CREAT. MATH. INFORM.
Volume **27** (2018), No. 2,
Pages 177 - 190

Online version at https://creative-mathematics.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 286X; Online Edition: ISSN 1843 - 441X
DOI: https://doi.org/10.37193/CMI.2018.02.10

Soft mappings on soft generalized topological spaces

TAHA YASIN OZTURK

ABSTRACT. In the present paper, we introduce $g-$open soft mapping, $g-$closed soft mapping, $g-$pseudo-open soft mapping, $g-$quotient soft mapping on soft generalized topological spaces. Furthermore, we discuss some characterizations and some applications of them.

1. INTRODUCTION

In the year 1999, Molodtsov [11] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties in economics, engineering, social science, medical science, etc.. Maji et al. [10] research deal with operations over soft set. The algebraic structure of set theories dealing with uncertainties is an important problem. Aktaş and Çağman [1] defined soft groups and derived their basic properties. In 2011, Shabir and Naz [13] defined soft topological spaces and introduced basic properties. There have been many studies in the field of soft topology by some authors [2],[3], [4], [7], [12], [14], [15].

Császár [5] defined the concept of generalized neighborhood systems and generalized topological spaces which are an important generalization of topological spaces. Ge et al. [6] introduced some properties of mappings on generalized topological spaces. Moreover, since the soft set structure is a more general concept than a classical set, it is very important that the definitions and theorems in classical topology are moved to soft generalized topological spaces. Thomas and John introduced soft generalized topology, soft generalized neighborhood systems and some properties in [16].

In this study, firstly we introduce soft subspace on the soft generalized topological spaces and its regarding soft closure and soft interior is discussed. Later, we define $g-$open soft mapping, $g-$closed soft mapping, $g-$pseudo-open soft mapping, $g-$quotient soft mapping on soft generalized topological spaces and introduce some properties of their. Finally, we discuss some characterizations and some applications of the soft mappings.

2. PRELIMINARIES

In this section we will introduce necessary definitions and theorems for soft sets. Molodtsov [11] defined the soft set in the following way:

Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A, B \subseteq E$.

Definition 2.1. [11] A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in A$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F, A), or as the set of

Received: 20.06.2017. In revised form: 12.01.2018. Accepted: 19.01.2018
2010 *Mathematics Subject Classification.* 03E75, 54A05, 54C05.
Key words and phrases. soft sets, soft generalized topological space, $g-$open soft mapping, $g-$closed soft mapping, $g-$pseudo-open soft mapping, $g-$quotient soft mapping.

$e-$approximate elements of the soft set, i.e.,

$$(F, A) = \{(e, F(e)) : e \in A \subseteq E, F : A \to P(X)\}.$$

Definition 2.2. [10] For two soft sets (F, A) and (G, B) over X, (F, A) is called soft subset of (G, B) if

(1) $A \subset B$ and
(2) $\forall e \in A$, $F(e)\widetilde{\subseteq}G(e)$.

This relationship is denoted by $(F, A)\widetilde{\subseteq}(G, B)$. Similarly, (F, A) is called a soft superset of (G, B) if (G, B) is a soft subset of (F, A). This relationship is denoted by $(F, A)\widetilde{\supseteq}(G, B)$. Two soft sets (F, A) and (G, B) over X are called soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.3. [10] The intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F, A) \widetilde{\cap} (G, B) = (H, C)$.

Definition 2.4. [10] The union of two soft sets (F, A) and (G, B) over X is the soft set, where $C = A \cup B$ and $\forall e \in C$,

$$H(\varepsilon) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

This relationship is denoted by $(F, A) \widetilde{\cup} (G, B) = (H, C)$.

Definition 2.5. [10] A soft set (F, A) over X is said to be a NULL soft set denoted by Φ if for all $e \in A$, $F(e) = \emptyset$ (null set).

Definition 2.6. [10] A soft set (F, A) over X is said to be an absolute soft set denoted by $\widetilde{X}$ if for all $e \in A$, $F(e) = X$.

Definition 2.7. [13] The difference (H, E) of two soft sets (F, E) and (G, E) over X , denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.

Definition 2.8. [13] Let Y be a non-empty subset of X , then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$.

In particular, (X, E) will be denoted by $\widetilde{X}$.

Definition 2.9. [13] The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined $(F, E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X\backslash F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F.

Definition 2.10. [9] Let (X, E) and $(Y, E^{'})$ be two soft sets, $f : X \to Y$ and $g : E \to E^{'}$ be two mappings. Then $(f_g) : (X, E) \to (Y, E^{'})$ is called a soft mapping and is defined as: for a soft set (F, A) in (X, E), $(f_g)((F, A)) = f(F)_{g(a)}$, $B = g(A) \subseteq E^{'}$ is a soft set in $(Y, E^{'})$ given by

$$f(F)(e^{'}) = \begin{cases} f\left(\underset{e \in g^{-1}(e^{'}) \cap A}{\cup} F(e)\right), & \text{if } g^{-1}(e^{'}) \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise,} \end{cases}$$

for $e^{'} \in B \subseteq E^{'}$. $(f(F), g(A))$ is called a soft image of a soft set (F, A).

Definition 2.11. [9] Let (X, E) and $(Y, E^{'})$ be two soft sets, $(f_g) : (X, E) \to (Y, E^{'})$ be a soft mapping and $(G, C)\widetilde{\subseteq}(Y, E^{'})$. Let $f : X \to Y$ and $g : E \to E^{'}$ be two mappings. Then $(f_g)^{-1}((G, C)) = f^{-1}(G)_{g^{-1}(C)}$, $D = g^{-1}(C)$, is a soft set in the soft set (X, E), defined as:

$$f^{-1}(G)(e) = \begin{cases} f^{-1}(G(g(e))), & \text{if } g(e) \in C, \\ \emptyset, & \text{otherwise,} \end{cases}$$

for $e \in D \subseteq E$. $(f_g)^{-1}((G, C))$ is called a soft inverse image of (G, C).

Definition 2.12. [8] Let $(f_g) : (X, E) \to (Y, E^{'})$ be a soft mapping from (X, E) to $(Y, E^{'})$. A soft mapping (f_g) is said to be injective if f, g are both injective. A soft mapping (f_g) is said to be surjective if f, g are both surjective. A soft mapping (f_g) is said to be bijective if f, g are both bijective.

Definition 2.13. [13] Let τ be the collection of soft set over X, then τ is said to be a soft topology on X if

(1) $\Phi, \widetilde{X}$ belongs to τ;
(2) the union of any number of soft sets in τ belongs to τ;
(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.14. [13] Let (X, τ, E) be a soft topological space over X. Then members of τ are said to be soft open sets in X.

Definition 2.15. [13] Let (X, τ, E) be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed in X, if its complement $(F, E)^c$ belongs to τ.

Proposition 2.1. [13] *Let (X, τ, E) be a soft topological space over X. Then the collection $\tau_e = \{F(e) : (F, E) \in \tau\}$ for each $e \in E$, defines a topology on X.*

Definition 2.16. [2] Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e^c) = \varnothing$ for all $e^c \in E - \{e\}$ (briefly denoted by x_e).

Definition 2.17. [2] *Two soft points (x_e, E) and $(y_{e'}, E)$ over a common universe X, we say that the points are different points if $x \neq y$ or $e \neq e'$.*

Definition 2.18. [2] The soft point x_e is said to be belonging to the soft set (F, E), denoted by $x_e\widetilde{\in}(F, E)$, if $x_e(e) \in F(e)$,i.e., $\{x\} \subseteq F(e)$.

Definition 2.19. [5] Let X be a nonempty set and g be a collection of subsets of X. Then g is called a generalized topology on X if and only if

(1) $\emptyset \in g$,
(2) $G_i \in g$ for $i \in I \neq \emptyset$ implies $\underset{i \in I}{\cup} G_i \in g$.

Definition 2.20. [16] Let $\widetilde{\mu}$ be the collection of soft set over X. Then $\widetilde{\mu}$ is said to be a soft generalized topology on X if

(1) Φ belongs to $\widetilde{\mu}$;
(2) the union of any number of soft sets in $\widetilde{\mu}$ belongs to $\widetilde{\mu}$.

The triplet $(X, \widetilde{\mu}, E)$ is called a soft generalized topological space (briefly $SGT-$space) over X.

Definition 2.21. [16] A soft generalized topology $\widetilde{\mu}$ on (X, E) is called strong if $\widetilde{X} \in \widetilde{\mu}$.

Definition 2.22. [16] Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space over (X, E). Then every element of $\widetilde{\mu}$ is called a $g_{\widetilde{\mu}}-$open soft set.

Definition 2.23. [16] Let $(X,\widetilde{\mu},E)$ be a $SGT-$space over (X,E) and $(F,E)\widetilde{\subseteq}(X,E)$. Then (F,E) is called a $g_{\widetilde{\mu}}-$closed soft set if its soft complement $(F,E)^c$ is a $g_{\widetilde{\mu}}-$open soft set.

Definition 2.24. [16] Let $(X,\widetilde{\mu},E)$ be a $SGT-$space over (X,E) and $(F,E)\widetilde{\subseteq}(X,E)$. Then the soft $g_{\widetilde{\mu}}-$interior of (F,E) denoted by $\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)$ is defined as the soft union of all $g_{\widetilde{\mu}}-$open soft subsets of (F,E). That is, $\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)$ is the largest $g_{\widetilde{\mu}}-$open soft set that is contained in (F,E).

Definition 2.25. [16] Let $(X,\widetilde{\mu},E)$ be a $SGT-$space over (X,E) and $(F,E)\widetilde{\subseteq}(X,E)$. Then the soft $g_{\widetilde{\mu}}-$closure of (F,E) denoted by $\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)$ is defined as the soft intersection of all $g_{\widetilde{\mu}}-$closed soft super sets of (F,E). That is, $\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)$ is the smallest $g_{\widetilde{\mu}}-$closed soft set that is containing (F,E).

