Neutrosophic generalized $\alpha$-contra-continuity

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ABSTRACT. In this paper we introduce and study the following new concepts: neutrosophic generalized $\alpha$-contra-continuous function, neutrosophic strongly generalized $\alpha$-contra-continuous function, neutrosophic generalized $\alpha$-contra-irresolute. Their interrelations are also established with illustrative examples.

1. INTRODUCTION AND PRELIMINARIES

Zadeh [14] introduced the concept of fuzzy set. Atanassov [2] introduced the notion of intuitionistic fuzzy set as a generalization of fuzzy set. Coker [4] introduced the notion of intuitionistic fuzzy topological space. The concepts of generalized intuitionistic fuzzy closed set was introduced by Dhavaseelan et al. [5] and also investigated generalized intuitionistic fuzzy contra-continuous functions [6]. F. Smadaranche introduced the notion of neutrosophy and the neutrosophic set [12, 13], and A. A. Salama and F. Smadaranche [11] first introduced the notions of crisp set and neutrosophic crisp topology. In this paper, we focus on some versions of Dontchev’s notion of contra-continuity [9] in the context of neutrosophic topology such as neutrosophic generalized $\alpha$-contra-continuous function, neutrosophic strongly generalized $\alpha$-contra-continuous function, neutrosophic generalized $\alpha$-contra-irresolute. Moreover, we establish their interrelations with some examples.

Definition 1.1. [12, 13] Let $T, I, F$ be real standard or non standard subsets of $]0^{-}, 1^{+}[,$ with $\sup T = t_{sup}, \inf T = t_{inf}$

$\sup I = i_{sup}, \inf I = i_{inf}$

$\sup F = f_{sup}, \inf F = f_{inf}$

$n - \sup = t_{sup} + i_{sup} + f_{sup}$

$n - \inf = t_{inf} + i_{inf} + f_{inf} \cdot T, I, F$ are neutrosophic components.

Definition 1.2. [12, 13] Let $X$ be a nonempty fixed set. A neutrosophic set [NS for short] $A$ is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set $A$.

Remark 1.1. [12, 13]

1. A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A \rangle$ in $]0^{-}, 1^{+}[ \cdot$ on $X$.

2. For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$. 

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Definition 1.3. [11] Let $X$ be a nonempty set and the neutrosophic sets $A$ and $B$ in the form

$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}, B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X\}$; [Complement of $A$]

(d) $A \cap B = \{\langle x, \mu_A(x) \land \mu_B(x), \sigma_A(x) \land \sigma_B(x), \gamma_A(x) \lor \gamma_B(x) \rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \lor \mu_B(x), \sigma_A(x) \lor \sigma_B(x), \gamma_A(x) \land \gamma_B(x) \rangle : x \in X\}$;

(f) $\forall A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) \rangle : x \in X\}$;

(g) $\forall A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$.

Definition 1.4. [11] Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in $X$. Then

(a) $\bigcap A_i = \{\langle x, \land \mu_{A_i}(x), \land \sigma_{A_i}(x), \lor \gamma_{A_i}(x) \rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \lor \mu_{A_i}(x), \lor \sigma_{A_i}(x), \land \gamma_{A_i}(x) \rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets $0_N$ and $1_N$ in $X$ as follows:

Definition 1.5. [11] $0_N = \{\langle x, 0, 0, 1 \rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0 \rangle : x \in X\}$.

Definition 1.6. [7] A neutrosophic topology (NT) on a nonempty set $X$ is a family $T$ of neutrosophic sets in $X$ satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\bigcup G_i \in T$ for arbitrary family $\{G_i | i \in \Lambda\} \subseteq T$.

In this case the ordered pair $(X, T)$ or simply $X$ is called a neutrosophic topological space (briefly NTS) and each neutrosophic set in $T$ is called a neutrosophic open set (briefly NOS). The complement $\bar{A}$ of a NOS $A$ in $X$ is called a neutrosophic closed set (briefly NCS) in $X$.

