_m - z_n\| \leq \epsilon.$$

Lemma 2.2. ([6]) *Let A be a nonempty closed and convex subset and B be nonempty closed subset of a uniformly convex Banach space. Let $\{x_n\}$ and $\{z_n\}$ be sequences in A and $\{y_n\}$ be a sequence in B satisfying:*

(i) $\|x_n - y_n\| \to D(A,B)$,
(ii) $\|z_n - y_n\| \to D(A,B)$.

Then $\|x_n - z_n\|$ converges to zero.

The following lemma is also an important tool used in the proofs the new results.

Lemma 2.3. (Petric [11]) *Let X be a uniformly convex Banach space, A and B be two nonempty, closed, convex subsets of X. Let $T : A \cup B \to A \cup B$ be a cyclic operator satisfying*

$$\|Tx - T^2x\| \leq \alpha\|x - Tx\| + (1-\alpha)D(A,B), \text{ for all } x \in A \cup B, \tag{2.4}$$

where $\alpha \in [0,1)$. Then

(i) $\|T^n x - T^{n+1}x\| \leq \alpha^n\|x - Tx\| + (1-\alpha^n)D(A,B)$, *for all $x \in A \cup B$ and $n \geq 0$.*
(ii) $\|T^n x - T^{n+1}x\| \to D(A,B)$, *as $n \to \infty$, for all $x \in A \cup B$.*
(iii) $\|T^{2n}x - T^{2n+\text{-}2}x\| \to 0$, *as $n \to \infty$, for all $x \in A \cup B$.*
(iv) z is a best proximity point if and only if z is a fixed point of T^2

Proof. The first statement follows immediately by induction, using (2.4). Letting $n \to \infty$ in (i) yields the second statement (ii).

To prove (iii) we will use Lemma 2.2 and (ii). Let $x \in A \cup B$. Then by (ii) follows that $\|T^{2n}x - T^{2n-1}x\| \to D(A,B)$ and $\|T^{2n-2}x - T^{2n-1}x\| \to D(A,B)$. From Lemma 2.2 it results

$$\|T^{2n}x - T^{2n-2}x\| \to D(A,B)$$

for any $x \in A \cup B$. Similarly, $\|T^{2n}x - T^{2n+1}x\| \to D(A,B)$ and $\|T^{2n+2}x - T^{2n+1}x\| \to D(A,B)$, and then by Lemma 2.2 we obtain

$$\|T^{2n}x - T^{2n+2}x\| \to D(A,B).$$

We next show the (iv). First we assume that z is a best proximity point, i.e. $\|z - Tz\| = D(A,B)$. Then from (i) we have that $\|T^2z - Tz\| = D(A,B)$. By Lemma 2.2, we obtain

$T^2x = z$. Now, assume that z is a fixed point of T^2 and z is not a best proximity point, i.e. $D(A,B) < \|z - Tz\|$. Then, using (2.4) we have:

$$\begin{aligned} \|z - Tz\| &= \|T^2z - Tz\| \le \alpha\|z - Tz\| + (1-\alpha)D(A,B) \\ &< \alpha\|z - Tz\| + (1-\alpha)\|z - Tz\| = \|z - Tz\| \end{aligned} \tag{2.5}$$

a contradiction. This completes the proof. □

3. Main results

Regarding Bianchini contractive condition we can define the following notion:

Definition 3.1. Let (X,d) be a metric space and let A and B be nonempty subsets of X. Then an cyclic operator $T : A \cup B \to A \cup B$ is called *weak cyclic Bianchini contraction* if it satisfies the following condition

$$d(Tx,Ty) \le h\max\{d(x,Tx), d(y,Ty)\} + (1-h)D(A,B), \forall x \in A, y \in B, \tag{3.6}$$

where $h \in [0,1)$.