For $x_e\widetilde{\in}SS(X,E)$, let $(U,E)_{x_e}$ be a $g_{\widetilde{\mu}}-$open soft subset containing x_e and

$$\widetilde{\mu}_{x_e}=\left((U,E):x_e\widetilde{\in}(U,E)\in\widetilde{\mu}\right).$$

Proposition 2.2. [16] *Let $(X,\widetilde{\mu},E)$ be a $SGT-$space over (X,E) and $(F,E)\widetilde{\subseteq}(X,E)$.Then the following statements hold.*

(1) $\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)\widetilde{\subseteq}(F,E)\widetilde{\subseteq}\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)$,

(2) $\widetilde{J}_{\widetilde{\mu}}\left(\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)\right)=\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)$ *and* $\widetilde{C}_{\widetilde{\mu}}\left(\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\right)=\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)$,

(3) *If* $(G,E)\widetilde{\subseteq}(F,E)$*, then* $\widetilde{J}_{\widetilde{\mu}}\left((G,E)\right)\widetilde{\subseteq}\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)$, $\widetilde{C}_{\widetilde{\mu}}\left((G,E)\right)\widetilde{\subseteq}\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)$,

(4) $\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)=(F,E)\Leftrightarrow(F,E)$ *is* $g_{\widetilde{\mu}}-$*open soft set in* $(X,E)\Leftrightarrow$ *for each* $x_e\widetilde{\in}(F,E)$, $(U,E)_{x_e}\widetilde{\subseteq}(F,E)$ *for some* $(U,E)_{x_e}\in\widetilde{\mu}_{x_e}$,

(5) $\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)=(F,E)\Leftrightarrow(F,E)$ *is* $g_{\widetilde{\mu}}-$*closed soft set in* $(X,E)\Leftrightarrow$*for each* $x_e\widetilde{\in}(X,E)\backslash(F,E)$, $(U,E)_{x_e}\widetilde{\cap}(F,E)=\Phi$ *for some* $(U,E)_{x_e}\in\widetilde{\mu}_{x_e}$,

(6) $\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)=(X,E)\backslash\widetilde{J}_{\widetilde{\mu}}\left((X,E)\backslash(F,E)\right)$ *and* $\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)=(X,E)\backslash\widetilde{C}_{\widetilde{\mu}}\left((X,E)\backslash(F,E)\right)$,

(7) $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\Leftrightarrow(U,E)_{x_e}\widetilde{\cap}(F,E)\neq\Phi$ *for each* $(U,E)_{x_e}\in\widetilde{\mu}_{x_e}$,

(8) $x_e\widetilde{\in}\widetilde{J}_{\widetilde{\mu}}\left((F,E)\right)\Leftrightarrow(U,E)_{x_e}\widetilde{\subseteq}(F,E)$ *for some* $(U,E)_{x_e}\in\widetilde{\mu}_{x_e}$.

Definition 2.26. [16] Let $(X,\widetilde{\mu},E)$ and $\left(Y,\widetilde{\upsilon},E^{'}\right)$ be two $SGT-$spaces and $(f_g):(X,\widetilde{\mu},E)\rightarrow\left(Y,\widetilde{\upsilon},E^{'}\right)$ be a soft mapping. Then (f_g) is called $g-$continuous soft mapping, if $(f_g)^{-1}\left((V,E^{'})\right)\in\widetilde{\mu}$ for each $(V,E^{'})\in\widetilde{\upsilon}$.

Proposition 2.3. [16] *Let $(X,\widetilde{\mu},E)$ and $\left(Y,\widetilde{\upsilon},E^{'}\right)$ be two $SGT-$spaces and $(f_g):(X,\widetilde{\mu},E)\rightarrow\left(Y,\widetilde{\upsilon},E^{'}\right)$ be a soft mapping. Then the following conditions are equivalent.*

(1) (f_g) *is* $g-$*continuous soft mapping.*

(2) $(f_g)^{-1}\left((G,E^{'})\right)$ *is* $g_{\widetilde{\mu}}-$*closed soft set in* (X,E) *for each* $g_{\widetilde{\upsilon}}-$*closed soft subset* $(G,E^{'})$ *of* $\left(Y,E^{'}\right)$.

(3) $(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\right)\widetilde{\subseteq}\widetilde{C}_{\widetilde{\upsilon}}\left((f_g)\left((F,E)\right)\right)$ *for each soft subset* (F,E) *of* (X,E).

(4) $\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)\widetilde{\subseteq}(f_g)^{-1}\left(\widetilde{C}_{\widetilde{\upsilon}}\left((G,E^{'})\right)\right)$ *for each soft subset* $(G,E^{'})$ *of* $\left(Y,E^{'}\right)$.

(5) $(f_g)^{-1}\left(\widetilde{J}_{\widetilde{\upsilon}}\left((G,E^{'})\right)\right)\widetilde{\subseteq}\widetilde{J}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)$ *for each soft subset* $(G,E^{'})$ *of* $\left(Y,E^{'}\right)$.

(6) *For each* $x_e\widetilde{\in}SS(X,E)$*, if* $(f_g)(x_e)\widetilde{\in}(V,E^{'})\in\widetilde{\upsilon}$*, then* $(f_g)\left((U,E)\right)\widetilde{\subseteq}(V,E^{'})$ *for some* $(U,E)\in\widetilde{\mu}_{x_e}$.

3. Soft mappings on soft generalized topological spaces

Given a soft subset in a soft generalized topological space, we can also induce a soft subspace of the soft generalized topological space. Morover, the relationship between a soft generalized topological space and its soft subspaces regarding soft closure and soft interior is discussed in the following propositon.

Proposition 3.4. *Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space over (X, E). If $(X^{'}, E)$ is a soft subset of (X, E), (G, E) is a soft subset of $(X^{'}, E)$ and put $\widetilde{\mu}^{'} = \{(F, E)\widetilde{\cap}(X^{'}, E) : (F, E) \in \widetilde{\mu}\}$. Then the following properties hold.*

(1) *$(X^{'}, \widetilde{\mu}^{'}, E)$ is a $SGT-$space, which is called a soft subspace of $(X, \widetilde{\mu}, E)$.*
(2) *$\widetilde{C}_{\widetilde{\mu}'}((G, E)) = \widetilde{C}_{\widetilde{\mu}}((G, E))\,\widetilde{\cap}(X^{'}, E)$, where $\widetilde{C}_{\widetilde{\mu}'}((G, E))$ is the soft closure of (G, E) in $(X^{'}, \widetilde{\mu}^{'}, E)$.*
(3) *$\widetilde{J}_{\widetilde{\mu}}((G, E))\,\widetilde{\subseteq}\widetilde{J}_{\widetilde{\mu}'}((G, E))\,\widetilde{\cap}\widetilde{J}_{\widetilde{\mu}}\left((X^{'}, E)\right)$, where $\widetilde{J}_{\widetilde{\mu}'}((G, E))$ is soft interior of (G, E) in $(X^{'}, \widetilde{\mu}^{'}, E)$.*

Proof. (1) Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space and $(X^{'}, E)\widetilde{\subseteq}(X, E)$. Since $\Phi \in \widetilde{\mu}$, then $\Phi\widetilde{\cap}(X^{'}, E)$ $= \Phi \in \widetilde{\mu}^{'}$. Assume that $\{(G_i, E)\}_{i\in\in I} \in \widetilde{\mu}^{'}$. Since each $(G_i, E) \in \widetilde{\mu}^{'}$, there exists $(F_i, E) \in \widetilde{\mu}$ such that $(G_i, E) = (F_i, E)\widetilde{\cap}(X^{'}, E)$. Now consider $\underset{i\in I}{\widetilde{\cup}}(G_i, E) = \underset{i\in I}{\widetilde{\cup}}\left((F_i, E)\widetilde{\cap}(X^{'}, E)\right)$ $= \left(\underset{i\in I}{\widetilde{\cup}}(F_i, E)\right)\widetilde{\cap}(X^{'}, E) \in \widetilde{\mu}^{'}$. Therefore, $(X^{'}, \widetilde{\mu}^{'}, E)$ is a soft $SGT-$space.

(2) Let $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}'}((G, E))$. For each $(U, E) \in \widetilde{\mu}_{x_e}$, we have $(U, E)\widetilde{\cap}(X^{'}, E) \in \widetilde{\mu}^{'}_{x_e}$. From Proposition 2.2.(7), $\left((U, E)\widetilde{\cap}(X^{'}, E)\right)\widetilde{\cap}(G, E) \neq \Phi$ and so $(U, E)\widetilde{\cap}(G, E) \neq \Phi$. Thus $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}}((G, E))$. Since $x_e\widetilde{\in}(X^{'}, E)$, then $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}}((G, E))\,\widetilde{\cap}(X^{'}, E)$.

Conversely, let $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}}((G, E))\,\widetilde{\cap}(X^{'}, E)$, then $x_e\widetilde{\in}(X^{'}, E)$ and $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}}((G, E))$. For each $(U^{'}, E) \in \widetilde{\mu}^{'}_{x_e}$, there exists $(U, E) \in \widetilde{\mu}_{x_e}$ such that $(U^{'}, E) = (U, E)\widetilde{\cap}(X^{'}, E)$. By Proposition 2.2.(7), we have $(U, E)\widetilde{\cap}(G, E) \neq \Phi$. Notice that $(G, E)\widetilde{\subseteq}(X^{'}, E)$. So $(U^{'}, E)\widetilde{\cap}(G, E) =$ $\left((U, E)\widetilde{\cap}(X^{'}, E)\right)\widetilde{\cap}(G, E) = (U, E)\widetilde{\cap}(G, E) \neq \Phi$ and so $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}'}((G, E))$.

