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A refinement of a Radon type inequality

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ABSTRACT. In this paper we prove a refinement of a Radon type inequality, previously considered by authors in [Bătineţu-Giurgiu, D. M. and Pop, O. T., *A generalization of Radon's inequality*, Creat. Math. Inform., **19** (2010), No. 2, 116–121] and [Bătineţu-Giurgiu, D. M., Mărghidanu, D. and Pop, O. T., *A new generalization of Radon's inequality and applications*, Creat. Math. Inform., **20** (2011), No. 2, 111–116].

1. INTRODUCTION

Let $\mathbb{N}$ be the set of positive integers, $\mathbb{N} = \{1, 2, \dots\}$. The inequality from (1.1) is usually known in literature as Bergström's inequality, see [3], [4], [5], [7], [9], [11], [12], [18].

Theorem 1.1. *If $n \in \mathbb{N}$, $x_k \in \mathbb{R}$ and $y_k > 0$, $k \in \{1, 2, \dots, n\}$, then*

$$\frac{x_1^2}{y_1} + \frac{x_2^2}{y_2} + \cdots + \frac{x_n^2}{y_n} \geq \frac{(x_1 + x_2 + \cdots + x_n)^2}{y_1 + y_2 + \cdots + y_n}, \tag{1.1}$$

with equality if and only if $\dfrac{x_1}{y_1} = \dfrac{x_2}{y_2} = \cdots = \dfrac{x_n}{y_n}$.

In [19], Radon proved the inequality from Theorem 1.2, currently called Radon's inequality.

Theorem 1.2. *If $n \in \mathbb{N}$, $x_k \geq 0$, $y_k > 0$, $k \in \{1, 2, \dots, n\}$ and $p \geq 0$, then*

$$\frac{x_1^{p+1}}{y_1^p} + \frac{x_2^{p+1}}{y_2^p} + \cdots + \frac{x_n^{p+1}}{y_n^p} \geq \frac{(x_1 + x_2 + \cdots + x_n)^{p+1}}{(y_1 + y_2 + \cdots + y_n)^p}. \tag{1.2}$$

In [1] we proved a generalization of Radon's inequality which is the inequality from the next Theorem 1.3.

Theorem 1.3. *If $n \in \mathbb{N}$, $x_k \geq 0$, $y_k > 0$, $k \in \{1, 2, \dots, n\}$, $p \geq 0$ and $r \geq 1$, then*

$$\frac{x_1^{p+r}}{y_1^p} + \frac{x_2^{p+r}}{y_2^p} + \cdots + \frac{x_n^{p+r}}{y_n^p} \geq \frac{\left(x_1 y_1^{r-1} + x_2 y_2^{r-1} + \cdots + x_n y_n^{r-1}\right)^{p+r}}{\left(y_1^r + y_2^r + \cdots + y_n^r\right)^{p+r-1}}, \tag{1.3}$$

with equality if and only if $\dfrac{x_1}{y_1} = \dfrac{x_2}{y_2} = \cdots = \dfrac{x_n}{y_n}$.

On the other hand, in [3] we proved another generalization of Radon's inequality.

Theorem 1.4. *If $n \in \mathbb{N}$, $x_k \geq 0$, $y_k > 0$, $k \in \{1, 2, \dots, n\}$ and $p \geq r \geq 0$,*

$$\frac{x_1^{p+1}}{y_1^r} + \frac{x_2^{p+1}}{y_2^r} + \cdots + \frac{x_n^{p+1}}{y_n^r} \geq n^{r-p} \frac{(x_1 + x_2 + \cdots + x_n)^{p+1}}{(y_1 + y_2 + \cdots + y_n)^r}, \tag{1.4}$$

with equality if and only if $x_1 = x_2 = \cdots = x_n$ and $y_1 = y_2 = \cdots = y_n$.

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The main aim of this note is to prove a refinement of the Radon type inequality (1.3).

2. Refinements of some Radon type inequalities

In this section, we prove a refinement of the inequality from (1.3). In the proof we use the idea from [11].