It ie easy to see that:

Remark 3.1. Let $x \in A \cup B$. Since T is cyclic we can take $y := Tx$ in (3.6). We obtain

$$d(Tx,T^2x) \le h\max\{d(x,Tx), d(Tx,T^2x)\} + (1-h)D(A,B).$$

If $d(Tx,T^2x) > d(x,Tx)$ then from the above inequality it follows that

$$d(Tx,T^2x) = D(A,B),$$

and hence $d(x,Tx) = D(A,B) = d(Tx,T^2x)$, a contradiction. Therefore

$$d(Tx,T^2x) \le d(x,Tx)$$

and (3.6) implies (2.4) with $\alpha = h \in [0,1)$. Hence we can apply Lemma 2.3.

Hawing in view this definition and remark we can prove now the following:

Theorem 3.5. *Let (X,d) be a metric space, let A and B be nonempty subsets of X and let $T : A \cup B \to A \cup B$ be a weak cyclic Bianchini contraction. Suppose that A is a convex set and let $x \in A$ such that a subsequence $\{T^{2n_k}x\}$ of $\{T^{2n}x\}$ converges to $z \in A$. Then z is the unique best proximity point of T.*

Proof. Let $x \in A$ such that $\lim_{k\to\infty} T^{2n_k}x = z \in A$. Suppose that $d(z,Tz) > D(A,B)$. By (3.6) and (ii) in Lemma 2.3, yields

$$\begin{aligned} d(z,Tz) &= \lim_{k\to\infty} d(T^{2n_k}x, Tz) \\ &\le \lim_{k\to\infty} h\max\{d(T^{2n_k-1}x, T^{2n_k}x), d(z,Tz)\} + (1-h)D(A,B) \\ &= h\max\{D(A,B), d(z,Tz)\} + (1-h)D(A,B) \\ &= hd(z,Tz) + (1-h)D(A,B) \\ &< d(z,Tz). \end{aligned}$$

a contradiction. Hence $d(z,Tz) = D(A,B)$.

We have to show that z is the unique best proximity point of T. Arguing by contradiction, suppose there exists $z' \in A$ another best proximity point of T. Then, by Lemma 2.3 $z' = T^2z'$, and using (3.6), it follows

$$\begin{aligned} D(A,B) \le d(z',Tz) &= d(T^2z', Tz) \\ &\le h\max\{d(z',Tz'), d(z,Tz)\} + (1-h)D(A,B) \\ &= D(A,B). \end{aligned}$$

Hence $d(z', Tz) = D(A,B)$. Since also $d(z, Tz) = D(A,B)$ and A is a convex set we obtain that $z' = z$. With this the uniqueness of the best proximity point is proved.

□

We will prove a convergence and existence result for weak cyclic Bianchini contractions.

Theorem 3.6. *Let A and B be nonempty closed convex subsets of a uniformly convex Banach space. Suppose $T : A \cup B \to A \cup B$ is a weak cyclic Bianchini contraction map. Then*

(i) T has a unique best proximity point z in A.
(ii) The sequence $\{T^{2n}x\}$ converges to z for any starting point $x \in A$.
(iii) z is the unique fixed point of T^2.
(iv) Tz is a best proximity point of T in B.

Proof. Suppose $D(A,B) = 0$, then $A \cap B \neq \emptyset$ and the theorem follows from cyclic Bianchini fixed point theorem 1.4, as T is a cyclic Bianchini operator.

Therefore assume $D(A,B) \neq 0$. Let $x \in A$. Since (3.6) implies (2.4), (see Remark 3.1), by (ii) from Lemma 2.3 we have that

$$\|T^{2n}x - T^{2n+1}x\| \to D(A,B).$$

If, for given $\epsilon > 0$, there exists an $N_0 \in \mathbb{N}$ such that for $m > n \geq N_0$,

$$\|T^{2m}x - T^{2n+1}x\| \leq D(A,B) + \epsilon,$$

then by Lemma 2.1, for given $\epsilon > 0$, there exists an N_1, such that, for $m > n \geq N_1$, $\|T^{2m}x - T^{2n}x\| \leq \epsilon$. Then $\{T^{2n}x\}$ is a Cauchy sequence and hence a convergent one. Let $z \in A$ such that $T^{2n}x \to z$, as $n \to \infty$. By Theorem 3.5, z is the unique best proximity point of T in A. By Lemma 2.3 z is the unique fixed point of T^2, since z is unique.