(3) Since $(G, E)\widetilde{\subseteq}(X^{'}, E)$, then $\widetilde{J}_{\widetilde{\mu}}((G, E))\,\widetilde{\subseteq}\widetilde{J}_{\widetilde{\mu}}\left((X^{'}, E)\right)$. So we show that $\widetilde{J}_{\widetilde{\mu}}((G, E))$ $\widetilde{\subseteq}\widetilde{J}_{\widetilde{\mu}'}((G, E))$. Let $x_e\widetilde{\in}\widetilde{J}_{\widetilde{\mu}}((G, E))$, then there exists $(U, E) \in \widetilde{\mu}_{x_e}$ such that $x_e\widetilde{\in}(U, E)\widetilde{\subseteq}(G, E)$ $\widetilde{\subseteq}(X^{'}, E)$. Since $(U, E)\widetilde{\cap}(X^{'}, E) = (U, E)$ and $(U, E) \in \widetilde{\mu}^{'}$, then $x_e\widetilde{\in}\widetilde{J}_{\widetilde{\mu}'}((G, E))$. That is, $x_e\widetilde{\in}\widetilde{J}_{\widetilde{\mu}'}((G, E))\,\widetilde{\cap}\widetilde{J}_{\widetilde{\mu}}\left((X^{'}, E)\right)$. □

Remark 3.1. In Proposition 3.4. (3), "$\widetilde{\supseteq}$" may not be provided.

Example 3.1. Let $X = \{x^1,\ x^2,\ x^3\}$ be any set and $E = \{e_1,\ e_2\}$ be a set of parameters. We consider the following soft set over X.

$$\begin{aligned}(F_1, E) &= \left\{\left(e_1,\ \{x^1,\ x^2\}\right),\ \left(e_2,\ \{x^1,\ x^2\}\right)\right\}\\(F_2, E) &= \left\{\left(e_1,\ \{x^2,\ x^3\}\right),\ \left(e_2,\ \{x^2,\ x^3\}\right)\right\}\end{aligned}$$

Then $(X, \widetilde{\mu}, E)$ is a $SGT-$space, where $\widetilde{\mu} = \left\{\Phi,\ \widetilde{X},\ (F_1, E),\ (F_2, E)\right\}$. Suppose that $(X^{'}, E) = \left\{\left(e_1,\ \{x^1,\ x^2\}\right),\ \left(e_2,\ \{x^1,\ x^2\}\right)\right\}\widetilde{\subseteq}(X, E)$. It is clear that,

$$\widetilde{\mu}^{'} = \left\{\Phi,\ \widetilde{X}^{'},\ (F_3, E) = (F_2, E)\widetilde{\cap}(X^{'}, E) = \left\{\left(e_1,\ \{x^2\}\right),\ \left(e_2,\ \{x^2\}\right)\right\}\right\}$$

and so $(X^{'}, \widetilde{\mu}^{'}, E)$ is a soft subspace of $(X, \widetilde{\mu}, E)$. In this case, $\widetilde{J_{\widetilde{\mu}}}((F_3, E)) = \Phi$. Similarly, $\widetilde{J}_{\widetilde{\mu}'}((F_3, E)) = (F_3, E)$ and $\widetilde{J}_{\widetilde{\mu}}\left((X^{'}, E)\right) = (X^{'}, E)$. So

$$\widetilde{J}_{\widetilde{\mu}'}((F_3, E)) \widetilde{\cap} \widetilde{J}_{\widetilde{\mu}}\left((X^{'}, E)\right) = (F_3, E) \neq \widetilde{J}_{\widetilde{\mu}}((F_3, E)) = \Phi.$$

Theorem 3.1. *Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space over (X, E) and $(X^{'}, \widetilde{\mu}^{'}, E)$ be a soft subspace of $(X, \widetilde{\mu}, E)$. (K, E) is a $g_{\widetilde{\mu}'}-$closed soft subset of $(X^{'}, E)$ if and only if there exists a $g_{\widetilde{\mu}}-$closed soft subset (F, E) of (X, E) such that $(K, E) = (F, E)\widetilde{\cap}(X^{'}, E)$.*

Proof. $\Rightarrow$ Suppose that (K, E) is a $g_{\widetilde{\mu}'}-$closed soft subset of $(X^{'}, E)$. Then $(X^{'}, E)\backslash(K, E)$ is a $g_{\widetilde{\mu}'}-$open soft subset of $(X^{'}, E)$. From Propositon 3.4., there exists a $g_{\widetilde{\mu}}-$open soft set (U, E) such that $(X^{'}, E)\backslash(K, E) = (X^{'}, E)\widetilde{\cap}(U, E)$. In this case,

$$\begin{aligned}(K, E) &= (X^{'}, E)\backslash\left((X^{'}, E)\backslash(K, E)\right) = (X^{'}, E)\backslash\left((X^{'}, E)\widetilde{\cap}(U, E)\right)\\ &= (X^{'}, E)\widetilde{\cap}\left((X^{'}, E)\widetilde{\cap}(U, E)\right)^{c} = (X^{'}, E)\widetilde{\cap}\left((X^{'}, E)^{c}\widetilde{\cup}(U, E)^{c}\right)\\ &= (X^{'}, E)\widetilde{\cap}(U, E)^{c} = (X^{'}, E)\widetilde{\cap}\left((X, E)\backslash(U, E)\right).\end{aligned}$$

Since (U, E) is a $g_{\widetilde{\mu}}-$open soft set, then $(X, E)\backslash(U, E) = (F, E)$ is a $g_{\widetilde{\mu}}-$closed soft set and $(K, E) = (F, E)\widetilde{\cap}(X^{'}, E)$.

$\Leftarrow$ Let $(F, E)\widetilde{\subseteq}(X, E)$ be a $g_{\widetilde{\mu}}-$closed soft set and $(K, E) = (F, E)\widetilde{\cap}(X^{'}, E)$. In this case,

$$\begin{aligned}(X^{'}, E)\backslash(K, E) &= (X^{'}, E)\backslash\left((X^{'}, E)\widetilde{\cap}(F, E)\right) = (X^{'}, E)\widetilde{\cap}\left((X^{'}, E)\widetilde{\cap}(F, E)\right)^{c}\\ &= (X^{'}, E)\widetilde{\cap}\left((X^{'}, E)^{c}\widetilde{\cup}(F, E)^{c}\right) = (X^{'}, E)\widetilde{\cap}(F, E)^{c}\\ &= (X^{'}, E)\widetilde{\cap}\left((X, E)\backslash(F, E)\right).\end{aligned}$$

Since $(X^{'}, E)\backslash(K, E) = (X^{'}, E)\widetilde{\cap}(U, E) \in \widetilde{\mu}^{'}$, then (K, E) is a $g_{\widetilde{\mu}'}-$closed soft set. □

We note that all mappings in this paper are assumed to be surjective and bijective.

Definition 3.27. Let $(X, \widetilde{\mu}, E)$ and $\left(Y, \widetilde{\upsilon}, E^{'}\right)$ be two $SGT-$spaces and $(f_g) : (X, \widetilde{\mu}, E) \rightarrow \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be a soft mapping. Then $(i_{\imath}) : (X^{'}, \widetilde{\mu}^{'}, E) \rightarrow (f(X^{'}), \widetilde{\upsilon}^{'}, g(E))$ is called a soft restriction of (f_g) on $(X^{'}, \widetilde{\mu}^{'}, E)$, if $(i_{\imath})(x_e) = i(x)_{\imath(e)} = (f_g)(x_e) = f(x)_{g(e)}$ for each soft point $x_e \in SS(X^{'})_E$, where $\widetilde{\mu}^{'} = \{(U, E)\widetilde{\cap}(X^{'}, E) : (U, E) \in \widetilde{\mu}\}$ and $\widetilde{\upsilon}^{'} = \{(V, E^{'})\widetilde{\cap}(f_g)\left((X^{'}, E)\right) : (V, E^{'}) \in \widetilde{\upsilon}\}$. A soft restriction of (f_g) on $(X^{'}, \widetilde{\mu}^{'}, E)$ is denoted by $(f_g)|_{(X^{'}, E)}$.

Proposition 3.5. *Let $(X, \widetilde{\mu}, E)$ and $\left(Y, \widetilde{\upsilon}, E^{'}\right)$ be two $SGT-$spaces, (f_g): $(X, \widetilde{\mu}, E) \rightarrow \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be a soft mapping and $(X^{'}, E)\widetilde{\subseteq}(X, E)$. If (f_g) is $g-$continuous soft mapping, then its soft restriction $(f_g)|_{(X^{'}, E)}$ on $(X^{'}, \widetilde{\mu}^{'}, E)$ is soft $g-$continuous.*

