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Wijsman quasi-invariant convergence

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ABSTRACT. In this study, we defined concepts of Wijsman quasi-invariant convergence, Wijsman quasi-strongly invariant convergence and Wijsman quasi-strongly q-invariant convergence. Also, we give the concept of Wijsman quasi-invariant statistically convergence. Then, we study relationships among these concepts. Furthermore, we investigate relationship between these concepts and some convergence types given earlier for sequences of sets, too.

1. INTRODUCTION AND BACKGROUND

The concept of statistical convergence was firstly introduced by Fast [5] and this concept has been studied by Šalát [15], Fridy [6], Connor [4] and many others, too.

A sequence $x = (x_k)$ is statistically convergent to L if for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n}\Big|\big\{k \le n : |x_k - L| \ge \varepsilon\big\}\Big| = 0,$$

where the vertical bars indicate the number of elements in the enclosed set.

Several authors have studied on the concepts of invariant mean and invariant convergent sequences (see, [7, 9, 10, 14, 16, 19]).

Let σ be a mapping of the positive integers into themselves. A continuous linear functional ϕ on ℓ_∞, the space of real bounded sequences, is said to be an invariant mean or a σ-mean if it satisfies following conditions:

(1) $\phi(x) \ge 0$, when the sequence (x_n) has $x_n \ge 0$ for all n,
(2) $\phi(e) = 1$, where $e = (1, 1, 1, ...)$, and
(3) $\phi(x_{\sigma(n)}) = \phi(x_n)$ for all $x \in \ell_\infty$.

The mappings σ are assumed to be one-to-one and such that $\sigma^m(n) \ne n$ for all positive integers n and m, where $\sigma^m(n)$ denotes the m th iterate of the mapping σ at n. Thus, ϕ extends the limit functional on c, the space of convergent sequences, in the sense that $\phi(x) = \lim x$ for all $x \in c$.

In the case σ is translation mappings $\sigma(n) = n + 1$, the σ-mean is often called a Banach limit.

It can be shown that

$$V_\sigma = \left\{ x = (x_n) \in \ell_\infty : \lim_{m\to\infty} \frac{1}{m} \sum_{k=1}^{m} x_{\sigma^k(n)} = L, \text{ uniformly in } n \right\}.$$

The concept of strongly σ-convergence was introduced by Mursaleen [8].

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A sequence $x = (x_k)$ is said to be strongly σ-convergent to L if

$$\lim_{m\to\infty} \frac{1}{m} \sum_{k=1}^{m} |x_{\sigma^k(n)} - L| = 0,$$

uniformly in n.

In [17], Savaş generalized the concept of strongly σ-convergence as below:

$$[V_\sigma]_p = \left\{ x = (x_k) : \lim_{m\to\infty} \frac{1}{m} \sum_{k=1}^{m} |x_{\sigma^k(n)} - L|^p = 0, \text{ uniformly in } n \right\},$$

where $0 < p < \infty$.

The concept of σ-statistically convergent sequence was introduced by Savaş and Nuray [18] as follows:

A sequence $x = (x_k)$ is said to be σ-statistically convergent to L if for every $\varepsilon > 0$

$$\lim_{m\to\infty} \frac{1}{m} \Big| \big\{ k \le m : |x_{\sigma^k(n)} - L| \ge \varepsilon \big\} \Big| = 0,$$

uniformly in n.

Let X be any non-empty set and $\mathbb{N}$ be the set of natural numbers. The function $f : \mathbb{N} \to P(X)$ is defined by $f(k) = A_k \in P(X)$ for each $k \in \mathbb{N}$, where $P(X)$ is power set of X. The sequence $\{A_k\} = (A_1, A_2, \ldots)$, which is the range's elements of f, is said to be sequences of sets.

Let (X, ρ) be a metric space. For any point $x \in X$ and any non-empty subset A of X, the distance from x to A is defined by $d(x, A) = \inf_{a\in A} \rho(x, a)$.