Theorem 2.5. *If* $p, r \in \mathbb{R}$, $p+r-2 \geq 0$, $a, b > 0$ *and* $x, y \geq 0$, *then*

$$\frac{x^{p+r}}{a^p} + \frac{y^{p+r}}{b^p} - \frac{(xa^{r-1}+yb^{r-1})^{p+r}}{(a^r+b^r)^{p+r-1}} \geq \frac{(p+r-1)(ab)^{r-2}(xa^{r-1}+yb^{r-1})^{p+r-2}(bx-ay)^2}{(a^r+b^r)^{p+r-1}}. \tag{2.5}$$

Proof. We have that

$$\frac{x^{p+r}}{a^p} + \frac{y^{p+r}}{b^p} - \frac{(xa^{r-1}+yb^{r-1})^{p+r}}{(a^r+b^r)^{p+r-1}} = \frac{b^p x\big((x(a^r+b^r))^{p+r-1}-(a(xa^{r-1}+yb^{r-1}))^{p+r-1}\big) - a^p y\big((b(xa^{r-1}+yb^{r-1}))^{p+r-1}-(y(a^r+b^r))^{p+r-1}\big)}{a^p b^p (a^r+b^r)^{p+r-1}}. \tag{2.6}$$

Without loss of generality, we can assume that $bx \geq ay$. From this inequality, it results that

$$x(a^r+b^r) \geq a(xa^{r-1}+yb^{r-1})$$

and

$$b(xa^{r-1}+yb^{r-1}) \geq y(a^r+b^r).$$

The following inequality is well-known: for any $\alpha \geq 1$ and $0 < u \leq v$, the inequality

$$\alpha u^{\alpha-1}(v-u) \leq v^\alpha - u^\alpha \leq \alpha v^{\alpha-1}(v-u)$$

holds.

By using these inequalities in the right member of (2.6), we obtain

$$\frac{b^p x\left((x(a^r+b^r))^{p+r-1}-(a(xa^{r-1}+yb^{r-1}))^{p+r-1}\right) - a^p y\left((b(xa^{r-1}+yb^{r-1}))^{p+r-1}-(y(a^r+b^r))^{p+r-1}\right)}{a^p b^p (a^r+b^r)^{p+r-1}}$$

$$\geq \frac{b^p x(p+r-1)(a(xa^{r-1}+yb^{r-1}))^{p+r-2}(xb^r-ayb^{r-1}) - a^p y(p+r-1)(b(xa^{r-1}+yb^{r-1})^{p+r-2}(bxa^{r-1}-ya^r)}{a^p b^p (a^r+b^r)^{p+r-1}}$$

$$= \frac{(p+r-1)(xa^{r-1}+yb^{r-1})^{p+r-2}(ab)^{p+r-2}(bx-ay)^2}{a^p b^p (a^r+b^r)^{p+r-1}}.$$

From the inequality above and (2.6), inequality (2.5) follows. The inequality in (2.5) holds if and only if $bx = ay$ or $p+r=2$. □

Theorem 2.6. *If* $n, m \in \mathbb{N}$, $n > m$, $x_k \geq 0$, $y_k > 0$, $k \in \{1, 2, \ldots, n\}$, $p \geq 0$ *and* $r \geq 1$, *then*

$$\sum_{k=m+1}^{n} \frac{x_k^{p+r}}{y_k^p} + \frac{\left(\sum\limits_{k=1}^{m} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{m} y_k^r\right)^{p+r-1}} \geq \frac{\left(\sum\limits_{k=1}^{n} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{n} y_k^r\right)^{p+r-1}}. \tag{2.7}$$

Proof. By using inequality (1.3), we have that

$$\sum_{k=m+1}^{n} \frac{x_k^{p+r}}{y_k^p} \geq \frac{\left(\sum\limits_{k=m+1}^{n} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=m+1}^{n} y_k^r\right)^{p+r-1}}$$

ad then

$$\sum_{k=m+1}^{n} \frac{x_k^{p+r}}{y_k^p} + \frac{\left(\sum\limits_{k=1}^{m} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{m} y_k^r\right)^{p+r-1}} \geq \frac{\left(\sum\limits_{k=1}^{m} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{m} y_k^r\right)^{p+r-1}} + \frac{\left(\sum\limits_{k=m+1}^{n} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=m+1}^{n} y_k^r\right)^{p+r-1}}. \tag{2.8}$$

By applying Radon's inequality, we have that

$$\frac{\left(\sum\limits_{k=1}^{m} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{m} y_k^r\right)^{p+r-1}} + \frac{\left(\sum\limits_{k=m+1}^{n} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=m+1}^{n} y_k^r\right)^{p+r-1}} \geq \frac{\left(\sum\limits_{k=1}^{m} x_k y_k^{r-1} + \sum\limits_{k=m+1}^{n} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{m} y_k^r + \sum\limits_{k=m+1}^{n} y_k^r\right)^{p+r-1}}$$

and by taking (2.8) into account, inequality (2.7) follows. □

Theorem 2.7. *If* $n, m \in \mathbb{N}$, $n > m$, $x_k \geq 0$, $y_k > 0$, $k \in \{1, 2, \ldots, n\}$, $p \geq 0$, $r \geq 1$ *and*