Proof. Let $(f_g) : (X, \widetilde{\mu}, E) \rightarrow \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be $g-$continuous soft mapping, $(F, E)\widetilde{\subseteq}(X^{'}, E)$ and the soft restriction on $(X^{'}, \widetilde{\mu}^{'}, E)$ be $(f_g)|_{(X^{'}, E)} : (X^{'}, \widetilde{\mu}^{'}, E) \rightarrow (f(X^{'}), \widetilde{\upsilon}^{'}, g(E))$, where $\widetilde{\mu}^{'} = \{(U, E)\widetilde{\cap}(X^{'}, E) : (U, E) \in \widetilde{\mu}\}$ and $\widetilde{\upsilon}^{'} = \{(V, E^{'})\widetilde{\cap}(f_g)\left((X^{'}, E)\right) : (V, E^{'}) \in \widetilde{\upsilon}\}$. By Propositons 3.4.(2) and 2.3.(3), we have

$$(i_{\imath})\left(\widetilde{C}_{\widetilde{\mu}'}((F, E))\right) = (i_{\imath})\left(\widetilde{C}_{\widetilde{\mu}}((F, E))\widetilde{\cap}(X^{'}, E)\right) = (f_g)\left(\widetilde{C}_{\widetilde{\mu}}((F, E))\widetilde{\cap}(X^{'}, E)\right)$$

$$\begin{aligned}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}((F,E))\widetilde{\cap}(X^{'},E)\right) &\widetilde{\subseteq} (f_g)\left(\widetilde{C}_{\widetilde{\mu}}((F,E))\right)\widetilde{\cap}(f_g)\left((X^{'},E)\right)\\ &\widetilde{\subseteq} \widetilde{C}_{\widetilde{v}}((f_g)((F,E)))\widetilde{\cap}(f_g)\left((X^{'},E)\right)\\ &= \widetilde{C}_{\widetilde{v}'}((f_g)((F,E))),\end{aligned}$$

where $\widetilde{C}_{\widetilde{\mu}'}((F,E))$ and $\widetilde{C}_{\widetilde{v}'}((f_g)((F,E)))$ are the soft closures of (F,E) in $(X^{'},\widetilde{\mu}^{'},E)$ and $(f_g)((F,E))$ in $(f(X^{'}),\widetilde{v}^{'},g(E))$ respectively. Therefore, $(f_g)|_{(X^{'},E)}$ is $g-$continuous soft mapping by Proposition 2.3.(3). □

There are many useful mappings defined on topological and soft topological spaces. Therefore, we define some of these mappings on soft generalized topological spaces.

Definition 3.28. Let $(X,\widetilde{\mu},E)$ and $\left(Y,\widetilde{v},E^{'}\right)$ be two $SGT-$spaces and $(f_g):(X,\widetilde{\mu},E)\to\left(Y,\widetilde{v},E^{'}\right)$ be a soft mapping. Then

1. (f_g) is called a $g-$open soft mapping if $(f_g)((U,E))\in\widetilde{v}$ for each $(U,E)\in\widetilde{\mu}$.
2. (f_g) is called a $g-$closed soft mapping if $(f_g)((K,E))$ is a $g_{\widetilde{v}}-$closed soft subset of $\left(Y,E^{'}\right)$ for each $g_{\widetilde{\mu}}-$closed soft subset (K,E) of (X,E).
3. (f_g) is called a $g-$pseudo-open soft mapping if for each soft point $y_{e^{'}}\widetilde{\in}SS(Y,E^{'})$ and $(f_g)^{-1}(y_{e^{'}})\widetilde{\subseteq}(U,E)\in\widetilde{\mu}$, then $y_{e^{'}}\widetilde{\in}\widetilde{J}_{\widetilde{v}}((f_g)((U,E)))$.
4. (f_g) is called a $g-$quotient soft mapping if for each $\left(V,E^{'}\right)\widetilde{\subseteq}\left(Y,E^{'}\right)$, $(f_g)^{-1}\left(\left(V,E^{'}\right)\right)\in\widetilde{\mu}$ implies $\left(V,E^{'}\right)\in\widetilde{v}$.

Moreover, we can generalize hereditary properties of soft mappings to the context of soft generalized topological spaces.

Definition 3.29. Let $(X,\widetilde{\mu},E)$ and $\left(Y,\widetilde{v},E^{'}\right)$ be two $SGT-$spaces and $(f_g):(X,\widetilde{\mu},E)\to\left(Y,\widetilde{v},E^{'}\right)$ be a soft mapping. Then (f_g) is called a hereditarily $g-$open soft mapping (resp. hereditarily $g-$closed soft mapping, hereditarily $g-$pseudo-open soft mapping, hereditarily $g-$quotient soft mapping) if

$$(f_g)|_{(f_g)^{-1}\left((Y^{'},E^{'})\right)}:\left(f^{-1}\left(Y^{'}\right),\widetilde{\mu}^{'},g^{-1}(E^{'})\right)\to\left(Y^{'},\widetilde{v}^{'},E^{'}\right)$$

is $g-$open soft mapping (resp. $g-$closed soft mapping, $g-$pseudo-open soft mapping, $g-$quotient soft mapping) for each $(Y^{'},E^{'})\widetilde{\subseteq}\left(Y,E^{'}\right)$, where $\widetilde{\mu}^{'}=\{(U,E)\widetilde{\cap}(f_g)^{-1}\left((Y^{'},E^{'})\right):(U,E)\in\widetilde{\mu}\}$ and $\widetilde{v}^{'}=\{(V,E^{'})\widetilde{\cap}(Y^{'},E^{'}):(V,E^{'})\in\widetilde{v}\}$.

Proposition 3.6. *Every $g-$open soft mapping (resp. $g-$closed soft mapping, $g-$pseudo-open soft mapping) is hereditarily $g-$open soft mapping (resp. $g-$closed soft mapping, $g-$pseudo-open soft mapping).*

Proof. Let $(f_g):(X,\widetilde{\mu},E)\to\left(Y,\widetilde{v},E^{'}\right)$ be a soft mapping. For each $(Y^{'},E^{'})\widetilde{\subseteq}\left(Y,E^{'}\right)$, put $(X^{'},E)=(f_g)^{-1}\left((Y^{'},E^{'})\right)$, $\widetilde{\mu}^{'}=\{(U,E)\widetilde{\cap}(X^{'},E):(U,E)\in\widetilde{\mu}\}$, $\widetilde{v}^{'}=\{(V,E^{'})\widetilde{\cap}(Y^{'},E^{'}):(V,E^{'})\in\widetilde{v}\}$ and $(i_\imath)=(f_g)|_{(X^{'},E)}:(X^{'},\widetilde{\mu}^{'},E)\to\left(Y^{'},\widetilde{v}^{'},E^{'}\right)$. Since all mappings in this paper are assumed to be surjective, hence $(f_g)\left((X^{'},E)\right)=(Y^{'},E^{'})$.

(1) Suppose that (f_g) is $g-$open soft mapping. Let $(W,E)\in\widetilde{\mu}'$, then there exists $(U,E)\in\widetilde{\mu}$ such that $(W,E)=(U,E)\widetilde{\sqcap}(X',E)$. So

$$\begin{aligned}(i_\imath)((W,E)) &= (i_\imath)\left((U,E)\widetilde{\sqcap}(X',E)\right)=(f_g)\left((U,E)\widetilde{\sqcap}(f_g)^{-1}\left((Y',E')\right)\right)\\ &= (f_g)((U,E))\widetilde{\sqcap}(f_g)\left((f_g)^{-1}\left((Y',E')\right)\right)=(f_g)((U,E))\widetilde{\sqcap}(Y',E').\end{aligned}$$

Since (f_g) is $g-$open soft mapping and $(f_g)((U,E))\in\widetilde{\upsilon}$ so $(i_\imath)((W,E))\in\widetilde{\upsilon}'$. This shows that $(i_\imath)$ is $g-$open soft mapping. That is, (f_g) is hereditarily $g-$open soft mapping.

(2) Suppose that (f_g) is $g-$closed soft mapping. Let (F,E) be a $g_{\widetilde{\mu}'}-$closed soft subset of (X',E). By Propositions 2.2.(5) and 3.4.(2), there exists a $g_{\widetilde{\mu}}-$closed soft subset (G,E) of (X,E) such that $(F,E)=(G,E)\widetilde{\sqcap}(X',E)$. Therefore,

$$\begin{aligned}(i_\imath)((F,E)) &= (i_\imath)\left((G,E)\widetilde{\sqcap}(X',E)\right)=(f_g)\left((G,E)\widetilde{\sqcap}(f_g)^{-1}\left((Y',E')\right)\right)\\ &= (f_g)((G,E))\widetilde{\sqcap}(Y',E').\end{aligned}$$

Since (f_g) is $g-$closed soft mapping, $(f_g)((G,E))$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $\left(Y,E'\right)$, so $(i_\imath)((F,E))$ is a $g_{\widetilde{\upsilon}'}-$closed soft subset of (Y',E'). Thus, $(i_\imath)$ is soft $g-$closed soft mapping. That is, (f_g) is hereditarily $g-$closed soft mapping.

(3) Suppose that (f_g) is $g-$pseudo-open soft mapping. Let $y_{e'}\widetilde{\in}SS(Y,E')$ and $(i_\imath)^{-1}(y_{e'})\widetilde{\sqsubseteq}(W,E)\in\widetilde{\mu}'$, then there exists $(U,E)\in\widetilde{\mu}$ such that $(W,E)=(U,E)\widetilde{\sqcap}(X',E)$. Since $(i_\imath)^{-1}(y_{e'})\widetilde{\sqsubseteq}(W,E)\widetilde{\sqsubseteq}(X',E)$ and $(i_\imath)=(f_g)|_{(X',E)}$, then $(f_g)^{-1}(y_{e'})=(i_\imath)^{-1}(y_{e'})$. So $(f_g)^{-1}(y_{e'})\widetilde{\sqsubseteq}(W,E)\widetilde{\sqsubseteq}(U,E)$. Since (f_g) is $g-$pseudo-open soft mapping and $y_{e'}\widetilde{\in}\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))$, then $y_{e'}\widetilde{\in}\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))\widetilde{\sqcap}(Y',E')$. Therefore, $\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))\widetilde{\sqcap}(Y',E')\in\widetilde{\upsilon}'$ and $\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))\widetilde{\sqcap}(Y',E')=\widetilde{J}_{\widetilde{\upsilon}'}\left(\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))\widetilde{\sqcap}(Y',E')\right)$. On the other hand, from Propositon 2.2.(3), we have

$$\begin{aligned}\widetilde{J}_{\widetilde{\upsilon}'}\left(\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))\widetilde{\sqcap}(Y',E')\right) &\widetilde{\sqsubseteq} \widetilde{J}_{\widetilde{\upsilon}'}\left((f_g)((U,E))\widetilde{\sqcap}(Y',E')\right)\\ &= \widetilde{J}_{\widetilde{\upsilon}'}\left((f_g)\left((U,E)\widetilde{\sqcap}(X',E)\right)\right)\\ &= \widetilde{J}_{\widetilde{\upsilon}'}\left((i_\imath)\left((U,E)\widetilde{\sqcap}(X',E)\right)\right)\\ &= \widetilde{J}_{\widetilde{\upsilon}'}((i_\imath)((W,E)))\end{aligned}$$

which gives $y_{e'}\widetilde{\in}\widetilde{J}_{\widetilde{\upsilon}'}((i_\imath)((W,E)))$. This proves that $(i_\imath)$ is $g-$pseudo-open soft mapping, hence (f_g) is hereditarily $g-$pseudo-open soft mapping. □

Remark 3.2. In the next theorems and propositions, we will show that a $g-$quotient soft mapping need not be hereditarily $g-$quotient soft mapping.