Definition 1.7. [7] Let $A$ be a neutrosophic set in a neutrosophic topological space $X$. Then

$Nint(A) = \bigcup \{G | G$ is a neutrosophic open set in $X$ and $G \subseteq A\}$ is called the neutrosophic interior of $A$;

$Ncl(A) = \bigcap \{G | G$ is a neutrosophic closed set in $X$ and $G \supseteq A\}$ is called the neutrosophic closure of $A$.

Definition 1.8. [8] Let $(X, T)$ and $(Y, S)$ be any two neutrosophic topological spaces.

(i) A function $f : (X, T) \rightarrow (Y, S)$ is called neutrosophic contra-continuous if the inverse image of every neutrosophic open set in $(Y, S)$ is a neutrosophic closed set in $(X, T)$.

Equivalently if the inverse image of every neutrosophic closed set in $(Y, S)$ is a neutrosophic open set in $(X, T)$.

(ii) A function $f : (X, T) \rightarrow (Y, S)$ is called generalized neutrosophic contra-continuous if the inverse image of every neutrosophic open set in $(Y, S)$ is a generalized neutrosophic closed set in $(X, T)$.

Equivalently if the inverse image of every neutrosophic closed set in $(Y, S)$ is a generalized neutrosophic open set in $(X, T)$.

Definition 1.9. [1] Let $f$ be a function from a neutrosophic topological spaces $(X, T)$ and $(Y, S)$. Then $f$ is called

(i) a neutrosophic open function if $f(A)$ is a neutrosophic open set in $Y$ for every neutrosophic open set $A$ in $X$. 

(ii) a neutrosophic \( \alpha \)-open function if \( f(A) \) is a neutrosophic \( \alpha \)-open set in \( Y \) for every neutrosophic open set \( A \) in \( X \).

(iii) a neutrosophic preopen function if \( f(A) \) is a neutrosophic preopen set in \( Y \) for every neutrosophic open set \( A \) in \( X \).

(iv) a neutrosophic semiopen function if \( f(A) \) is a neutrosophic semiopen set in \( Y \) for every neutrosophic open set \( A \) in \( X \).

2. NEUTROSOPTHIC GENERALIZED \( \alpha \)-CONTRA-CONTINUOUS FUNCTION

In this section we introduce neutrosophic generalized \( \alpha \)-contra-continuous function and studied some of its properties.

**Definition 2.10.** A neutrosophic set \( A \) in \( (X,T) \) is said to be a neutrosophic generalized \( \alpha \)-closed set if \( N\text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is a neutrosophic \( \alpha \) open set in \( (X,T) \).

**Definition 2.11.** A function \( f : (X,T) \to (Y,S) \) is called a neutrosophic generalized \( \alpha \)-contra-continuous if \( f^{-1}(B) \) is a neutrosophic generalized \( \alpha \)-closed set in \( (X,T) \) for every neutrosophic open set \( B \) in \( (Y,S) \).

**Definition 2.12.** A function \( f : (X,T) \to (Y,S) \) is called a neutrosophic strongly generalized \( \alpha \)-continuous if \( f^{-1}(B) \) is a neutrosophic open set in \( (X,T) \) for every neutrosophic generalized \( \alpha \)-open set \( B \) in \( (Y,S) \).

**Definition 2.13.** A function \( f : (X,T) \to (Y,S) \) is called a neutrosophic generalized \( \alpha \)-contra-continuous if \( f^{-1}(B) \) is a neutrosophic closed set in \( (X,T) \) for every neutrosophic generalized \( \alpha \)-open set \( B \) in \( (Y,S) \).

**Definition 2.14.** A function \( f : (X,T) \to (Y,S) \) is called a neutrosophic generalized \( \alpha \)-contra-irresolute if \( f^{-1}(B) \) is a neutrosophic generalized \( \alpha \)-closed set in \( (X,T) \) for every neutrosophic generalized \( \alpha \)-open set \( B \) in \( (Y,S) \).

