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Conformable fractional integral inequalities for some convex functions

ERHAN SET, BARIŞ ÇELİK and ABDURRAHMAN GÖZPINAR

ABSTRACT. The main purpose of this paper is to present new Hermite-Hadamard's type inequalities for functions that belongs to the classes of $Q(I)$, $P(I)$, $SX(h, I)$ and r-convex via conformable fractional integrals . The results presented here would provide extensions of those given in earlier works.

1. INTRODUCTION

The following inequality is well known in the literature as the Hermite-Hadamard integral inequality (see, e.g., [3]):

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}$$

where $f : I \subset \mathbb{R} \to \mathbb{R}$ is a convex function on the interval I of real numbers and $a, b \in I$ with $a < b$.

Now, we give the concept of the Godunova–Levin function and P function introduced by Godunova–Levin and Dragomir *et al.* respectively.

Definition 1.1. ([5]) A function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is said to belong to the class of $Q(I)$ if it is nonnegative and for all $x, y \in I$ and $\lambda \in (0, 1)$ satisfies the inequality:

$$f(\lambda x + (1-\lambda)y) \leq \frac{f(x)}{\lambda} + \frac{f(y)}{(1-\lambda)}. \tag{1.1}$$

Definition 1.2. ([4]) A function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is P function or that f belongs to the class of $P(I)$, if it is nonnegative and for all $x, y \in I$ and $\lambda \in [0, 1]$, satisfies the following inequality:

$$f(\lambda x + (1-\lambda)y) \leq f(x) + f(y). \tag{1.2}$$

In [14], Varošanec defined the following class of functions.

Definition 1.3. Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a positive function. We say that $f : I \subset \mathbb{R} \to \mathbb{R}$ is h-convex function or that f belongs to the class $S(X) \in I$, if f is nonnegative and for all $x, y \in I$ and $\lambda \in (0, 1)$ we have

$$f(\lambda x + (1-\lambda)y) \leq h(\lambda)f(x) + h(1-\lambda)f(y). \tag{1.3}$$

Recall that a positive function f is *r*-convex on an interval $I = [a, b]$ or (a, b), if for all $x, y \in I$ and $\lambda \in [0, 1]$,

$$f(\lambda x + (1-\lambda)y) \leq \begin{cases} [\lambda f^r(x) + (1-\lambda)f^r(y)]^{\frac{1}{r}}, & \text{if } r \neq 0, \\ f^{\lambda}(x)f^{1-\lambda}(y), & \text{if } r = 0. \end{cases} \tag{1.4}$$

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Corresponding author: Barış Çelik; bariscelik15@hotmail.com

The definition of *r*-convexity naturally complements the concept of *r*-concavity, in which the inequality is reversed (see [13]) and which plays an important role in statistics.

The Beta function defined as follows:

$$B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}=\int_0^1 t^{a-1}(1-t)^{b-1}\,dt, \qquad a,b>0$$

where $\Gamma(\alpha)$ is Gamma function.

Some Hermite-Hadamard type inequalities for Godunova-Levin functions, *P* functions and *h*-convex functions are obtained by Özdemir *et al.* in [8] as follows:

Theorem 1.1. *Let $f\in Q(I)$, $a,b\in I$ with $0\le a<b$ and $f\in L_1[a,b]$. Then the following inequality for fractional integrals hold:*

$$f\left(\frac{a+b}{2}\right)\le\frac{2\Gamma(\alpha+1)}{(b-a)^\alpha}\left[J_{a^+}^\alpha(b)+J_{b^-}^\alpha f(a)\right] \tag{1.5}$$

with $\alpha>0$.

Theorem 1.2. *Let $f\in P(I)$, $a,b\in I$ with $a<b$ and $f\in L_1[a,b]$. Then one has following inequality for fractional integrals hold:*

$$f\left(\frac{a+b}{2}\right)\le\frac{\Gamma(\alpha+1)}{(b-a)^\alpha}\left[J_{a^+}^\alpha(b)+J_{b^-}^\alpha f(a)\right]\le 2[f(a)+f(b)] \tag{1.6}$$

with $\alpha>0$.