Throughout the paper we take (X, ρ) as a metric space and A, A_k as any non-empty closed subsets of X.

There are different convergence notions for sequence of sets. One of them handled in this paper is the concept of Wijsman convergence (see, [1, 2, 3, 20, 21, 22]).

A sequence $\{A_k\}$ is said to be Wijsman convergent to A if for each $x \in X$,

$$\lim_{k\to\infty} d(x, A_k) = d(x, A)$$

and it is denoted by $A_k \stackrel{W}{\to} A$.

A sequence $\{A_k\}$ is said to be bounded if for each $x \in X$, there exists an $M > 0$ such that $|d(x, A_k)| < M$ for all k, i.e., if $\sup_k \big\{ d(x, A_k) \big\} < \infty$.

The set of all bounded sequences of sets is denoted by L_∞.

Nuray and Rhoades [11] defined the concept of Wijsman statistical convergence for sequences of sets.

A sequence $\{A_k\}$ is Wijsman statistically convergent to A if for each $x \in X$ and every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} \Big| \big\{ k \le n : |d(x, A_k) - d(x, A)| \ge \varepsilon \big\} \Big| = 0$$

and it is denoted by $st - \lim_W A_k = A$.

Using the invariant mean, the concepts of Wijsman invariant convergence (WV_σ), Wijsman strongly invariant convergence $[WV_\sigma]$ and Wijsman invariant statistical convergence (WS_σ) were also introduced by Pancaroğlu and Nuray [13].

A sequence $\{A_k\}$ is said to be Wijsman invariant convergent to A if for each $x \in X$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} d(x, A_{\sigma^k(m)}) = d(x, A),$$

uniformly in m and it is denoted by $A_k \stackrel{WV_\sigma}{\longrightarrow} A$.

A sequence $\{A_k\}$ is said to be Wijsman strongly invariant convergent to A if for each $x \in X$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} |d(x, A_{\sigma^k(m)}) - d(x, A)| = 0,$$

uniformly in m and it is denoted by $A_k \stackrel{[WV_\sigma]}{\longrightarrow} A$.

A sequence $\{A_k\}$ is Wijsman invariant statistically convergent to A if for each $x \in X$ and every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} \Big| \big\{ k \le n : |d(x, A_{\sigma^k(m)}) - d(x, A)| \ge \varepsilon \big\} \Big| = 0,$$

uniformly in m and it is denoted by $A_k \stackrel{(WS_\sigma)}{\longrightarrow} A$.

Nuray [12] studied concepts of quasi-invariant convergence and quasi-invariant statistical convergence for real sequences in a normed space.

2. Wijsman quasi-invariant convergence

In this section, we defined concepts of Wijsman quasi-invariant convergence, Wijsman quasi-strongly invariant convergence and Wijsman quasi-strongly q-invariant convergence. Also, we give the concept of Wijsman quasi-invariant statistically convergence. Then, we study relationships among these concepts. Furthermore, we investigate relationship between these concepts and some convergences types given earlier for sequences of sets.

Definition 2.1. A sequence $\{A_k\}$ is Wijsman quasi-invariant convergent to A if for each $x \in X$

$$\lim_{p\to\infty} \left| \frac{1}{p} \sum_{k=0}^{p-1} d_x(A_{\sigma^k(np)}) - d_x(A) \right| = 0,$$

uniformly in $n = 1, 2, \ldots$ where $d_x(A_{\sigma^k(np)}) = d(x, A_{\sigma^k(np)})$ and $d_x(A) = d(x, A)$. In this case, we write $A_k \stackrel{WQV_\sigma}{\longrightarrow} A$.