4. THE CARACTERIZATIONS OF SOFT MAPPINGS ON SOFT GENERALIZED TOPOLOGICAL SPACES

Theorem 4.2. *Let $(X,\widetilde{\mu},E)$ and $\left(Y,\widetilde{\upsilon},E'\right)$ be two $SGT-$spaces and $(f_g):(X,\widetilde{\mu},E)\to\left(Y,\widetilde{\upsilon},E'\right)$ be a soft mapping. Then the following conditions are equivalent.*

(1) *(f_g) is a $g-$open soft mapping.*
(2) *If $(G,E')\widetilde{\sqsubseteq}(Y,E')$, then $(f_g)^{-1}\left(\widetilde{C}_{\widetilde{\upsilon}}\left((G,E')\right)\right)\widetilde{\sqsubseteq}\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E')\right)\right)$.*
(3) *If $(F,E)\widetilde{\sqsubseteq}(X,E)$, then $(f_g)\left(\widetilde{J}_{\widetilde{\mu}}((F,E))\right)\widetilde{\sqsubseteq}\widetilde{J}_{\widetilde{\upsilon}}((f_g)((F,E)))$.*

(4) *For each* $x_e \widetilde{\in} SS(X,E)$, *if* $x_e \widetilde{\in} (U,E) \in \widetilde{\mu}$, *then* $(f_g)(x_e)\widetilde{\in}(V,E')\widetilde{\subseteq}(f_g)((U,E))$ *for some* $(V,E') \in \widetilde{\upsilon}$.

Proof. (1)⇒(2) Suppose that (f_g) is a $g-$open soft mapping. Let $x_e \widetilde{\in} (f_g)^{-1}\left(\widetilde{C}_{\widetilde{\upsilon}}\left((G,E')\right)\right)$, then $(f_g)(x_e)\widetilde{\in}(f_g)\left((f_g)^{-1}\left(\widetilde{C}_{\widetilde{\upsilon}}\left((G,E')\right)\right)\right) = \widetilde{C}_{\widetilde{\upsilon}}\left((G,E')\right)$. On the other hand, whenever $(U,E)_{x_e} \in \widetilde{\mu}_{x_e}$ implies $(f_g)(x_e)\widetilde{\in}(f_g)((U,E)_{x_e})$, then $(f_g)((U,E)_{x_e}) \in \widetilde{\upsilon}_{(f_g)(x_e)}$ because (f_g) is a $g-$open soft mapping. It follows from Proposition 2.2.(7) that $(f_g)((U,E)_{x_e})$ $\widetilde{\cap}(G,E') \neq \Phi$. Choose $y_{e'}\widetilde{\in}\left((f_g)((U,E)_{x_e})\widetilde{\cap}(G,E')\right)$. Then there exists $x'_{e_0}\widetilde{\in}(U,E)_{x_e}$ such that $y_{e'} = (f_g)\left(x'_{e_0}\right)\widetilde{\in}(G,E')$, i.e., $x'_{e_0}\widetilde{\in}(f_g)^{-1}\left((G,E')\right)$. Therefore, $x'_{e_0}\widetilde{\in}\left((U,E)_{x_e}\widetilde{\cap}(f_g)^{-1}\left((G,E')\right)\right) \neq \Phi$ which gives $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E')\right)\right)$ by using Proposition 2.2.(7).

(2)⇒(3) Assume that the condition (2) holds. Let $(F,E)\widetilde{\subseteq}(X,E)$. From the Proposition 2.2.(1), $\widetilde{J}_{\widetilde{\mu}}((F,E))\widetilde{\subseteq}(F,E)\widetilde{\subseteq}(f_g)^{-1}((f_g)((F,E)))$, so $\widetilde{J}_{\widetilde{\mu}}((F,E))\widetilde{\subseteq}\widetilde{J}_{\widetilde{\mu}}\left((f_g)^{-1}((f_g)((F,E)))\right)$. By Proposition 2.2.(6),

$$\begin{aligned}\widetilde{J}_{\widetilde{\mu}}\left((f_g)^{-1}((f_g)((F,E)))\right) &= (X,E)\backslash\widetilde{C}_{\widetilde{\mu}}\left((X,E)\backslash\left((f_g)^{-1}((f_g)((F,E)))\right)\right)\\ &= (X,E)\backslash\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((Y,E')\backslash((f_g)((F,E)))\right)\right)\end{aligned}$$

so $\widetilde{J}_{\widetilde{\mu}}((F,E))\widetilde{\subseteq}(X,E)\backslash\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((Y,E')\backslash((f_g)((F,E)))\right)\right)$. Since the condition (2) holds and $(f_g)^{-1}\left(\widetilde{C}_{\widetilde{\upsilon}}\left((Y,E')\backslash((f_g)((F,E)))\right)\right) = \widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((Y,E')\backslash((f_g)((F,E)))\right)\right)$, hence

$$\begin{aligned}\widetilde{J}_{\widetilde{\mu}}((F,E)) &\widetilde{\subseteq} (X,E)\backslash(f_g)^{-1}\left(\widetilde{C}_{\widetilde{\upsilon}}\left((Y,E')\backslash((f_g)((F,E)))\right)\right)\\ &= (f_g)^{-1}\left((Y,E')\backslash\widetilde{C}_{\widetilde{\upsilon}}\left((Y,E')\backslash((f_g)((F,E)))\right)\right)\\ &= (f_g)^{-1}\left(\widetilde{J}_{\widetilde{\upsilon}}((f_g)((F,E)))\right).\end{aligned}$$

Therefore, $(f_g)\left(\widetilde{J}_{\widetilde{\mu}}((F,E))\right)\widetilde{\subseteq}(f_g)\left((f_g)^{-1}\left(\widetilde{J}_{\widetilde{\upsilon}}((f_g)((F,E)))\right)\right) = \widetilde{J}_{\widetilde{\upsilon}}((f_g)((F,E)))$.

(3)⇒(1) Suppose that condition (3) holds. Let $(U,E) \in \widetilde{\mu}$, then $\widetilde{J}_{\widetilde{\mu}}((U,E)) = (U,E)$ and $(f_g)((U,E)) = (f_g)\left(\widetilde{J}_{\widetilde{\mu}}((U,E))\right)\widetilde{\subseteq}\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))$. On the other hand, $\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E)))$ $\widetilde{\subseteq}(f_g)((U,E))$ by using Proposition 2.2.(1). Thus, $\widetilde{J}_{\widetilde{\upsilon}}((f_g)((U,E))) = (f_g)((U,E))$ which means $(f_g)((U,E)) \in \widetilde{\upsilon}$ by Proposition 2.2.(4). This proves that (f_g) is a $g-$open soft mapping.

(1)⇒(4) Assume that (f_g) is a $g-$open soft mapping. For each $x_e\widetilde{\in}SS(X,E)$, if $x_e\widetilde{\in}(U,E)$ $\in \widetilde{\mu}$, then $(f_g)(x_e)\widetilde{\in}(f_g)((U,E)) \in \widetilde{\upsilon}$, for $(V,E') = (f_g)((U,E))$.

(4)⇒(1) Suppose that the condition (4) holds. Let $(U,E) \in \widetilde{\mu}$. For each $y_{e'}\widetilde{\in}(f_g)((U,E))$, there exists $(x_e)_{y_{e'}}\widetilde{\in}(U,E)$ such that $y_{e'} = (f_g)\left((x_e)_{y_{e'}}\right)$. So there exists $(V,E')_{y_{e'}} \in \widetilde{\upsilon}$ such that $y_{e'}\widetilde{\in}(V,E')_{y_{e'}}\widetilde{\subseteq}(f_g)((U,E))$. Therefore, $(f_g)((U,E))$ $= \cup\left\{(V,E')_{y_{e'}} : y_{e'}\widetilde{\in}(f_g)((U,E))\right\} \in \widetilde{\upsilon}$. This proves that (f_g) is a $g-$open soft mapping. □

From Theorem 4.2. and Proposition 2.3., the following Corollary holds.

Corollary 4.1. *Let* $(X,\widetilde{\mu},E)$ *and* $\left(Y,\widetilde{\upsilon},E^{'}\right)$ *be two* $SGT-$*spaces and* $(f_g):(X,\widetilde{\mu},E)\to\left(Y,\widetilde{\upsilon},E^{'}\right)$ *be a* $g-$*continuous soft mapping. Then the following conditions are equivalent.*

(1) (f_g) *is a* $g-$*open soft mapping.*
(2) *If* $(G,E^{'})\widetilde{\subseteq}(Y,E^{'})$, *then* $(f_g)^{-1}\left(\widetilde{C}_{\widetilde{\upsilon}}\left((G,E^{'})\right)\right)\widetilde{\subseteq}\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)$.