**Proposition 2.1.** For any two neutrosophic topological spaces \( (X,T) \) and \( (Y,S) \), if \( f : (X,T) \to (Y,S) \) is a neutrosophic contra-continuous function then \( f \) is a neutrosophic generalized \( \alpha \)-contra-continuous function.

**Proof.**

Let \( B \) be a neutrosophic open set in \( (Y,S) \). Since \( f \) is a neutrosophic contra-continuous function, \( f^{-1}(B) \) is a neutrosophic closed set in \( (X,T) \). Since every neutrosophic closed set is a neutrosophic generalized \( \alpha \)-closed set, \( f^{-1}(B) \) is a neutrosophic generalized \( \alpha \)-closed set in \( (X,T) \). Hence \( f \) is a neutrosophic generalized \( \alpha \)-contra-continuous function. \( \square \)

The converse of Proposition 2.1 need not be true as shown in Example 2.1.

**Example 2.1.** Let \( X = \{a, b\} \). Define the neutrosophic sets \( G_1 \) and \( G_2 \) in \( X \) as follows:

\[
G_1 = \langle x, (0.6, 0.6, 0.6), (0.4, 0.4, 0.4) \rangle \quad \text{and} \quad G_2 = \langle x, (0.2, 0.2, 0.3), (0.8, 0.8, 0.7) \rangle.
\]

Then the families \( T = \{0_N, 1_N, G_1\} \) and \( S = \{0_N, 1_N, G_2\} \) are neutrosophic topologies on \( X \). Define a function \( f : (X,T) \to (Y,S) \) as follow \( f(a) = a, f(b) = b \). Then \( f \) is a neutrosophic generalized \( \alpha \)-contra-continuous function, but \( f^{-1}(G_2) \) is not a neutrosophic closed set in \( (X,T) \). Hence \( f \) is not a neutrosophic contra-continuous function.

**Proposition 2.2.** For any two neutrosophic topological spaces \( (X,T) \) and \( (Y,S) \), if \( f : (X,T) \to (Y,S) \) is a neutrosophic \( \alpha \)-contra-continuous function then \( f \) is a neutrosophic generalized \( \alpha \)-contra-continuous function.
Proposition 2.4. Let $B$ be a neutrosophic open set in $(Y, S)$. Since $f$ is a neutrosophic $\alpha$-contra-continuous function, $f^{-1}(B)$ is a neutrosophic $\alpha$-closed set in $(X, T)$. Since every neutrosophic $\alpha$-closed set is a neutrosophic generalized $\alpha$-closed set, $f^{-1}(B)$ is a neutrosophic generalized $\alpha$-closed set in $(X, T)$. Hence $f$ is a neutrosophic generalized $\alpha$-contra-continuous function.

The converse of Proposition 2.2 need not be true as shown in Example 2.2.

Example 2.2. Let $X = \{a, b\}$. Define the neutrosophic sets $G_1$ and $G_2$ in $X$ as follows: $G_1 = \langle x, (0.5, 0.5, 0.5), (0.5, 0.5, 0.5) \rangle$ and $G_2 = \langle x, (0.4, 0.4, 0.4), (0.6, 0.6, 0.6) \rangle$. Then the families $T = \{0_N, 1_N, G_1\}$ and $S = \{0_N, 1_N, G_2\}$ are neutrosophic topologies on $X$. Define a function $f : (X, T) \rightarrow (X, S)$ as follow $f(a) = a, f(b) = b$. Then $f$ is a neutrosophic generalized $\alpha$-contra-continuous function, but $f^{-1}(G_2)$ is not a neutrosophic $\alpha$-closed set in $(X, T)$. Hence $f$ is not a neutrosophic $\alpha$-contra-continuous function.

Proposition 2.3. For any two neutrosophic topological spaces $(X, T)$ and $(Y, S)$, if $f : (X, T) \rightarrow (Y, S)$ is a neutrosophic strongly generalized $\alpha$-contra-continuous function then $f$ is a neutrosophic generalized $\alpha$-contra-continuous function.