Theorem 2.1. *If a sequence $\{A_k\}$ is Wijsman invariant convergent to A, then $\{A_k\}$ is Wijsman quasi-invariant convergent to A.*

Proof. Suppose that the sequence $\{A_k\}$ is Wijsman invariant convergent to A. Then, for each $x \in X$ and every $\varepsilon > 0$ there exists an integer $p_0 > 0$ such that for all $p > p_0$

$$\left| \frac{1}{p} \sum_{k=0}^{p-1} d_x(A_{\sigma^k(m)}) - d_x(A) \right| < \varepsilon,$$

for all m. If m is taken as $m = np$, then we get

$$\left| \frac{1}{p} \sum_{k=0}^{p-1} d_x(A_{\sigma^k(np)}) - d_x(A) \right| < \varepsilon$$

for all n. Since $\varepsilon > 0$ is an arbitrary, we have

$$\lim_{p\to\infty} \left| \frac{1}{p} \sum_{k=0}^{p-1} d_x(A_{\sigma^k(np)}) - d_x(A) \right| = 0$$

uniformly in n. Therefore, the sequence $\{A_k\}$ is Wijsman quasi-invariant convergent to A. □

Definition 2.2. A sequence $\{A_k\}$ is Wijsman quasi-invariant statistically convergent to A if for each $x \in X$ and every $\varepsilon > 0$

$$\lim_{p\to\infty} \frac{1}{p}\Big|\big\{k \le p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| = 0,$$

uniformly in n. In this case, we write $A_k \overset{WQS_\sigma}{\longrightarrow} A$.

The set of all Wijsman quasi-invariant statistically convergent sequences will be denoted by (WQS_σ).

Theorem 2.2. *If a sequence $\{A_k\}$ is Wijsman invariant statistically convergent to A, then $\{A_k\}$ is Wijsman quasi-invariant statistically convergent to A.*

Proof. Suppose that the sequence $\{A_k\}$ is Wijsman invariant statistically convergent to A. In this case, when $\delta > 0$ is given, for each $x \in X$ and every $\varepsilon > 0$ there exists an integer $p_0 > 0$ such that for all $p > p_0$

$$\frac{1}{p}\Big|\big\{k \le p : |d_x(A_{\sigma^k(m)}) - d_x(A)| \ge \varepsilon\big\}\Big| < \delta,$$

for all m. If m is taken as $m = np$, then we get

$$\frac{1}{p}\Big|\big\{k \le p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| < \delta$$

for all n. Since $\delta > 0$ is an arbitrary, we have

$$\lim_{p\to\infty} \frac{1}{p}\Big|\big\{k \le p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| = 0$$

uniformly in n. Therefore, the sequence A_k is Wijsman quasi-invariant satatistically convergent to A. □

Definition 2.3. A sequence $\{A_k\}$ is Wijsman quasi-strongly invariant convergent to A if for each $x \in X$

$$\lim_{p\to\infty} \frac{1}{p}\sum_{k=0}^{p-1} |d_x(A_{\sigma^k(np)}) - d_x(A)| = 0,$$

uniformly in n. In this case, we write $A_k \overset{[WQV_\sigma]}{\longrightarrow} A$.

Definition 2.4. Let $0 < q < \infty$. A sequence $\{A_k\}$ is Wijsman quasi-strongly q-invariant convergent to A if for each $x \in X$

$$\lim_{p\to\infty} \frac{1}{p}\sum_{k=0}^{p-1} |d_x(A_{\sigma^k(np)}) - d_x(A)|^q = 0,$$

uniformly in n. In this case, we write $A_k \overset{[WQV_\sigma]^q}{\longrightarrow} A$.

The set of all Wijsman quasi-strongly q-invariant convergence sequences will be denoted by $[WQV_\sigma]^q$.