Theorem 1.3. *Let $f\in SX(h,I)$, $a,b\in I$ with $a<b$ and $f\in L_1[a,b]$. Then one has inequality for h-convex functions via fractional integrals*

$$\begin{aligned}\frac{1}{h\left(\frac{1}{2}\right)}f\left(\frac{a+b}{2}\right) &\le \frac{\Gamma(\alpha)}{(b-a)^\alpha}\left[J_{a^+}^\alpha(b)+J_{b^-}^\alpha f(a)\right] \\ &\le [f(a)+f(b)]\int_0^1 t^{\alpha-1}[h(t)+h(1-t)]dt.\end{aligned} \tag{1.7}$$

In [8], Özdemir *et al.* established following theorem for *r*-convex functions:

Theorem 1.4. *Let $f\in[a,b]\to(0,\infty)$ be r-convex function on $[a,b]$ with $a<b$ and $0<r<1$. Then the following inequality for fractional integrals holds:*

$$\begin{aligned}&\frac{\Gamma(\alpha+1)}{(b-a)^\alpha}\left[J_{a^+}^\alpha(b)+J_{b^-}^\alpha f(a)\right] \\ \le\ & \left[\left(\frac{1}{\alpha+\frac{1}{r}}\right)^r[f(a)]^r+\left(\beta(\alpha,\frac{r+1}{r})\right)^r[f(b)]^r\right]^{\frac{1}{r}} \\ &+\left[\left(\beta(\alpha,\frac{r+1}{r})\right)^r[f(a)]^r+\left(\frac{1}{\alpha+\frac{1}{r}}\right)^r[f(b)]^r\right]^{\frac{1}{r}}.\end{aligned} \tag{1.8}$$

In the following, we give some definitions and properties of conformable fractional integrals which helps to obtain main identity and results. Recently, some authors started to study on conformable fractional integrals. In [7], Khalil *et al.* defined the fractional integral of order $0<\alpha\le 1$ only. In [1], Abdeljawad gave the definition of left and right conformable fractional integrals of any order $\alpha>0$. Let $\mathbb{N}$ be the set of positive integers and $\mathbb{N}_0:=\mathbb{N}\cup\{0\}$.

Definition 1.4. Let $\alpha \in (n, n+1]$ and set $\beta = \alpha - n$ then the left conformable fractional integral starting at a if order α is defined by

$$(I_\alpha^a f)(t) = \frac{1}{n!} \int_a^t (t-x)^n (x-a)^{\beta-1} f(x) dx$$

Analogously, the right conformable fractional integral is defined by

$$({}^b I_\alpha f)(t) = \frac{1}{n!} \int_t^b (x-t)^n (b-x)^{\beta-1} f(x) dx.$$

Notice that if $\alpha = n+1$ then $\beta = \alpha - n = n+1-n = 1$ where $n = 0, 1, 2, ...$ and hence $(I_\alpha^a f)(t) = (J_{n+1}^a f)(t)$. Some recent result and properties concerning the fractional integral operators can be found [2, 6, 8, 9, 10, 11, 12, 15].

The aim of this paper, is to establish some new Hermite-Hadamard type inequalities for the classes of $Q(I)$, $P(I)$, $SX(h, I)$ and *r*-convex functions via conformable fractional integrals.

2. Main results

Theorem 2.5. *Let $f \in Q(I)$, $a, b \in I$ with $0 \leq a < b$ and $f \in L_1[a, b]$. Then, for $\alpha \in (n, n+1]$ $(n \in \mathbb{N}_0)$, the following inequality for conformable fractional integrals holds:*

$$B(n+1, \alpha - n) f\left(\frac{a+b}{2}\right) \leq \frac{2n!}{(b-a)^\alpha} \left[I_\alpha^a f(b) +^b I_\alpha f(a)\right] \tag{2.9}$$

where $B(.,.)$ is Euler beta function.

Proof. Since $f \in Q(I)$, we have

$$f\left(\frac{x}{2} + \frac{y}{2}\right) \leq \frac{f(x)}{\frac{1}{2}} + \frac{f(y)}{\frac{1}{2}} = 2\left[f(x) + f(y)\right]$$

for all $x, y \in I$ with $\lambda = \frac{1}{2}$ in (1.1).