Proof. Let $B$ be a neutrosophic open set in $(Y, S)$. Every neutrosophic open set is a neutrosophic generalized $\alpha$-open set. Now, $B$ is a neutrosophic generalized $\alpha$-open set in $(Y, S)$. Since $f$ is a neutrosophic strongly generalized $\alpha$-contra continuous function, $f^{-1}(B)$ is a neutrosophic closed set in $(X, T)$. Since every neutrosophic closed set is a neutrosophic generalized $\alpha$-closed set, $f^{-1}(B)$ is a neutrosophic generalized $\alpha$-closed set in $(X, T)$. Hence $f$ is a neutrosophic generalized $\alpha$-contra-continuous function.

The converse of Proposition 2.3 need not be true as shown in Example 2.3.

Example 2.3. Let $X = \{a, b\}$. Define the neutrosophic sets $G_1$ and $G_2$ in $X$ as follows: $G_1 = \langle x, (0.4, 0.4, 0.4), (0.3, 0.3, 0.3) \rangle$ and $G_2 = \langle x, (0.2, 0.2, 0.3), (0.8, 0.8, 0.7) \rangle$. Then the families $T = \{0_N, 1_N, G_1\}$ and $S = \{0_N, 1_N, G_2\}$ are neutrosophic topologies on $X$. Define a function $f : (X, T) \rightarrow (X, S)$ as follow $f(a) = a, f(b) = b$. Then $f$ is a neutrosophic generalized $\alpha$-contra-continuous function. Let $A = \langle x, (0.4, 0.4, 0.4), (0.6, 0.6, 0.6) \rangle$ is a neutrosophic generalized $\alpha$-open set in $(X, S)$, but $f^{-1}(A)$ is not a neutrosophic closed set in $(X, T)$. Hence $f$ is not a neutrosophic strongly generalized $\alpha$-contra-continuous function.

Proposition 2.4. For any two neutrosophic topological spaces $(X, T)$ and $(Y, S)$, if $f : (X, T) \rightarrow (Y, S)$ is a neutrosophic strongly generalized $\alpha$-contra-continuous function then $f$ is a neutrosophic contra-continuous function.

Proof. Let $B$ be a neutrosophic open set in $(Y, S)$. Every neutrosophic open set is a neutrosophic generalized $\alpha$-open set. Now, $B$ is a neutrosophic generalized $\alpha$-open set in $(Y, S)$. Since $f$ is a neutrosophic strongly generalized $\alpha$-contra continuous function, $f^{-1}(B)$ is a neutrosophic closed set in $(X, T)$. Hence $f$ is a neutrosophic contra-continuous function.

The converse of Proposition 2.4 need not be true as shown in Example 2.4.

Example 2.4. Let $X = \{a, b\}$. Define the neutrosophic sets $G_1$ and $G_2$ in $X$ as follows: $G_1 = \langle x, (0.3, 0.3, 0.3), (0.7, 0.7, 0.7) \rangle$ and $G_2 = \langle x, (0.7, 0.7, 0.7), (0.3, 0.3, 0.3) \rangle$. Then the families $T = \{0_N, 1_N, G_1\}$ and $S = \{0_N, 1_N, G_2\}$ are neutrosophic topologies on $X$. Define a function $f : (X, T) \rightarrow (X, S)$ as follow $f(a) = a, f(b) = b$. Then $f$ is a neutrosophic contra-continuous function. Let $A = \langle x, (0.35, 0.35, 0.4), (0.5, 0.5, 0.6) \rangle$ is a neutrosophic generalized $\alpha$-closed set in $(X, S)$, but $f^{-1}(A)$ is not a neutrosophic open set in $(X, T)$. Hence $f$ is not a neutrosophic strongly generalized $\alpha$-contra-continuous function.
3. INTERRELATIONS

From the above results proved, we have a diagram of implications as shown below.

In the diagram, A, B, C and D denote a neutrosophic contra continuous function, neutrosophic generalized α-contra-continuous function, neutrosophic α-contra-continuous function and neutrosophic strongly generalized α-contra-continuous function respectively.