Theorem 4.3. *Let* $(X,\widetilde{\mu},E)$ *and* $\left(Y,\widetilde{\upsilon},E^{'}\right)$ *be two* $SGT-$*spaces and* $(f_g):(X,\widetilde{\mu},E)\to\left(Y,\widetilde{\upsilon},E^{'}\right)$ *be a soft mapping. Then the following conditions are equivalent.*

(1) (f_g) *is a* $g-$*closed soft mapping.*
(2) *If* $(F,E)\widetilde{\subseteq}(X,E)$, *then* $\widetilde{C}_{\widetilde{\upsilon}}\left((f_g)\left((F,E)\right)\right)\widetilde{\subseteq}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\right)$.
(3) *If* $(G,E^{'})\widetilde{\subseteq}(Y,E^{'})$ *and* $(U,E)\in\widetilde{\mu}$ *such that* $(f_g)^{-1}\left((G,E^{'})\right)\widetilde{\subseteq}(U,E)$, *then there exists* $(V,E^{'})\in\widetilde{\upsilon}$ *such that* $(G,E^{'})\widetilde{\subseteq}(V,E^{'})$ *and* $(f_g)^{-1}\left((V,E^{'})\right)\widetilde{\subseteq}(U,E)$.
(4) *If* $y_{e'}\widetilde{\in}SS(Y,E^{'})$ *and* $(U,E)\in\widetilde{\mu}$ *such that* $(f_g)^{-1}(y_{e'})\widetilde{\subseteq}(U,E)$, *then there exists* $(V,E^{'})\in\widetilde{\upsilon}$ *such that* $y_{e'}\widetilde{\subseteq}(V,E^{'})$ *and* $(f_g)^{-1}\left((V,E^{'})\right)\widetilde{\subseteq}(U,E)$.

Proof. (1)⇒(2) Assume that (f_g) is a $g-$closed soft mapping. Let $(F,E)\widetilde{\subseteq}(X,E)$, then $\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)$ is a $g_{\widetilde{\mu}}-$closed soft subset of (X,E) and so $(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\right)$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y,E^{'})$. Since $(F,E)\widetilde{\subseteq}\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)$ and $(f_g)\left((F,E)\right)\widetilde{\subseteq}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\right)$, hence $\widetilde{C}_{\widetilde{\upsilon}}\left((f_g)\left((F,E)\right)\right)\widetilde{\subseteq}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\right)$ by using Proposition 2.2.(3).

(2)⇒(1) Suppose that the condition (2) holds. Let (F,E) be a $g_{\widetilde{\mu}}-$closed soft subset of (X,E). Then $\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)=(F,E)$ and $(f_g)\left((F,E)\right)=(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((F,E)\right)\right)=\widetilde{C}_{\widetilde{\upsilon}}\left((f_g)\left((F,E)\right)\right)$. From Proposition 2.2.(5), $(f_g)\left((F,E)\right)$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y,E^{'})$. This proves that (f_g) is a $g-$closed soft mapping.

(1)⇒(3) Assume that (f_g) is a $g-$closed soft mapping. Let $(G,E^{'})\widetilde{\subseteq}(Y,E^{'})$ and $(U,E)\in\widetilde{\mu}$ such that $(f_g)^{-1}\left((G,E^{'})\right)\widetilde{\subseteq}(U,E)$. Put $(V,E^{'})=(Y,E^{'})\setminus\left((f_g)\left((X,E)\setminus(U,E)\right)\right)$. It sufficies to check the following three cases.

Case1. $(V,E^{'})\in\widetilde{\upsilon}$.

Since $(X,E)\setminus(U,E)$ is a $g_{\widetilde{\mu}}-$closed soft subset of (X,E) and (f_g) is a $g-$closed soft mapping, then $(f_g)\left((X,E)\setminus(U,E)\right)$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y,E^{'})$. It follows that $(V,E^{'})=(Y,E^{'})\setminus\left((f_g)\left((X,E)\setminus(U,E)\right)\right)\in\widetilde{\upsilon}$.

Case2. $(G,E^{'})\widetilde{\subseteq}(V,E^{'})$.

$(f_g)^{-1}\left((G,E^{'})\right)\widetilde{\subseteq}(U,E)$ implies $(X,E)\setminus(U,E)\widetilde{\subseteq}(X,E)\setminus(f_g)^{-1}\left((G,E^{'})\right)$, hence

$$\begin{aligned}(f_g)\left((X,E)\setminus(U,E)\right)\widetilde{\subseteq}(f_g)\left((X,E)\setminus(f_g)^{-1}\left((G,E^{'})\right)\right)&=(f_g)\left((f_g)^{-1}\left((Y,E^{'})\setminus(G,E^{'})\right)\right)\\&=(Y,E^{'})\setminus(G,E^{'}).\end{aligned}$$

Therefore, $(G,E^{'})\widetilde{\subseteq}(Y,E^{'})\setminus\left((f_g)\left((X,E)\setminus(U,E)\right)\right)=(V,E^{'})$.

Case3. $(f_g)^{-1}\left((V,E^{'})\right)\widetilde{\subseteq}(U,E)$.

$$\begin{aligned}(f_g)^{-1}\left((V,E^{'})\right)&=(f_g)^{-1}\left((Y,E^{'})\setminus\left((f_g)\left((X,E)\setminus(U,E)\right)\right)\right)\\&=(f_g)^{-1}\left((Y,E^{'})\right)\setminus(f_g)^{-1}\left(\left((f_g)\left((X,E)\setminus(U,E)\right)\right)\right)\\&\widetilde{\subseteq}(X,E)\setminus\left((X,E)\setminus(U,E)\right)=(U,E).\end{aligned}$$

(3)⇒(4) It is clear.

(4)⇒(1) Suppose that the condition (3) holds. Let (F, E) be a $g_{\widetilde{\mu}}-$closed soft subset of (X, E). We need to show that $(f_g)((F, E))$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y, E^{'})$. Let $y_{e^{'}}\widetilde{\in}(Y, E^{'})\backslash(f_g)((F, E))$, then $(f_g)^{-1}(y_{e^{'}})\widetilde{\sqcap}(F, E) = \Phi$, i.e., $(f_g)^{-1}(y_{e^{'}})\widetilde{\sqsubseteq}(X, E)\backslash(F, E)$. Notice that $(X, E)\backslash(F, E) \in \widetilde{\mu}$, then by the condition (4) there exists $(V, E^{'}) \in \widetilde{\upsilon}$ such that $y_{e^{'}}\widetilde{\in}(V, E^{'})$ and $(f_g)^{-1}\left((V, E^{'})\right)\widetilde{\sqsubseteq}(X, E)\backslash(F, E)$, i.e., $(f_g)^{-1}\left((V, E^{'})\right)\widetilde{\sqcap}(F, E) = \Phi$. Therefore,

$$\begin{aligned}(V, E^{'})\widetilde{\sqcap}(f_g)((F, E)) &= (f_g)\left((f_g)^{-1}\left((V, E^{'})\right)\right)\widetilde{\sqcap}(f_g)((F, E)) \\ &= (f_g)\left((f_g)^{-1}\left((V, E^{'})\right)\widetilde{\sqcap}(F, E)\right) = (f_g)(\Phi) = \Phi.\end{aligned}$$

From Proposition 2.2.(5), $(f_g)((F, E))$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y, E^{'})$. □

From Theorem 4.3. and Proposition 2.3., the following Corollary holds.

Corollary 4.2. *Let $(X, \widetilde{\mu}, E)$ and $\left(Y, \widetilde{\upsilon}, E^{'}\right)$ be two $SGT-$spaces and $(f_g) : (X, \widetilde{\mu}, E) \to \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be a soft mapping. Then the following conditions are equivalent.*

(1) *(f_g) is a $g-$closed soft mapping.*

(2) *If $(F, E)\widetilde{\sqsubseteq}(X, E)$, then $\widetilde{C}_{\widetilde{\upsilon}}((f_g)((F, E))) = (f_g)\left(\widetilde{C}_{\widetilde{\mu}}((F, E))\right)$.*

Theorem 4.4. *Let $(X, \widetilde{\mu}, E)$ and $\left(Y, \widetilde{\upsilon}, E^{'}\right)$ be two $SGT-$spaces and $(f_g) : (X, \widetilde{\mu}, E) \to \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be a soft mapping. Then the following conditions are equivalent.*

(1) *(f_g) is a $g-$quotient soft mapping.*

(2) *For each soft subset $(G, E^{'})\widetilde{\sqsubseteq}(Y, E^{'})$, if $(f_g)^{-1}\left((G, E^{'})\right)$ is a $g_{\widetilde{\mu}}-$closed soft subset of (X, E), then $(G, E^{'})$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y, E^{'})$.*

Proof. (1)⇒(2) Assume that (f_g) is a $g-$quotient soft mapping. Let $(G, E^{'})\widetilde{\sqsubseteq}(Y, E^{'})$ such that $(f_g)^{-1}\left((G, E^{'})\right)$ is a $g_{\widetilde{\mu}}-$closed soft subset of (X, E). Then

$$(f_g)^{-1}\left((Y, E^{'})\backslash(G, E^{'})\right) = (X, E)\backslash(f_g)^{-1}\left((G, E^{'})\right) \in \widetilde{\mu}.$$

Since (f_g) is a $g-$quotient soft mapping, $(Y, E^{'})\backslash(G, E^{'}) \in \widetilde{\upsilon}$, and so $(G, E^{'})$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y, E^{'})$.