Thus, assume the contrary: there exists an $\epsilon_0 > 0$, such that for every $k \in \mathbb{N}$, there exists $m_k > n_k \geq k$ such that

$$\|T^{2m_k}x - T^{2n_k+1}x\| > D(A,B) + \epsilon_0. \tag{3.7}$$

Let m_k be the smallest integer greater than n_k, to satisfy the above inequality, i.e.

$$\|T^{2(m_k-1)}x - T^{2n_k+1}x\| \leq D(A,B) + \epsilon_0.$$

Now, by triangle rule, we have

$$D(A,B) + \epsilon_0 < \|T^{2m_k}x - T^{2n_k+1}x\| \leq \|T^{2m_k}x - T^{2m_k-2}x\| + \|T^{2m_k-1}x - T^{2n_k+1}x\|.$$

By Lemma 2.3 (iii), $\|T^{2m_k}x - T^{2m_k-2}x\| \to 0$, as $k \to \infty$. Therefore,

$$D(A,B) + \epsilon_0 \leq \lim_{k\to\infty} \|T^{2m_k}x - T^{2n_k+1}x\| \leq D(A,B) + \epsilon_0.$$

So, $\lim\limits_{k\to\infty} \|T^{2m_k}x - T^{2n_k+1}x\| = D(A,B) + \epsilon_0$. On the other hand, by triangle inequality, it follows

$$\|T^{2m_k}x - T^{2n_k+1}x\| \leq \|T^{2m_k}x - T^{2m_k+2}x\| + \|T^{2m_k+2}x - T^{2n_k+3}x\| + \\ + \|T^{2n_k+3}x - T^{2n_k+1}x\|.$$

Now, letting $k \to \infty$, and using (iii) from Lemma 2.3, (3.6), and (ii) in Lemma 2.3, from the above inequality, we obtain

$$\begin{aligned} D(A,B) + \epsilon_0 &\leq \lim_{k\to\infty} \|T^{2m_k+2}x - T^{2n_k+3}x\| \\ &\leq \lim_{k\to\infty} h \max\{\|T^{2m_k+1}x - T^{2m_k+2}x\|, \|T^{2n_k+2}x - T^{2n_k+3}x\|\} \\ &+ (1-h)D(A,B) = D(A,B). \end{aligned}$$

Hence $\epsilon_0 \leq 0$, a contradiction. We still have to prove that Tz is a best proximity point of T in B. Since z is the unique best proximity point we have that $\|z - Tz\| = D(A, B)$. Now, by (i) from Lemma 2.3, it follows:

$$D(A, B) \leq \|Tz - T^2z\| \leq \alpha\|z - Tz\| + (1 - \alpha)D(A, B) = D(A, B)$$

where $\alpha = h$. Therefore, $\|Tz - T^2z\| = D(A, B)$, i.e. Tz is a best proximity point of T in B. This completes the proof. □

4. Remarks

It is possible to reformulate this result as a common best proximity point theorem for two mappings:

Corollary 4.1. *Let A and B be nonempty closed convex subsets of a uniformly convex Banach space. Suppose $f : aA \to B$ and $g : B \to A$ be two functions such that:*

$$d\left(f(x), g(y)\right) \leq h \max\{d(x, f(x)), d(y, g(y))\} + (1 - h)D(A, B), \forall x \in A, y \in B.$$

Then

(i) f has a unique best proximity point z in A.
(ii) The sequence $\{f^n x\}$ converges to z for any starting point $x \in A$.
(iii) z is the unique fixed point of $f \circ g$.

Proof. Apply Theorem 3.6 to the mapping $T : A \cup B \to A \cup B$ defined by setting

$$Tx = \begin{cases} f(x),\ x \in A \\ g(x),\ x \in B \end{cases} .$$

□

For a sample of other recent results related to the topic of this paper, we refer to [21]-[35], randomly selected from MathSciNet.

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