**Proposition 3.5.** Let \((X,T),(Y,S)\) and \((Z,R)\) be any three neutrosophic topological spaces. If a function \(f : (X,T) \rightarrow (Y,S)\) is a neutrosophic strongly generalized α-continuous function and \(g : (Y,S) \rightarrow (Z,R)\) is a neutrosophic generalized α-contra-continuous function then \(g \circ f\) is a neutrosophic contra-continuous function.

**Proof.** Let \(V\) be a neutrosophic open set of \((Z,R)\). Since \(g\) is a neutrosophic generalized α-contra-continuous function, \(g^{-1}(V)\) is neutrosophic generalized α-closed set in \((Y,S)\). Since \(f\) is a neutrosophic strongly generalized α-continuous function, \(f^{-1}(g^{-1}(V))\) is a neutrosophic closed set in \((X,T)\). Hence \(g \circ f\) is a neutrosophic contra-continuous function. \(\square\)

**Proposition 3.6.** Let \((X,T),(Y,S)\) and \((Z,R)\) be any three neutrosophic topological spaces. Then the following statements hold:

(i) If \(f\) is a neutrosophic generalized α-contra-continuous function and \(g\) is a neutrosophic continuous function, then \(g \circ f\) is a neutrosophic generalized α-contra-continuous function.

(ii) If \(f\) is a neutrosophic generalized α-contra-continuous function and \(g\) is a neutrosophic contra-continuous function, then \(g \circ f\) is a neutrosophic generalized α-continuous function.

(iii) If \(f\) is a neutrosophic generalized α-contra-irresolute function and \(g\) is a neutrosophic generalized α-contra-continuous function, then \(g \circ f\) is a neutrosophic generalized α-continuous function.

(iv) If \(f\) is a neutrosophic generalized α-irresolute function and \(g\) is a neutrosophic generalized α-contra-continuous function, then \(g \circ f\) is a neutrosophic generalized α-contra-continuous function.

**Proof.**

(i) Let \(B\) be a neutrosophic open set of \((Z,R)\). Since \(g\) is a neutrosophic continuous function, \(g^{-1}(B)\) is neutrosophic open set in \((Y,S)\). Since \(f\) is a neutrosophic generalized α-contra-continuous function, \(f^{-1}(g^{-1}(B))\) is a neutrosophic generalized α-closed set in \((X,T)\). Hence \(g \circ f\) is a neutrosophic generalized α-contra-continuous function.
(ii) Let $B$ be a neutrosophic open set of $(Z, R)$. Since $g$ is a neutrosophic contra-continuous function, $g^{-1}(B)$ is neutrosophic closed set in $(Y, S)$. Since $f$ is a neutrosophic generalized $\alpha$-contra-continuous function, $f^{-1}(g^{-1}(B))$ is a neutrosophic generalized $\alpha$-open set in $(X, T)$. Hence $g \circ f$ is a neutrosophic generalized $\alpha$-continuous function.

(iii) Let $B$ be a neutrosophic open set of $(Z, R)$. Since $g$ is a neutrosophic generalized $\alpha$-contra -continuous function, $g^{-1}(B)$ is neutrosophic generalized $\alpha$-closed set in $(Y, S)$. Since $f$ is a neutrosophic generalized $\alpha$-contra-irresolute function, $f^{-1}(g^{-1}(B))$ is a neutrosophic generalized $\alpha$-open set in $(X, T)$. Hence $g \circ f$ is a neutrosophic generalized $\alpha$-continuous function.

(iv) Let $B$ be a neutrosophic open set of $(Z, R)$. Since $g$ is a neutrosophic generalized $\alpha$-contra -continuous function, $g^{-1}(B)$ is neutrosophic generalized $\alpha$-closed set in $(Y, S)$. Since $f$ is a neutrosophic generalized $\alpha$-irresolute function, $f^{-1}(g^{-1}(B))$ is a neutrosophic generalized $\alpha$-closed set in $(X, T)$. Hence $g \circ f$ is a neutrosophic generalized $\alpha$-contra-continuous function.

\[\square\]

**Definition 3.15.** Let $(X, T)$ and $(Y, S)$ be any two neutrosophic topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a function. The graph $g : X \rightarrow X \times Y$ of $f$ is defined by $g(x) = (x, f(x)), \forall x \in X$.