$$a_i = \sum_{k=1}^{i} \frac{x_k^{p+r}}{y_k^p} - \frac{\left(\sum\limits_{k=1}^{i} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{i} y_k^r\right)^{p+r-1}}, \; i \in \mathbb{N}, i \leq n, \tag{2.9}$$

then

$$a_n \geq a_m. \tag{2.10}$$

Proof. If in inequality (2.7), we add in each member $\sum\limits_{k=1}^{m} \frac{x_k^{p+r}}{y_k^p}$, we obtain

$$\sum_{k=1}^{m} \frac{x_k^{p+r}}{y_k^p} + \frac{\left(\sum\limits_{k=1}^{m} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{m} y_k^r\right)^{p+r-1}} \geq \sum_{k=1}^{m} \frac{x_k^{p+r}}{y_k^p} + \frac{\left(\sum\limits_{k=1}^{n} x_k y_k^{r-1}\right)^{p+r}}{\left(\sum\limits_{k=1}^{n} y_k^r\right)^{p+r-1}},$$

from where, inequality (2.10) is obtained. □

Corollary 2.1. *In the conditions of Theorem* 2.7, *we have*

$$a_n \geq a_{n-1} \geq \cdots \geq a_2 \geq a_1 = 0. \tag{2.11}$$

Proof. It results from Theorem 2.7. □

Theorem 2.8. *If $n \in \mathbb{N}$, $x_k \geq 0$, $y_k > 0$, $k \in \{1, 2, \ldots, n\}$, $p \geq 0$, $r \geq 1$ and $p + r - 2 \geq 0$, then*

$$\frac{x_1^{p+r}}{y_1^p} + \frac{x_2^{p+r}}{y_1^p} + \cdots + \frac{x_n^{p+r}}{x_n^p} - \frac{(x_1 y_1^{r-1} + x_2 y_2^{r-1} + \cdots + x_n y_n^{r-1})^{p+r}}{(y_1^r + y_2^r + \cdots + y_n^r)^{p+r-1}} \tag{2.12}$$
$$\geq \max_{1 \leq i < j \leq n} \left(\frac{x_i^{p+r}}{y_i^p} + \frac{x_j^{p+r}}{y_j^p} - \frac{(x_i y_i^{r-1} + x_j y_j^{r-1})^{p+r}}{(y_i^r + y_j^r)^{p+r-1}} \right)$$
$$\geq (p + r - 1) \max_{1 \leq i < j \leq n} \frac{(y_i y_j)^{r-2} (x_i y_i^{r-1} + x_j y_j^{r-1})^{p+r-2} (x_i y_j - x_j y_i)^2}{(y_i^r + y_j^r)^{p+r-1}} .$$

Proof. It results from Corollary 2.1 and Theorem 2.5. □

Remark 2.1. In the conditions of Theorem 2.8, for $r = 1$ we obtain the main result from [15].

In the following, we give some applications. These applications are refinements of some inequalities from [1].

Application 2.1. If $n \in \mathbb{N}$, $y_k > 0$, $k \in \{1, 2, \ldots, n\}$, $p \geq 0$, $r \geq 1$ and $p + r - 2 \geq 0$, then

$$\frac{1}{y_1^p} + \frac{1}{y_2^p} + \cdots + \frac{1}{y_n^p} - \frac{(y_1^{r-1} + y_2^{r-1} + \cdots + y_n^{r-1})^{p+r}}{(y_1^r + y_2^r + \cdots + y_n^r)^{p+r-1}} \tag{2.13}$$
$$\geq (p + r - 1) \max_{1 \leq i < j \leq n} \frac{(y_i y_j)^{r-2} (y_i^{r-1} + y_j^{r-1})^{p+r-2} (y_i - y_j)^2}{(y_i^r + y_j^r)^{p+r-1}} .$$

Solution. In inequality (2.12) we consider $x_k = 1$, $k \in \{1, 2, \ldots, n\}$.