Theorem 2.3.

i) *If a sequence $\{A_k\}$ is Wijsman quasi-strongly q-invariant convergent to A, then this sequence is Wijsman quasi-invariant statistically convergent to A.*
ii) *If a sequence $\{A_k\} \in L_\infty$ and Wijsman quasi-invariant statistically convergent to A, then this sequence is Wijsman quasi-strongly q-invariant convergent to A.*
iii) $(WQS_\sigma) \cap L_\infty = [WQV_\sigma]^q$

Proof. **i)** Suppose that the sequence $\{A_k\}$ is Wijsman quasi-strongly q-invariant convergent to A. For each $x \in X$ and every $\varepsilon > 0$, following inequality is provided:

$$\sum_{k=0}^{p-1} |d_x(A_{\sigma^k(np)}) - d_x(A)|^q \geq \varepsilon^q \left|\left\{k \leq p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \geq \varepsilon\right\}\right|,$$

for all n. If the both side of the above inequality are multipled by $\frac{1}{p}$ and after that the limit is taken for $p \to \infty$, we get

$$\lim_{p\to\infty} \frac{1}{p} \sum_{k=0}^{p-1} |d_x(A_{\sigma^k(np)}) - d_x(A)|^q \geq \varepsilon^q \lim_{p\to\infty} \frac{1}{p}\left|\left\{k \leq p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \geq \varepsilon\right\}\right|. \quad (2.1)$$

Since the sequence $\{A_k\}$ is Wijsman quasi-strongly q-invariant convergent to A, the left side of inequality (2.1) is equal to 0. Hence, we have

$$\lim_{p\to\infty} \frac{1}{p}\left|\left\{k \leq p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \geq \varepsilon\right\}\right| = 0$$

uniformly in n. So, the proof is completed.

ii) Suppose that the sequence $\{A_k\} \in L_\infty$ and Wijsman quasi-invariant statistically convergent to A. Since $\{A_k\}$ is bounded, there exists an $M > 0$ such that for each $x \in X$

$$\left|d_x(A_{\sigma^k(np)}) - d_x(A)\right| \leq M.$$

Also, since $\{A_k\}$ is Wijsman quasi-invariant statistically convergent to A, for each $x \in X$ and every $\varepsilon > 0$ there exists a number $N_\varepsilon \in \mathbb{N}$ such that for all $p > N_\varepsilon$

$$\frac{1}{p}\left|\left\{k \leq p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \geq \left(\frac{\varepsilon}{2}\right)^{1/q}\right\}\right| < \frac{\varepsilon}{2M^q},$$

for all n. Now, we take the set

$$G_p = \left\{k \leq p : |d_x(A_{\sigma^k(np)}) - d_x(A)| \geq \left(\frac{\varepsilon}{2}\right)^{1/q}\right\}.$$

Thus, for each $x \in X$ we get

$$\begin{aligned}
\frac{1}{p} \sum_{k=0}^{p-1} |d_x(A_{\sigma^k(np)}) - d_x(A)|^q &= \frac{1}{p}\Bigg(\sum_{\substack{k\leq p \\ k\in G_p}} |d_x(A_{\sigma^k(np)}) - d_x(A)|^q \\
&\quad + \sum_{\substack{k\leq p \\ k\notin G_p}} |d_x(A_{\sigma^k(np)}) - d_x(A)|^q \Bigg) \\
&< \frac{1}{p}\, p\, \frac{\varepsilon}{2M^q} M^q + \frac{1}{p}\, p\, \frac{\varepsilon}{2} \\
&= \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon,
\end{aligned}$$

for all n. So, the proof is completed.

iii) If (i) and (ii) are considered together, we handle $(WQS_\sigma) \cap L_\infty = [WQV_\sigma]^q$. $\square$

Lemma 2.1. *If for each $x \in X$ and every $\varepsilon > 0$ there exists numbers p_0 and n_0 such that for all $p \geq p_0$ and $n \geq n_0$*

$$\frac{1}{p}\sum_{k=0}^{p-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| < \varepsilon,$$

then the sequence $\{A_k\}$ is Wijsman quasi-strongly invariant convergent to A.