Changing the variables with $x = ta + (1-t)b$ and $y = (1-t)a + tb$ in the above inequality, we get

$$f\left(\frac{a+b}{2}\right) \leq 2\left[f(ta + (1-t)b) + f((1-t)a + tb)\right].$$

Multiplying both sides with $\frac{1}{n!} t^n (1-t)^{\alpha-n-1}$ and integrating the resulting inequality with respect to t over $[0, 1]$, we obtain

$$\begin{aligned}
f\left(\frac{a+b}{2}\right) \frac{1}{n!} \int_0^1 t^n (1-t)^{\alpha-n-1} dt &\leq \frac{2}{n!} \Bigg[\int_0^1 t^n (1-t)^{\alpha-n-1} f(ta + (1-t)b) dt \\
&\quad + \int_0^1 t^n (1-t)^{\alpha-n-1} f((1-t)a + tb) dt\Bigg] \\
\frac{B(n+1, \alpha-n)}{n!} f\left(\frac{a+b}{2}\right) &\leq 2\Bigg[\frac{1}{n!} \int_b^a \left(\frac{x-b}{a-b}\right)^n \left(\frac{a-x}{a-b}\right)^{\alpha-n-1} \frac{f(x)}{a-b} dx \\
&\quad + \frac{1}{n!} \int_a^b \left(\frac{y-a}{b-a}\right)^n \left(\frac{b-y}{b-a}\right)^{\alpha-n-1} \frac{f(y)}{b-a} dy\Bigg] \\
&= 2\frac{1}{(b-a)^\alpha} \left[I_\alpha^a f(b) +^b I_\alpha f(a)\right]
\end{aligned}$$

which completes the proof. $\square$

Remark 2.1. If we choose $\alpha = n+1$, then inequality (2.9) becomes inequality (1.5).

Theorem 2.6. *Let $f \in P(I)$, $a, b \in I$ with $a < b$ and $f \in L_1[a,b]$. Then, for $\alpha \in (n, n+1]$ $(n \in \mathbb{N}_0)$, the following inequalies for conformable fractional integrals holds:*

$$B(n+1, \alpha - n) f\left(\frac{a+b}{2}\right) \leq \frac{n!}{(b-a)^\alpha}\left[I_\alpha^a f(b) +^b I_\alpha f(a)\right] \leq 2[f(a)+f(b)] \tag{2.10}$$

where $B(.,.)$ is the Euler beta function.

Proof. Since $f \in P(I)$, we have

$$f(\lambda x + (1-\lambda)y) \leq f(x) + f(y)$$

for all $x, y \in I$.

Changing the variables with $x = ta + (1-t)b$, $y = (1-t)a + tb$ and taking $\lambda = \frac{1}{2}$ in the above inequality, we get

$$f\left(\frac{a+b}{2}\right) \leq \left[f(ta + (1-t)b) + f((1-t)a + tb)\right].$$

Again multiplying both sides with $\frac{1}{n!}t^n(1-t)^{\alpha-n-1}$ and integrating over $[0,1]$ with respect to t, we obtain

$$\begin{aligned} f\left(\frac{a+b}{2}\right)\frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}dt \quad &\leq \quad \frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}f(ta+(1-t)b)dt \\ &\quad + \frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}f((1-t)a+tb)dt \end{aligned}$$

then

$$f\left(\frac{a+b}{2}\right)\frac{B(n+1,\alpha-n)}{n!} \leq \frac{1}{(b-a)^\alpha}\left[I_\alpha^a f(b) +^b I_\alpha f(a)\right]$$

i.e

$$B(n+1,\alpha-n) f\left(\frac{a+b}{2}\right) \leq \frac{n!}{(b-a)^\alpha}\left[I_\alpha^a f(b) +^b I_\alpha f(a)\right].$$

So the first inequality is proved.

Since $f \in P(I)$, we have

$$f(ta + (1-t)b) \leq f(a) + f(b)$$

and

$$f((1-t)a + tb) \leq f(a) + f(b).$$

By adding two inequalities we get

$$f(ta + (1-t)b) + f((1-t)a + tb) \leq 2[f(a) + f(b)].$$

Then multiplying above inequalities with $\frac{1}{n!}t^n(1-t)^{\alpha-n-1}$ and integrating with respect to t over $[0,1]$

$$\begin{aligned} &\frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}f(ta+(1-t)b)dt + \frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}f((1-t)a+tb)dt \\ \leq \quad & 2\frac{f(a)+f(b)}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}dt. \end{aligned}$$

Thus

$$\frac{1}{(b-a)^\alpha}\left[I_\alpha^a f(b) +^b I_\alpha f(a)\right] \leq \frac{B(n+1,\alpha-n)}{n!}2[f(a)+f(b)]$$

which completes the proof. □

Remark 2.2. If we choose $\alpha = n+1$, then inequality (2.10) becomes inequality (1.6).