Application 2.2. If $a, b, c > 0$, prove that

$$\frac{a^5}{b^2} + \frac{b^5}{c^2} + \frac{c^5}{a^2} - \frac{(ab^2 + bc^2 + ca^2)^5}{(a^3 + b^3 + c^3)^4} \tag{2.14}$$
$$\geq \max \left(\frac{a^5}{b^2} + \frac{b^5}{c^2} - \frac{(ab^2 + bc^2)^5}{(b^3 + c^3)^4}, \frac{b^5}{c^2} + \frac{c^5}{a^2} - \frac{(bc^2 + ca^2)^5}{(c^3 + a^3)^4}, \frac{c^5}{a^2} + \frac{a^5}{b^2} - \frac{(ca^2 + ab^2)^5}{(a^3 + b^3)^4} \right)$$
$$\geq 4 \max \left(\frac{bc(ab^2 + bc^2)^3 (ac - b^2)^2}{(b^3 + c^3)^4}, \frac{ca(bc^2 + ca^2)^3 (ba - c^2)^2}{(c^3 + a^3)^4} \frac{ab(ca^2 + ab^2)^3 (cb - a^2)^2}{(a^3 + b^3)^4} \right).$$

Solution. If $n = 3$, $p = 2$ and $r = 3$, from (2.12), inequality (2.14) follows.

Application 2.3. If $a, b, c > 0$, prove that

$$\frac{1}{a(b+c)^3} + \frac{1}{b(c+a)^3} + \frac{1}{c(a+b)^3} - \frac{1}{(a^2+b^2+c^2)^2} \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \right)^3 = 0 \tag{2.15}$$

if and only if $a = b = c$.

Solution. If $n=3$, $x_1=\dfrac{1}{b+c}$, $x_2=\dfrac{1}{c+a}$, $x_3=\dfrac{1}{a+b}$, $y_1=a$, $y_2=b$, $y_3=c$, $p=1$ and $r=2$, from (2.12), after calculus, we have

$$\frac{1}{a(b+c)^3}+\frac{1}{b(c+a)^3}+\frac{1}{c(a+b)^3}-\frac{1}{(a^2+b^2+c^2)^2}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)^3$$
$$\geq 2\max\left(\frac{c^2(a^2+b^2+ac+bc)(b-a)^2}{(b+c)^3(c+a)^3(a^2+b^2)^2},\frac{a^2(b^2+c^2+ba+ca)(c-b)^2}{(c+a)^3(a+b)^3(b^2+c^2)^2},\right.$$
$$\left.\frac{b^2(c^2+a^2+cb+ab)(a-c)^2}{(a+b)^3(b+c)^3(c^2+a^2)^2}\right).$$

From the inequality above, the conclusion of this application is obtained.

3. The main results about the inequality from (1.4)

In this section, we prove a refinement of the inequality from (1.4).

Theorem 3.9. *If $n,m\in\mathbb{N}$, $n>m$, $x_k\geq 0$, $y_k>0$, $k\in\{1,2,\dots,n\}$ and $p\geq r\geq 0$, then*

$$\sum_{k=m+1}^{n}\frac{x_k^{p+1}}{y_k^r}+m^{r-p}\frac{\left(\sum\limits_{k=1}^{m}x_k\right)^{p+1}}{\left(\sum\limits_{k=1}^{m}y_k\right)^{r}}\geq n^{r-p}\frac{\left(\sum\limits_{k=1}^{n}x_k\right)^{p+1}}{\left(\sum\limits_{k=1}^{n}y_k\right)^{r}}. \tag{3.16}$$

Proof. We note $X_i=\sum\limits_{k=1}^{i}x_k$, $Y_j=\sum\limits_{k=1}^{i}y_k$, where $i\in\{1,2,\dots,n\}$. By using inequality (1.4), we have that

$$\sum_{k=m+1}^{n}\frac{x_k^{p+1}}{y_k^r}+m^{r-p}\frac{X_m^{p+1}}{Y_m^r}=\sum_{k=m+1}^{n}\frac{x_k^{p+1}}{y_k^r}+m\frac{\left(\frac{1}{m}X_m\right)^{p+1}}{\left(\frac{1}{m}Y_m\right)^{r}}$$
$$=\sum_{k=m+1}^{n}\frac{x_k^{p+1}}{y_k^r}+\underbrace{\frac{\left(\frac{1}{m}X_m\right)^{p+1}}{\left(\frac{1}{m}Y_m\right)^{r}}+\frac{\left(\frac{1}{m}X_m\right)^{p+1}}{\left(\frac{1}{m}Y_m\right)^{r}}+\dots+\frac{\left(\frac{1}{m}X_m\right)^{p+1}}{\left(\frac{1}{m}Y_m\right)^{r}}}_{m\ times}$$
$$\geq n^{r-p}\frac{\left(\sum\limits_{k=m+1}^{n}x_k+X_m\right)^{p+1}}{\left(\sum\limits_{k=m+1}^{n}y_k+Y_m\right)^{r}},$$