(2)⇒(1) It can be proved by the same method. □

Theorem 4.5. *Let $(X, \widetilde{\mu}, E)$ and $\left(Y, \widetilde{\upsilon}, E^{'}\right)$ be two $SGT-$spaces and $(f_g) : (X, \widetilde{\mu}, E) \to \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be a soft mapping. Then the following conditions are equivalent.*

(1) *(f_g) is a $g-$pseudo-open soft mapping.*

(2) *(f_g) is a hereditarily $g-$quotient soft mapping.*

(3) *If $(G, E^{'})\widetilde{\sqsubseteq}(Y, E^{'})$, then $\widetilde{C}_{\widetilde{\upsilon}}\left((G, E^{'})\right)\widetilde{\sqsubseteq}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G, E^{'})\right)\right)\right)$.*

Proof. (1)⇒(2) Assume that (f_g) is a $g-$pseudo-open soft mapping. Proposition 3.6. states that every $g-$pseudo-open soft mapping is hereditarily $g-$pseudo-open soft mapping. It follows that $(f_g)|_{(f_g)^{-1}\left((Y^{'}, E^{'})\right)}$ is $g-$pseudo-open soft mapping for each $(Y^{'}, E^{'})\widetilde{\sqsubseteq}(Y, E^{'})$. Therefore, it sufficies to prove that every $g-$pseudo-open soft mapping (f_g) is a $g-$quotient soft mapping so that $(f_g)|_{(f_g)^{-1}\left((Y^{'}, E^{'})\right)}$ is $g-$quotient soft mapping for each $(Y^{'}, E^{'})\widetilde{\sqsubseteq}(Y, E^{'})$.

Let $(V,E^{'})\widetilde{\subseteq}(Y,E^{'})$ such that $(f_g)^{-1}\left((V,E^{'})\right)\in\widetilde{\mu}$. If $y_{e'}\widetilde{\in}(V,E^{'})$, then $(f_g)^{-1}(y_{e'})$ $\widetilde{\subseteq}(f_g)^{-1}\left((V,E^{'})\right)\in\widetilde{\mu}$. Since (f_g) is a $g-$pseudo-open soft mapping, thus $y_{e'}\widetilde{\in}\widetilde{J}_{\widetilde{v}}\left((f_g)\left((f_g)^{-1}\left((V,E^{'})\right)\right)\right)=\widetilde{J}_{\widetilde{v}}\left((V,E^{'})\right)$ for each $y_{e'}\widetilde{\in}(V,E^{'})$. It follows that $(V,E^{'})$ $\in\widetilde{v}$. This proves that (f_g) is a $g-$quotient soft mapping.

(2)⇒(3) Assume that (f_g) is a hereditarily $g-$quotient soft mapping. Let $(G,E^{'})\widetilde{\subseteq}(Y,E^{'})$ and $y_{e'}\widetilde{\in}\widetilde{C}_{\widetilde{v}}\left((G,E^{'})\right)=(G,E^{'})\widetilde{\cup}\left(\widetilde{C}_{\widetilde{v}}\left((G,E^{'})\right)\setminus(G,E^{'})\right)$.

i) If $y_{e'}\widetilde{\in}(G,E^{'})$, then $y_{e'}\widetilde{\in}(G,E^{'})=(f_g)\left((f_g)^{-1}\left((G,E^{'})\right)\right)\widetilde{\subseteq}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'}!)\right)\right)\right)$.

ii) If $y_{e'}\widetilde{\in}\widetilde{C}_{\widetilde{v}}\left((G,E^{'})\right)\setminus(G,E^{'})$, let $(Y^{'},E^{'})=(G,E^{'})\widetilde{\cup}\{y_{e'}\}$ and $\widetilde{v}^{'}=\{(V,E^{'})\widetilde{\cap}(Y^{'},E^{'}):(V,E^{'})\in\widetilde{v}\}$ and let $(X^{'},E)=(f_g)^{-1}\left((Y,E^{'})\right)=(f_g)^{-1}\left((G,E^{'})\right)\widetilde{\cup}(f_g)^{-1}(y_{e'})$, $\widetilde{\mu}^{'}=\{(U,E)\widetilde{\cap}(X^{'},E):(U,E)\in\widetilde{\mu}\}$ and $(i_\imath)=(f_g)|_{(X^{'},E)}:(X^{'},\widetilde{\mu}^{'},E)\to\left(Y^{'},\widetilde{v}^{'},E^{'}\right)$, then $(i_\imath)$ is a $g-$quotient soft mapping because (f_g) is a hereditarily $g-$quotient soft mapping. Since $\widetilde{C}_{\widetilde{v}}\left((G,E^{'})\right)$ is the smallest $g_{\widetilde{v}}-$closed soft subset of $(Y,E^{'})$ containing $(G,E^{'})$ and $\widetilde{C}_{\widetilde{v}}\left((G,E^{'})\right)\widetilde{\cap}(Y^{'},E^{'})=(G,E^{'})\widetilde{\cup}\{y_{e'}\}$, $(G,E^{'})$ is not a $g_{\widetilde{v}'}-$closed soft subset of $(Y^{'},E^{'})$, it follows from Theorem 4.4. that $(i_\imath)^{-1}\left((G,E^{'})\right)$ is not a $g_{\widetilde{\mu}'}-$closed soft subset of $(X^{'},E)$, and hence there exists $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}'}\left((i_\imath)^{-1}\left((G,E^{'})\right)\right)\setminus(i_\imath)^{-1}\left((G,E^{'})\right)$. Notice that $(i_\imath)^{-1}\left((G,E^{'})\right)=(f_g)^{-1}\left((G,E^{'})\right)$, so $\widetilde{C}_{\widetilde{\mu}'}\left((i_\imath)^{-1}\left((G,E^{'})\right)\right)=\widetilde{C}_{\widetilde{\mu}'}\left((f_g)^{-1}\left((G,E^{'})\right)\right)$ $\widetilde{\subseteq}\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)$ which gives $x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)$ and hence $(f_g)(x_e)$ $\widetilde{\in}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)\right)$. On the other hand, since $\widetilde{C}_{\widetilde{\mu}'}\left((i_\imath)^{-1}\left((G,E^{'})\right)\right)\widetilde{\subseteq}(X^{'},E)$ and $(i_\imath)^{-1}\left((G,E^{'})\right)=(f_g)^{-1}\left((G,E^{'})\right)$,

$$\begin{aligned}&x_e\widetilde{\in}\widetilde{C}_{\widetilde{\mu}'}\left((i_\imath)^{-1}\left((G,E^{'})\right)\right)\setminus(i_\imath)^{-1}\left((G,E^{'})\right)\widetilde{\subseteq}(X^{'},E)\setminus(f_g)^{-1}\left((G,E^{'})\right)\\ =\ &(f_g)^{-1}\left((G,E^{'})\right)\widetilde{\cup}(f_g)^{-1}(y_{e'})\setminus(f_g)^{-1}\left((G,E^{'})\right)=(f_g)^{-1}(y_{e'})\end{aligned}$$

and hence $y_{e'}=(f_g)(x_e)\widetilde{\in}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)\right)$.

Finally, (i) and (ii) imply $\widetilde{C}_{\widetilde{v}}\left((G,E^{'})\right)\widetilde{\subseteq}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G,E^{'})\right)\right)\right)$.

(3)⇒(1) Suppose that the condition (3) holds. Let $y_{e'}\widetilde{\in}SS(Y,E^{'})$ and $(f_g)^{-1}(y_{e'})$ $\widetilde{\subseteq}(U,E)\widetilde{\in}\widetilde{\mu}$. Then $(f_g)^{-1}(y_{e'})\widetilde{\cap}((X,E)\setminus(U,E))=\Phi$, so $y_{e'}\notin(f_g)((X,E)\setminus(U,E))$, i.e., $y_{e'}\widetilde{\in}(Y,E^{'})\setminus(f_g)((X,E)\setminus(U,E))$. Since the condition (3) holds, $\widetilde{C}_{\widetilde{v}}\left((Y,E^{'})\setminus(f_g)((U,E))\right)$ $=(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((Y,E^{'})\setminus(f_g)((U,E))\right)\right)\right)=(f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((Y,E^{'})\right)\setminus(f_g)^{-1}((f_g)((U,E)))\right)\right)$ $\widetilde{\subseteq}(f_g)\left(\widetilde{C}_{\widetilde{\mu}}((X,E)\setminus(U,E))\right)=(f_g)((X,E)\setminus(U,E))$. By Proposition 2.2.(6), $\widetilde{J}_{\widetilde{v}}((f_g)((U,E)))=$ $(Y,E^{'})\setminus\widetilde{C}_{\widetilde{v}}\left((Y,E^{'})\setminus(f_g)((U,E))\right)\widetilde{\supseteq}(Y,E^{'})\setminus(f_g)((X,E)\setminus(U,E))$. So $y_{e'}\widetilde{\in}(Y,E^{'})\setminus(f_g)$ $((X,E)\setminus(U,E))$ implies $y_{e'}\widetilde{\in}\widetilde{J}_{\widetilde{v}}((f_g)((U,E)))$. This proves that (f_g) is a $g-$pseudo-open soft mapping. □

It is easy to see that the following Corollary holds from Theorem 4.5. and Proposition 2.3.

Corollary 4.3. *Let $(X, \widetilde{\mu}, E)$ and $\left(Y, \widetilde{\upsilon}, E^{'}\right)$ be two $SGT-$spaces and $(f_g) : (X, \widetilde{\mu}, E) \to \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be a soft mapping. Then the following conditions are equivalent.*

(1) *(f_g) is a $g-$pseudo-open soft mapping.*
(2) *If $(G, E^{'})\widetilde{\subseteq}(Y, E^{'})$, then $\widetilde{C}_{\widetilde{\upsilon}}\left((G, E^{'})\right) = (f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G, E^{'})\right)\right)\right)$.*

Finally, we can establish relationships among $g-$open soft mappings, $g-$closed soft mappings, $g-$pseudo-open soft mappings and $g-$quotitent soft mappings on soft generalized topological spaces.