Theorem 2.7. *Let $f \in [a,b] \to (0,\infty)$ be r-convex function on $[a,b]$ with $a<b$ and $0<r\le 1$. Then, for $\alpha \in (n,n+1]$ $(n \in \mathbb{N}_0)$, the following inequality for conformable fractional integrals holds:*

$$\begin{aligned}
&\frac{n!}{(b-a)^{\alpha}}\left[I_{\alpha}^{a}f(b)+^{b}I_{\alpha}f(a)\right] \qquad (2.11)\\
\le\;& \left[\left(B(n+\frac{1}{r}+1,\alpha-n)[f(a)]\right)^{r}+\left(B(n+1,\alpha-n+\frac{1}{r})[f(b)]\right)^{r}\right]^{\frac{1}{r}}\\
&+\left[\left(B(n+1,\alpha-n+\frac{1}{r})[f(a)]\right)^{r}+\left(B(n+\frac{1}{r}+1,\alpha-n)[f(b)]\right)^{r}\right]^{\frac{1}{r}}
\end{aligned}$$

where $B(.,.)$ is the Euler beta function.

Proof. Let f is an *r*-convex function with $r>0$, i.e

$$f(ta+(1-t)b)\le\left(t[f(a)]^{r}+(1-t)[f(b)]^{r}\right)^{\frac{1}{r}}$$

and

$$f((1-t)a+tb)\le\left((1-t)[f(a)]^{r}+t[f(b)]^{r}\right)^{\frac{1}{r}}$$

for all $t\in[0,1]$.

Adding these inequalities, multiplying both sides with $\frac{1}{n!}t^{n}(1-t)^{\alpha-n-1}$ and integrating with respect to t over $[0,1]$ we get

$$\begin{aligned}
&\frac{1}{n!}\int_0^1 t^{n}(1-t)^{\alpha-n-1}f(ta+(1-t)b)dt+\frac{1}{n!}\int_0^1 t^{n}(1-t)^{\alpha-n-1}f((1-t)a+tb)dt\\
\le\;&\frac{1}{n!}\int_0^1 t^{n}(1-t)^{\alpha-n-1}\left(t[f(a)]^{r}+(1-t)[f(b)]^{r}\right)^{\frac{1}{r}}dt\\
&+\frac{1}{n!}\int_0^1 t^{n}(1-t)^{\alpha-n-1}\left((1-t)[f(a)]^{r}+t[f(b)]^{r}\right)^{\frac{1}{r}}dt.
\end{aligned}$$

After some simple calculation, we observe that

$$\begin{aligned}
&\frac{1}{n!}\int_0^1 t^{n}(1-t)^{\alpha-n-1}f(ta+(1-t)b)dt+\frac{1}{n!}\int_0^1 t^{n}(1-t)^{\alpha-n-1}f((1-t)a+tb)dt\\
=\;&\frac{1}{(b-a)^{\alpha}}\left[I_{\alpha}^{a}f(b)+^{b}I_{\alpha}f(a)\right].
\end{aligned}$$

Using Minkowski inequality, we have

$$\begin{aligned}
&M_1=\frac{1}{n!}\int_0^1 t^{n}(1-t)^{\alpha-n-1}\left(t[f(a)]^{r}+(1-t)[f(b)]^{r}\right)^{\frac{1}{r}}dt\\
\le\;&\frac{1}{n!}\left[\left(\int_0^1 t^{n+\frac{1}{r}}(1-t)^{\alpha-n-1}[f(a)]dt\right)^{r}+\left(\int_0^1 t^{n}(1-t)^{\alpha-n-1+\frac{1}{r}}[f(b)]dt\right)^{r}\right]^{\frac{1}{r}}\\
=\;&\frac{1}{n!}\left[\left(B(n+\frac{1}{r}+1,\alpha-n)[f(a)]\right)^{r}+\left(B(n+1,\alpha-n+\frac{1}{r})[f(b)]\right)^{r}\right]^{\frac{1}{r}}
\end{aligned}$$