from where, inequality (3.16) follows. □

Theorem 3.10. *If $n,m\in\mathbb{N}$, $n>m$, $x_k\geq 0$, $y_k>0$, $k\in\{1,2,\dots,n\}$, $p\geq r\geq 0$ and*

$$b_i=\sum_{k=1}^{i}\frac{x_k^{p+1}}{y_k^r}-i^{r-p}\frac{\left(\sum\limits_{k=1}^{i}x_k\right)^{p+1}}{\left(\sum\limits_{k=1}^{i}y_k\right)^{r}},\ i\in\mathbb{N},i\leq n, \tag{3.17}$$

then

$$b_n\geq b_m. \tag{3.18}$$

Proof. If in inequality (3.16), we add in each member the sum

$$\sum_{k=1}^{m} \frac{x_k^{p+1}}{y_k^r},$$

we obtain the desired inequality (3.18). □

Corollary 3.2. *In the conditions of Theorem* 3.10*, we have*

$$b_n \geq b_{n-1} \geq \cdots \geq b_2 \geq b_1 = 0. \tag{3.19}$$

Proof. It results from Theorem 3.10. □

Theorem 3.11. *If $n \in \mathbb{N}$, $x_k \geq 0$, $y_k > 0$, $k \in \{1, 2, \ldots, n\}$ and $p \geq r \geq 0$, then*

$$\frac{x_1^{p+1}}{y_1^r} + \frac{x_2^{p+1}}{y_2^r} + \cdots + \frac{x_n^{p+1}}{y_n^r} - n^{r-p} \frac{(x_1 + x_2 + \cdots + x_n)^{p+1}}{(y_1 + y_2 + \cdots + y_n)^r} \tag{3.20}$$
$$\geq \max_{1 \leq i < j \leq n} \left(\frac{x_i^{p+1}}{y_i^r} + \frac{x_j^{p+1}}{y_j^r} - 2^{r-p} \frac{(x_i + x_j)^{p+1}}{(y_i + y_j)^r} \right) \geq 0.$$

Proof. It results from Corollary 3.1. □

The following applications are refinements of some inequalities from [3].

Application 3.1. If $a, b, c > 0$, prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{(ab+bc+ca)^3}{6a^3b^3c^3(a+b+c)} + A \tag{3.21}$$
$$\geq \frac{ab+bc+ca}{2a^2b^2c^2} + A \geq \frac{3}{2\sqrt[3]{(abc)^4}} + A \geq \frac{3}{2\sqrt[3]{(abc)^4}},$$

where

$$A = \max\left(\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} - \frac{(a+b)^3}{2a^3b^3(a+b+2c)}, \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \right.$$
$$\left. - \frac{(b+c)^3}{2b^3c^3(2a+b+c)}, \frac{1}{c^3(a+b)} + \frac{1}{a^3(b+c)} - \frac{(c+a)^3}{2c^3a^3(a+2b+c)} \right).$$

Solution. The first inequality is obtained from (3.20) for $n = 3$, $p = 2$, $r = 1$, $x_1 = \frac{1}{a}$, $x_2 = \frac{1}{b}$, $x_3 = \frac{1}{c}$, $y_1 = b + c$, $y_2 = c + a$ and $y_3 = a + b$. For the other inequalities, we use the inequalities

$$\frac{(ab+bc+cd)^3}{6a^3b^3c^3(a+b+c)} \geq \frac{ab+bc+ca}{2a^2b^2c^2} \geq \frac{3}{2\sqrt[3]{(abc)^4}},$$

see [3].

Remark 3.1. This applications is a refinement of the following problem given in the 26^{th} International Mathematical Olympiad, Canada, 1995:

If $a, b, c > 0$ with $abc = 1$, then

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}. \tag{3.22}$$

Application 3.2. If $a, b, c > 0$, then the inequality

$$\frac{a^5}{b^2} + \frac{b^5}{c^2} + \frac{c^5}{a^2} - \frac{1}{9}(a+b+c)^3 \tag{3.23}$$
$$\geq \max\left(\frac{a^5}{b^2} + \frac{b^5}{c^2} - \frac{(a+b)^5}{4(b+c)^2} \frac{b^5}{c^2} + \frac{c^5}{a^2} - \frac{(b+c)^5}{4(c+a)^2}, \frac{c^5}{a^2} + \frac{a^5}{b^2} - \frac{(c+a)^5}{4(a+b)^2}\right) \geq 0$$

holds.

Solution. In (3.20) we consider $n = 3$, $p = 4$, $r = 2$, $x_1 = a$, $x_2 = b$, $x_3 = c$, $y_1 = b$, $y_2 = c$ and $y_3 = a$.

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