Theorem 4.6. *Let $(X, \widetilde{\mu}, E)$ and $\left(Y, \widetilde{\upsilon}, E^{'}\right)$ be two $SGT-$spaces and (f_g): $(X, \widetilde{\mu}, E) \to \left(Y, \widetilde{\upsilon}, E^{'}\right)$ be a soft mapping. Then the following conditions are equivalent.*

(1) *(f_g) is a $g-$open soft mapping.*
(2) *(f_g) is a $g-$closed soft mapping.*
(3) *(f_g) is a $g-$pseudo-open soft mapping.*
(4) *(f_g) is a $g-$quotient soft mapping.*

Then (1)⇒(3)⇒(4) and (2)⇒(3).

Proof. (1)⇒(3) Assume that (f_g) is a $g-$open soft mapping. Let $y_{e'}\widetilde{\in}SS(Y, E^{'})$ such that $(f_g)^{-1}(y_{e'})\,\widetilde{\subseteq}(U, E)\widetilde{\in}\widetilde{\mu}$. Then $y_{e'}\widetilde{\in}\,(f_g)\left((U, E)\right)$ and $(U, E) = \widetilde{J}_{\widetilde{\mu}}\left((U, E)\right)$. By Theorem 4.1.,

$$y_{e'}\widetilde{\in}\,(f_g)\left((U, E)\right) = (f_g)\left(\widetilde{J}_{\widetilde{\mu}}\left((U, E)\right)\right)\widetilde{\subseteq}\widetilde{J}_{\widetilde{\upsilon}}\left((f_g)\left((U, E)\right)\right).$$

This proves that (f_g) is a $g-$pseudo-open soft mapping.

(3)⇒(4) Suppose that (f_g) is a $g-$pseudo-open soft mapping. Let $(G, E^{'})\widetilde{\subseteq}(Y, E^{'})$ such that $(f_g)^{-1}\left((G, E^{'})\right)$ is a $g_{\widetilde{\mu}}-$closed soft subset of (X, E). Then $\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G, E^{'})\right)\right) = (f_g)^{-1}\left((G, E^{'})\right)$. From the Theorem 4.3., $\widetilde{C}_{\widetilde{\upsilon}}\left((G, E^{'})\right) = (f_g)\left(\widetilde{C}_{\widetilde{\mu}}\left((f_g)^{-1}\left((G, E^{'})\right)\right)\right) = (f_g)\left((f_g)^{-1}\left((G, E^{'})\right)\right) = (G, E^{'})$. So $(G, E^{'})$ is a $g_{\widetilde{\upsilon}}-$closed soft subset of $(Y, E^{'})$. This proves that (f_g) is a $g-$quotient soft mapping.

(2)⇒(3) Suppose that (f_g) is a $g-$closed soft mapping.. Let $y_{e'}\widetilde{\in}SS(Y, E^{'})$ such that $(f_g)^{-1}(y_{e'})\,\widetilde{\subseteq}(U, E)\widetilde{\in}\widetilde{\mu}$. From Theorem 4.2., there exists $(H, E)\widetilde{\in}\widetilde{\mu}$ such that $(f_g)^{-1}(y_{e'})\,\widetilde{\subseteq}(H, E)\widetilde{\subseteq}(U, E)$ and $(f_g)\left((H, E)\right)\widetilde{\in}\widetilde{\upsilon}$.

So $y_{e'}\widetilde{\in}\,(f_g)\left((H, E)\right) = \widetilde{J}_{\widetilde{\upsilon}}\left((f_g)\left((H, E)\right)\right)\widetilde{\subseteq}\widetilde{J}_{\widetilde{\upsilon}}\left((f_g)\left((U, E)\right)\right)$. This proves that (f_g) is a $g-$pseudo-open soft mapping. □

Example 4.2. Let $X = \{x^1, x^2, x^3, x^4\}$ and $Y = \{y^1, y^2, y^3, y^4\}$ be two order sets and $E = \{e_1, e_2, e_3\}$ and $E^{'} = \{e_1^{'}, e_2^{'}, e_3^{'}\}$ be two sets of parameters. Here, the soft sets over (X, E) and $(Y, E^{'})$ defined as follows:

$$\begin{aligned}(F_1, E) &= \left\{(e_1, \{x^1, x^2\}), (e_2, \{x^2, x^3\}), (e_3, \{x^1, x^2, x^3\})\right\};\\ (F_2, E) &= \left\{(e_1, \{x^3\}), (e_2, \{x^1, x^4\}), (e_3, \{x^2, x^4\})\right\};\\ (F_3, E) &= \left\{(e_1, \{x^1, x^2, x^3\}), (e_2, X), (e_3, X)\right\};\end{aligned}$$

and

$$\begin{aligned}(G_1, E^{'}) &= \left\{(e_1^{'}, \{y^1, y^3\}), (e_2^{'}, \{y^2\}), (e_3^{'}, \{y^3, y^4\})\right\};\\ (G_2, E^{'}) &= \left\{(e_1^{'}, \{y^1, y^2, y^4\}), (e_2^{'}, \{y^1, y^4\}), (e_3^{'}, \{y^1, y^2\})\right\};\\ (G_3, E^{'}) &= \left\{(e_1^{'}, Y), (e_2^{'}, \{y^1, y^2, y^4\}), (e_3^{'}, Y)\right\}.\end{aligned}$$

Then $\widetilde{\mu}=\left\{\Phi,\ \widetilde{X},\ (F_1,E),\ (F_2,E),\ (F_3,E)\right\}$ and $\widetilde{\upsilon}=\left\{\Phi,\ \widetilde{Y},\ (G_1,E^{'}),\ (G_2,E^{'}),\ (G_3,E^{'})\right\}$ are two soft generated topology over (X,E) and $(Y,E^{'})$, respectively and $(X,\widetilde{\mu},E)$ and $\left(Y,\widetilde{\upsilon},E^{'}\right)$ are two $SGT-$spaces. If the soft mapping $(f_g):(X,\widetilde{\mu},E)\rightarrow\left(Y,\widetilde{\upsilon},E^{'}\right)$ defined as

$$\begin{aligned} f(x^1) &= y^4,\ f(x^2)=y^1,\ f(x^3)=y^2,\ f(x^4)=y^3 \\ g(e_1) &= e_2^{'},\ g(e_2)=e_3^{'},\ g(e_3)=e_1^{'}. \end{aligned}$$

Then (f_g) is a $g-$open soft mapping ($g-$pseudo-open soft mapping, $g-$quotient soft mapping).

5. Conclusions

In this paper, the concepts of $g-$open soft mapping, $g-$closed soft mapping, $g-$pseudo-open soft mapping, $g-$quotient soft mapping on soft generalized topological spaces are defined. We introduce their basic theorems and some interesting results. We hope that the findings in this paper will help researcher enhance and promote the further study on soft generalized topological spaces.

6. Compliance with ethical standards

Conflict of interest The author declares he has no conflict of interest.

References

[1] Aktas, H. and Cagman, N., *Soft sets and soft group*, Inf. Sci., **177** (2007), 2726–2735

[2] Bayramov, S. and Gunduz (Aras), C., *Soft locally compact spaces and soft paracompact spaces*, J. Math. Sys. Sci., **3** (2013), 122–130

[3] Bayramov, S., Gunduz (Aras), C. and Demirci, N., *A new approach to inverse and direct systems of soft topological spaces*, Maejo Int. J. Sci. Tech., **10** (2016), No. 1, 51–65

[4] Cagman, N., Karatas S. and Enginoğlu, S., *Soft topology*, Comput. Math. Appl., (2011), 351–358

[5] Császár, Á., *Generalized topology, generalized continuity*, Acta. Math. Hungar., **96** (2002), 351–357

[6] Ge, X., Gong, J. and Reilly, I., *Some properties of mappings on generalized topological spaces*, arXiv:1501.06388v1 [math. GN] 2015

[7] Gunduz (Aras), C., Sonmez, A. and Çakallı, H., *On soft Mappings*, Proceed. 13th Int. Conf. Comput. Math. Methods Sci. Eng., (2013)

[8] Hazra, H., Majumdar, P. and Samanta, S. K., *Soft topology*, Fuzzy Inf. Eng., **1** (2012), 105–115

[9] Kharal, A. and Ahmad, B., *Mapping on soft classes*, New Math. & Natur. Comput., **7** (2011), No. 3, 471–481

[10] Maji, P. K., Bismas, R. and Roy, A. R., *Soft Set Theory*, Comput. Math. Appl., **45** (2003), 555–562

[11] Molodtsov, D., *Soft Set Theory-First Results*, Comput. Math. Appl.,**37** (1999), 19–31

[12] Ozturk, T. Y. and Bayramov, S., *Soft mapping spaces*, Sci. World J., Article ID 307292, (2014) 8p.

[13] Shabir, M. and Naz, M., *On soft topological spaces*, Comput. Math. Appl., **61** (2011), 1786–1799

[14] Shabir, M. and Bashir, A., *Some properties of soft topological spaces*, Comput. Math. Appl., **62** (2011), 4058–4067

[15] Sahin, R. and Küçük, A., *Soft filters and their convergence properties*, Ann. Fuzzy Math. Inform., **6**(3) (2013), No. 3, 529–543

[16] Thomas, J. and John, J. S., *On soft generalized topological spaces*, J. New Results Sci., **4** (2014), 01–15

Department of Mathematics, Faculty of Science and Letters
Kafkas University
Kars, 36100, Turkey
Email address: taha36100@hotmail.com