**Definition 3.16.** Let $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ and $B = \{(y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y\}$ be neutrosophic sets of $X$ and $Y$ respectively. The product of two neutrosophic sets $A$ and $B$ is defined as $(A \times B)(x, y) = (x, y, \min(\mu_A(x), \mu_B(y)), \min(\sigma_A(x), \sigma_B(y)), \max(\gamma_A(x), \gamma_B(y)))$ for all $(x, y) \in X \times Y$.

**Proposition 3.7.** Let $(X, T)$ and $(Y, S)$ be any two neutrosophic topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a function. If the graph $g : X \rightarrow X \times Y$ of $f$ is a neutrosophic generalized $\alpha$-contra-continuous function then $f$ is also a neutrosophic generalized $\alpha$-contra-continuous function.

**Proof.** Let $B$ be a neutrosophic closed set in $(Y, S)$. By definition 3.15., $f^{-1}(B) = 1_N \cap f^{-1}(B) = g^{-1}(1_N \times B)$. Since $g$ is a neutrosophic generalized $\alpha$-contra-continuous function, $g^{-1}(1_N \times B)$ is a neutrosophic generalized $\alpha$-open set in $(X, T)$. Now, $f^{-1}(B)$ is a neutrosophic generalized $\alpha$-open set in $(X, T)$. Hence $f$ is a neutrosophic generalized $\alpha$-contra-continuous function. \[\square\]

**Proposition 3.8.** Let $(X, T)$ and $(Y, S)$ be any two neutrosophic topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a function. If the graph $g : X \rightarrow X \times Y$ of $f$ is a neutrosophic strongly generalized $\alpha$-contra-continuous function then $f$ is also a neutrosophic strongly generalized $\alpha$-contra-continuous function.

**Proof.** Let $B$ be a neutrosophic generalized $\alpha$-open set in $(Y, S)$. By definition 3.15., $f^{-1}(B) = 1_N \cap f^{-1}(B) = g^{-1}(1_N \times B)$. Since $g$ is a neutrosophic strongly generalized $\alpha$-contra-continuous function, $g^{-1}(1_N \times B)$ is a neutrosophic closed set in $(X, T)$. Now, $f^{-1}(B)$ is a neutrosophic closed set in $(X, T)$. Hence $f$ is an a neutrosophic strongly generalized $\alpha$-contra-continuous function. \[\square\]

**Proposition 3.9.** Let $(X, T)$ and $(Y, S)$ be any two neutrosophic topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a function. If the graph $g : X \rightarrow X \times Y$ of $f$ is a neutrosophic generalized $\alpha$-contra-irresolute function then $f$ is also a neutrosophic generalized $\alpha$-contra-irresolute function.
Proof. Let $B$ be a neutrosophic generalized $\alpha$-closed set in $(Y, S)$. By definition 3.15., $f^{-1}(B) = 1_N \cap f^{-1}(B) = g^{-1}(1_N \times B)$. Since $g$ is a neutrosophic generalized $\alpha$-contra-irresolute function, $g^{-1}(1_N \times B)$ is a neutrosophic generalized $\alpha$-open set in $(X, T)$. Now, $f^{-1}(B)$ is a neutrosophic generalized $\alpha$-open set in $(X, T)$. Hence $f$ is an a neutrosophic generalized $\alpha$-contra-irresolute function. □

4. Conclusion

We have introduced different types of contra continuity in neutrosophic topological spaces and discussed interesting properties and their interrelation along with necessary examples. Further we can continue the work on some concepts like Neutrosophic Almost Generalized $\alpha$-contra continuity, Neutrosophic Generalized $\alpha$-compact and connectedness.

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