Proof. Let $\varepsilon > 0$ be given. Because of the hypothesis, for each $x \in X$ we can choose numbers p_0' and n_0 such that

$$\frac{1}{p}\sum_{k=0}^{p-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| < \frac{\varepsilon}{2}, \tag{2.2}$$

for all $p \geq p_0'$ and $n \geq n_0$. It is enough to prove that there exists a number p_0'' such that

$$\frac{1}{p}\sum_{k=0}^{p-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| < \varepsilon,$$

for all $p \geq p_0''$ and $0 \leq n \leq n_0$. If p_0 is taken as $p_0 = \max\{p_0', p_0''\}$, then the following inequality is hold:

$$\frac{1}{p}\sum_{k=0}^{p-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| < \varepsilon,$$

for all $p \geq p_0$ and n. The number n_0 is a constant due to the its selection. Thus, we can take as

$$\sum_{k=0}^{n_0-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| = T.$$

Now, when considering the inequality (2.2) for $0 \leq n \leq n_0$ and $p \geq n_0$, we get

$$\begin{aligned}
\frac{1}{p}\sum_{k=0}^{p-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| &= \frac{1}{p}\sum_{k=0}^{n_0-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| \\
&\quad + \frac{1}{p}\sum_{k=n_0}^{p-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| \\
&= \frac{T}{p} + \frac{1}{p}\sum_{k=n_0}^{p-1}|d_x(A_{\sigma^k(np)}) - d_x(A)| \\
&\leq \frac{T}{p} + \frac{\varepsilon}{2}.
\end{aligned}$$

If p is taken sufficiently large, we can write

$$\frac{T}{p} + \frac{\varepsilon}{2} < \varepsilon.$$

So, the sequence $\{A_k\}$ is Wijsman quasi-strongly invariant convergent to A. □

Lemma 2.2. *If for each $x \in X$ and every $\varepsilon, \delta > 0$ there exists numbers p_0 and n_0 such that for all $p \geq p_0$ and $n \geq n_0$*

$$\frac{1}{p}\Big|\big\{0 \leq k \leq p-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \geq \varepsilon\big\}\Big| \leq \delta,$$

then the sequence $\{A_k\}$ is Wijsman quasi-invariant statistically convergent to A.

Proof. Let $\varepsilon, \delta > 0$ be given. Because of the hypothesis, for each $x \in X$ we can choose numbers p_0' and n_0 such that

$$\frac{1}{p}\Big|\big\{0 \le k \le p-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| < \frac{\delta}{2}, \tag{2.3}$$

for all $p \ge p_0'$ ve $n \ge n_0$. It is enough to prove that there exists a number p_0'' such that

$$\frac{1}{p}\Big|\big\{0 \le k \le p-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| < \delta,$$

for all $p \ge p_0''$ and $0 \le n \le n_0$. If p_0 is taken as $p_0 = \max\{p_0', p_0''\}$, then the following inequality is hold:

$$\frac{1}{p}\Big|\big\{0 \le k \le p-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| < \delta,$$

for all $p \ge p_0$ and n. The number n_0 is a constant due to the its selection. Thus, we can take as

$$\Big|\big\{0 \le k \le n_0-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| = H.$$

Now, when considering the inequality (2.3) for $0 \le n \le n_0$ and $p \ge n_0$, we get

$$\begin{aligned}
&\frac{1}{p}\Big|\big\{0 \le k \le p-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| \\
&\qquad \le \frac{1}{p}\Big|\big\{0 \le k \le n_0-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| \\
&\qquad\quad + \frac{1}{p}\Big|\big\{n_0 \le k \le p-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| \\
&\qquad = \frac{H}{p} + \frac{1}{p}\Big|\big\{n_0 \le k \le p-1 : |d_x(A_{\sigma^k(np)}) - d_x(A)| \ge \varepsilon\big\}\Big| \\
&\qquad \le \frac{H}{p} + \frac{\delta}{2}.
\end{aligned}$$

If p is taken sufficiently large, we can write

$$\frac{H}{p} + \frac{\delta}{2} < \delta.$$

So, the sequence $\{A_k\}$ is Wijsman quasi-invariant statistically convergent to A. □

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