and analogously

$$\begin{aligned} M_2 &= \frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}\left((1-t)[f(a)]^r+t[f(b)]^r\right)^{\frac{1}{r}}dt \\ &\le \frac{1}{n!}\left[\left(\int_0^1 t^n(1-t)^{\alpha-n-1+\frac{1}{r}}[f(a)]dt\right)^r+\left(\int_0^1 t^{n+\frac{1}{r}}(1-t)^{\alpha-n-1}[f(b)]dt\right)^r\right]^{\frac{1}{r}} \\ &= \frac{1}{n!}\left[\left(B(n+1,\alpha-n+\frac{1}{r})[f(a)]\right)^r+\left(B(n+\frac{1}{r}+1,\alpha-n)[f(b)]\right)^r\right]^{\frac{1}{r}}. \end{aligned}$$

Thus

$$\frac{1}{(b-a)^\alpha}\left[I^a_\alpha f(b)+^b I_\alpha f(a)\right]\le M_1+M_2$$

which completes the proof. □

Remark 2.3. If we choose $\alpha=n+1$, then inequality (2.11) becomes inequality (1.8).

Theorem 2.8. *Let $f\in SX(h,I)$, $a,b\in I$ with $a<b$ and $f\in L_1[a,b]$. Then, for $\alpha\in(n,n+1]$ $(n\in\mathbb{N}_0)$, one has inequality for h-convex functions via conformable fractional integrals*

$$\begin{aligned} &\frac{B(n+1,\alpha-n)}{h\left(\frac{1}{2}\right)}f\left(\frac{a+b}{2}\right) \\ \le\ & \frac{n!}{(b-a)^\alpha}\left[I^a_\alpha f(b)+^b I_\alpha f(a)\right] \\ \le\ & [f(a)+f(b)]\frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}[h(t)+h(1-t)]dt \end{aligned} \tag{2.12}$$

where $B(.,.)$ is Euler beta function.

Proof. Taking $x=ta+(1-t)b$, $y=(1-t)a+tb$ and $\lambda=\frac{1}{2}$ in (1.3)

$$\begin{aligned} f\left(\frac{a+b}{2}\right) &\le h\left(\frac{1}{2}\right)f(ta+(1-t)b)+h\left(\frac{1}{2}\right)f((1-t)a+tb) \\ &= h\left(\frac{1}{2}\right)\left[f(ta+(1-t)b+f((1-t)a+tb))\right]. \end{aligned}$$

Again in a same way i.e multiplying $\frac{1}{n!}t^n(1-t)^{\alpha-n-1}$ and integrating with respect to t over $[0,1]$, we get

$$\frac{B(n+1,\alpha-n)}{h\left(\frac{1}{2}\right)}f\left(\frac{a+b}{2}\right)\le\frac{n!}{(b-a)^\alpha}\left[I^a_\alpha f(b)+^b I_\alpha f(a)\right].$$

Since $f\in SX(h,I)$, we have

$$f(tx+(1-t)y)\le h(t)f(x)+h(1-t)f(y)$$

and

$$f((1-t)x+ty)\le h(1-t)f(x)+h(t)f(y).$$

By adding two inequalities we get

$$f(tx+(1-t)y)+f((1-t)x+ty)\le[h(t)+h(1-t)][f(x)+f(y)]. \tag{2.13}$$

By using (2.13) with $x=a$ and $y=b$, we have

$$f(ta+(1-t)b)+f((1-t)a+tb)\le[h(t)+h(1-t)][f(a)+f(b)].$$

Then multiplying both sides with $\frac{1}{n!}t^n(1-t)^{\alpha-n-1}$ and integrating with respect to t over $[0,1]$ we get

$$\begin{aligned} & \int_0^1 t^n(1-t)^{\alpha-n-1}f(ta+(1-t)b)dt + \frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}f((1-t)a+tb)dt \\ \leq \quad & [f(a)+f(b)]\frac{1}{n!}\int_0^1 t^n(1-t)^{\alpha-n-1}[h(t)+h(1-t)]dt \end{aligned}$$

which completes proof. □

Remark 2.4. If we choose $\alpha = n+1$, then the inequality (2.12) reduces the inequality (1.7).

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DEPARTMENT OF MATHEMATICS
ORDU UNIVERSITY
FACULTY OF SCIENCE AND ARTS, ORDU, TURKEY
Email address: erhanset@yahoo.com
Email address: bariscelik15@hotmail.com
Email address: abdurrahmangozpinar79@gmail.com